Student name____________________

Class ____________

YEAR 12

ASSESSMENT TEST 1
TERM 4, 2018

MATHEMATICS EXTENSION 1

Time Allowed – 90 Minutes

(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- NESA approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- NESA Reference sheets are provided

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc.

Each question must show (in the top right hand corner) your Candidate Number.

Question 1 (9 marks)

(a) Find $\int \frac{1}{\sqrt{4-x^2}}\,dx$ **1**

(b) Find the exact value of $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{2dx}{1+4x^2}$ **2**

(c) Find $\int x\sqrt{3x+2}\,dx$ using the substitution $u = 3x + 2$. **3**

(d) Find the value of the constant term in the expansion of $\left(3x + \frac{2}{\sqrt{x}}\right)^6$. **3**

Question 2 (9 marks) [START A NEW PAGE]

(a) For the function $y = \pi + 2\tan^{-1}(3x)$

(i) State the range of the function. **1**

(ii) Find the value of y when $x = \frac{1}{3}$ **1**

(iii) Sketch the curve of the function showing all information from previous parts. **2**

(b) Find the volume when the area between the curves $y = x^3$ and $y = 4x$, in the first quadrant is revolved about $y - axis$. The two curves intersect at points (0,0) and (2,8). **3**

(c) When $(5 + 3x)^n$ is written out as a polynomial in x, the coefficients of x^5 and x^6 have the same value. Find n. **2**

Question 3 (9 marks) [START A NEW PAGE]

(a) Find the exact value of:

(i) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ **1**

(i) $\sin\left[\tan^{-1}\left(-\frac{1}{2}\right)\right]$ **2**

(b) Determine the co-ordinates of the point where the curve $y = 3e^{\sin^{-1}x}$ cuts the y axis. Find the gradient of the tangent to the curve at this point. **3**

(c) Determine the coefficient of a^3 in the expansion of $(1 - 3a)^2(1 + 2a)^5$ **3**

Question 4 (9 marks) [START A NEW PAGE]

(a) Let $f(x) = x^2 - 6x + 2$.

(i) By expressing $f(x)$ in the form $(x-h)^2 + k$, or otherwise, find the range of $f(x)$. **1**

(ii) The domain of $f(x)$ is restricted to the largest domain such that it has an inverse. The domain contains $x = 5$. Find the equation of the inverse function $f^{-1}(x)$. **2**

(iii) Find the point(s) of intersection of $y = f(x)$ and $y = f^{-1}(x)$, given that $f(x)$ maintains its restricted domain. **2**

(b) (i) T_k is the K^{th} term in the expression $\left(\frac{1}{3} + 2x\right)^{18}$. Find $\frac{T_{k+1}}{T_k}$ and simplify. **2**

(ii) Hence, find the greatest co-efficient in the expansion $\left(\frac{1}{3} + 2x\right)^{18}$. **2**

Question 5 (9 marks) [START A NEW PAGE]

(a) A region R is bounded by the curve $y = 2\sin^{-1} x$, the $x-axis$ and the line $x = 1$. Show this information on a diagram and use Simpson's rule with five function values to give an approximation for the area R.
Give your answer correct to two decimal places. **4**

(b) (i) Prove that $\frac{d}{dx}\left[log_e x - \frac{1}{2}log_e(1 + x^2) - \frac{\tan^{-1} x}{x}\right] = \frac{\tan^{-1} x}{x^2}$. **3**

(ii) Hence, find $\int_2^3 \frac{\tan^{-1} x}{x^2}\,dx$

Give your answer to two decimal places. **2**

Question 6 (9 marks) [START A NEW PAGE]

(a) (i) Write down the general solution for $sin^2\,\theta = \frac{3}{4}$ **2**

(b) Use the substitution $x = u - 2$ to evaluate $\int_0^1 \frac{2-x}{(2+x)^3}\,dx$ **4**

(c) Find the derivative of $\sin^{-1}\sqrt{1-x}$. Simplify your answer. **3**

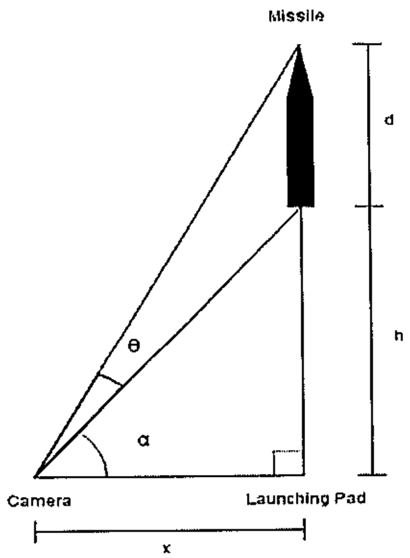
Question 7 (9 marks) [START A NEW PAGE]

(a) (i) Given that $\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$, find an expression for θ. **1**

(ii) Find the derivative $\frac{d\theta}{d\alpha}$ and solve $\frac{d\theta}{d\alpha} = 0$. Leave your answer in the exact form. **2**

(b) A camera is located x metres from the base of a missile launching pad.

Let α be the angle of elevation of the base of the missile from the camera lens.



(i) If a missile of length d metres is launched vertically upwards, show that when the base of the missile is h metres above the camera lens level, the angle θ subtended at the lens by the missile is given by:

$\theta = \tan^{-1}\left(\frac{h+d}{x}\right) - \tan^{-1}\left(\frac{h}{x}\right).$ **1**

(ii) You are given that $h = 60\ m$ and $d = 20m$. Show that θ is maximised when the camera is $40\sqrt{3}$ metres from the launching pad. **3**

(iii) Deduce that the maximum angle subtended by the missile at the camera lens is $\tan^{-1}\left(\frac{1}{4\sqrt{3}}\right)$ **2**

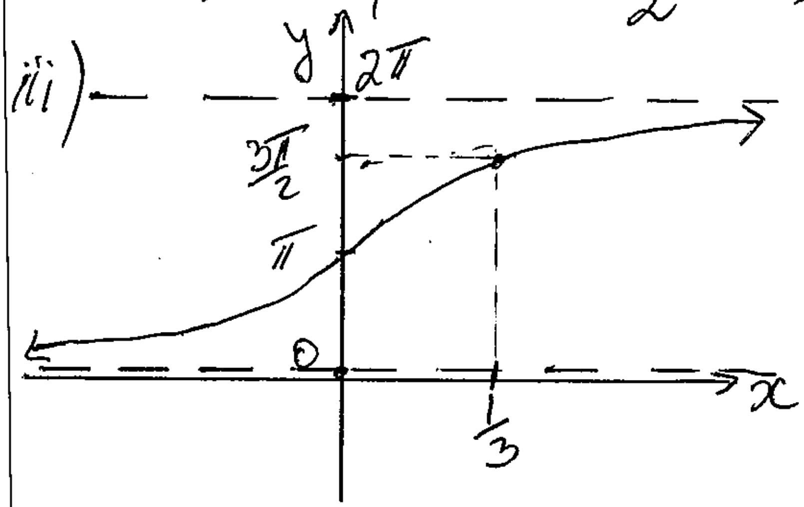
END OF EXAMINATION

MATHEMATICS: Question

Suggested Solutions	Marks	Marker's Comments
1. a) $\sin^{-1}\frac{x}{2} + C$		1
b) $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{2}{1+4x^2}\,dx$		
$= \frac{1}{4}\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{2}{\frac{1}{4}+x^2}\,dx$		
$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{\frac{1}{2}}{\frac{1}{4}+x^2}\,dx$		1 for recognising $\tan^{-1}(nx)$
$= \left[\tan^{-1}(2x)\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$		but <u>NO</u> marks if $\tan^{-1}(x^2)$!
$= \tan^{-1}\sqrt{3} - \tan^{-1}1$		
$= \frac{\pi}{3} - \frac{\pi}{4}$		
$= \frac{\pi}{12}$		
c) $I = \int x\sqrt{3x+2}\,dx$ let $u = 3x+2$, $\frac{du}{dx} = 3$, $dx = \frac{du}{3}$		
$= \frac{1}{9}\int \sqrt{u}\,(u-2)\,du$		
$= \frac{1}{9}\int u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\,du$ $x = \frac{u-2}{3}$		1
$= \frac{1}{9}\left[\frac{2u^{5/2}}{5} - \frac{4u^{\frac{3}{2}}}{3}\right] + C$		1
$= \frac{1}{9}\left[\frac{2(3x+2)^{5/2}}{5} - \frac{4(3x+2)^{3/2}}{3}\right] + C$		1

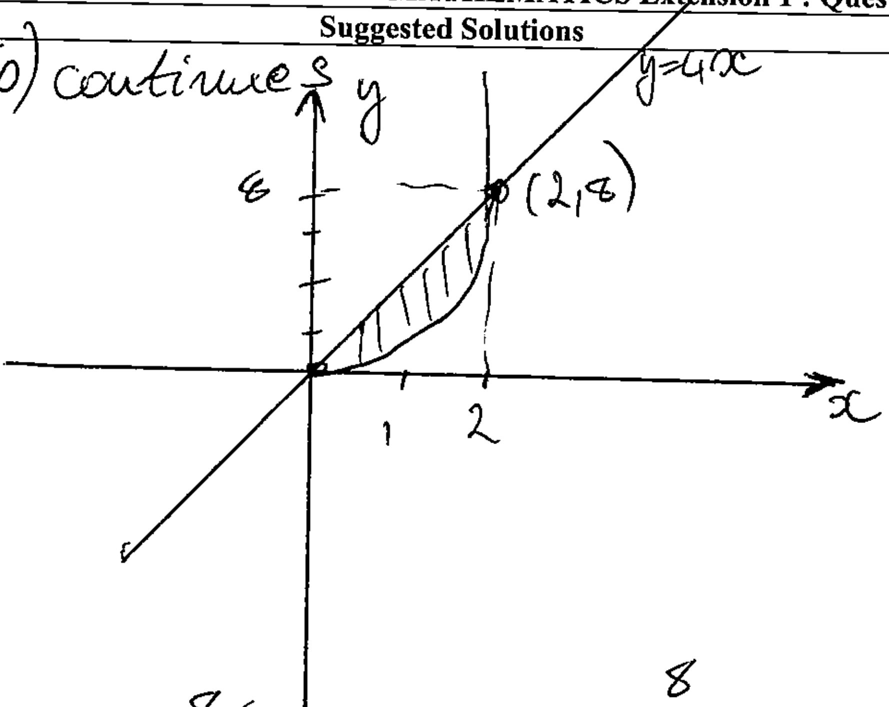
MATHEMATICS: Question		
Suggested Solutions	**Marks**	**Marker's Comments**
d) $\left(3x + \frac{2}{\sqrt{x}}\right)^6$ $= \sum_{r=0}^{6} {}^6C_r (3x)^{6-r} (2x^{-\frac{1}{2}})^r$ constant $\longrightarrow x^{6-r} x^{-\frac{r}{2}} = x^0$ $x^{6-\frac{3r}{2}} = x^0$ $\therefore 6 - \frac{3r}{2} = 0$ $\frac{3r}{2} = 6$ $3r = 12$ $r = 4$	1	
$\therefore$ constant $= {}^6C_4 \, 3^2 \, 2^4$ $= 2160$	1	
<u>Careful</u> $\left(3x + \frac{2}{\sqrt{x}}\right)^6 \neq {}^6C_r (3x)^{6-r} (2x^{\frac{1}{2}})^r$ You need a summation sign or say "general term equals"!		!!!

MATHEMATICS Extension 1 : Question..2...

Suggested Solutions	Marks	Marker's Comments
a) i) $\pi + 2\tan^{-1}(3x)$ $-\frac{\pi}{2} < \tan^{-1}3x < \frac{\pi}{2}$ $-\pi < 2\tan^{-1}3x < \pi$ $0 < \pi + 2\tan^{-1}3x < 2\pi$ $\therefore$ Range is $0 < y < 2\pi$	1	* done very well * if $\leq$ given on graph, is fixed * if made mistake here, CFE for the graph
ii) $y = \pi + 2\tan^{-1}(3 \times \frac{1}{3})$ $= \pi + 2\frac{\pi}{4} = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$	1	* another easy mark
iii) 	1 1	* for full 2 marks: * correct asympt. * correct points of intersection * correct shape
b) $y = x^3$ 1st quadrant $y = 4x$ about y-axis $y^{1/3} = x$ $V = \pi\int_a^b x^2\,dy$ $\frac{y}{4} = x$	1	

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
b) continues	*	* if done over x axis only 1 mark awarded as: you make it easier & did not read question correctly



$$V=\pi\int^{8}\left(y^{1/3}\right)^2 dx-\pi\int^{8}\left(\frac{y}{4}\right)^2 dy$$ (1)

$$=\pi\int_0^8\left(y^{2/3}-\frac{y^2}{16}\right)dy$$

$$=\pi\left[\frac{3y^{5/3}}{5}-\frac{y^3}{48}\right]_0^8$$

$$=\pi\left[\left(\frac{3}{5}8^{5/3}-\frac{8^3}{48}\right)-0\right]=$$

$$=\frac{128}{15}\pi\ u^3$$ (1)

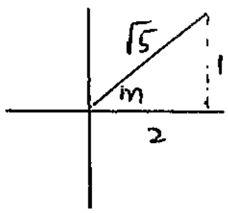
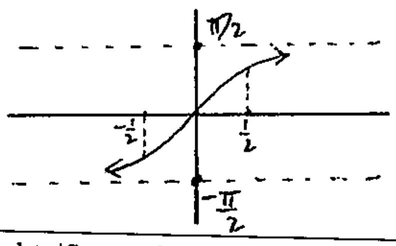
Marker's Comments: * number of you find the $\left(y^{1/3}-\frac{y}{4}\right)^2$ which is incorrect, just imagine your 3D shape

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
c) $(5+3x)^n = \sum_{r=0}^{n} 5^{n-r}(3x)^r\,{}^nC_r$		
$T_7 = \binom{n}{6} 5^{n-6}\, 3^6\, x^6$	1	
$T_6 = \binom{n}{5} 5^{n-5}\, 3^5\, x^5$		
equating		
$\binom{n}{6} 5^{n-6}\, 3^6 = \binom{n}{5} 5^{n-5}\, 3^5$		
$\frac{\cancel{n!}}{(n-6)!\,6!} \times \frac{\cancel{5^n}\, 3^{\cancel{6}}}{5^{\cancel{6}}} = \frac{\cancel{n!}}{(n-5)!\,5!}\, \frac{\cancel{5^n} \times \cancel{3^5}}{\cancel{5^5}}$		
$\frac{3}{5 \times \cancel{(n-6)}\cancel{(n-7)}\cdots 1 \; 6 \times \cancel{5 \times \cdots \times 1}} =$		* number of you have
$= \frac{1}{(n-5)\cancel{(n-6)}\cancel{(n-7)}\cdots \cancel{1 \times 5 \times 4 \times \cdots \times 1}}$		$n-5=10$ and got $n=5$!!!???
$\frac{3}{6} = \frac{5}{n-5}$		
$3n - 15 = 30$		
$3n = 45 \Rightarrow n = 15$	1	

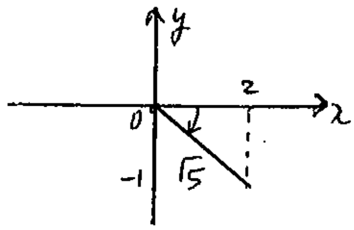
JRAHS : TASK 1 : 3U : 2018 T4

MATHEMATICS Extension 1: Question 3

Suggested Solutions	Marks Awarded	Marker's Comments
a) (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = x$ $\therefore \cos x = -\frac{1}{\sqrt{2}}$ $\Rightarrow x = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$ But $0 \le \cos^{-1} m \le \pi$ $\therefore \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$ only	1	For correct, in radians
(ii) <u>Method I</u>: $\sin\left(\tan^{-1}\left(-\frac{1}{2}\right)\right)$ $= \sin\left(-\tan^{-1}\frac{1}{2}\right)$ as $\tan^{-1}(-x) = -\tan^{-1}x$ $= -\sin\left(\tan^{-1}\frac{1}{2}\right)$ as $\sin^{-1}(-x) = -\sin^{-1}x$ Let $m = \tan^{-1}\frac{1}{2}$ $\quad \boxed{-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}}$ $\Rightarrow \tan m = \frac{1}{2}$	(1)	$\longrightarrow$ restriction must be stated, <u>OR</u> $0 < \tan^{-1}\frac{1}{2} < \frac{\pi}{2}$
 $\Rightarrow \sin m = \frac{1}{\sqrt{5}}$		
$\therefore \sin\left(\tan^{-1}\left(-\frac{1}{2}\right)\right) = -\frac{1}{\sqrt{5}}$	(1)	$\longrightarrow$ sign and correct exact value
		

JRAHS TASK 1 : 3u : TERM 4 : 2018

MATHEMATICS Extension 1: Question 3

Suggested Solutions	Marks Awarded	Marker's Comments
Method II:		
Let $\tan^{-1}\left(-\frac{1}{2}\right) = x$		
$\therefore \tan x = -\frac{1}{2}$		
But $-\frac{\pi}{2} < \tan^{-1}\left(-\frac{1}{2}\right) < 0$	1	For correct restriction
		
$\therefore \sin x = -\frac{1}{\sqrt{5}}$		
i.e $\sin\left(\tan^{-1}\left(-\frac{1}{2}\right)\right) = -\frac{1}{\sqrt{5}}$	1	For correct exact value
b) $y = 3e^{\sin^{-1}x}$		
when $x = 0$, $y = 3e^{\sin^{-1}0}$		
$= 3e^0$		
$= 3$		
$\therefore$ curve cuts the y-axis at $(0, 3)$	1	Coordinate of y-intercept
$y' = \frac{3}{\sqrt{1-x^2}} e^{\sin^{-1}x}$	1	For derivative
when $x = 0$, $y' = \frac{3}{\sqrt{1-0}} \cdot e^0 = 3$		
$\therefore$ gradient $= 3$	1	For gradient

JRAHS TASK 1 : 3U : TERM 4 2018

MATHEMATICS Extension 1: Question 3

Suggested Solutions	Marks Awarded	Marker's Comments
c) Method I: $(1-3a)^2(1+2a)^5$ $= (1-6a+9a^2)(1+5(2a)^1+10(2a)^2+10(2a)^3+\cdots)$ ... using Pascal's Δ	1	For applying Pascal's Δ to expand
Terms in a^3 are: $1\times 80a^3$, $-6\times 10\times 4a^3$ $9\times 10a^3$.	1	For correct Terms
$\therefore$ Coefficient of a^3 is $80-240+90=-70$	1	For correct coefficient
Method II: $(1-3a)^2(1+2a)^5 = (1-6a+9a^2)\sum_{k=0}^{5} {}^5C_k (2a)^k$ $= \sum_{k=0}^{5} {}^5C_k\, 2^k (a^k - 6a^{k+1} + 9a^{k+2})$ —— 1		
For $k=1$, Coefficient of a^3 is ${}^5C_1 \cdot 2^1 \cdot 9 = 90$ $k=2$, " " " " ${}^5C_2 \cdot 2^2 \cdot (-6) = -240$ —— 1 $k=3$, " " " " ${}^5C_3 \cdot 2^3 \cdot 1 = 80$		
$\therefore$ Coefficient of a^3 in $(1-3a)^2(1+2a)^5$ is -70 —— 1		

JRAHS Task 1 3u : Task 4 2018

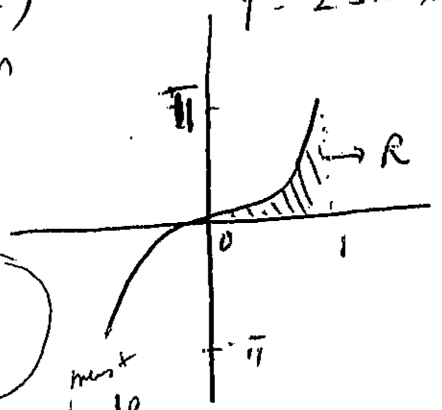
MATHEMATICS Extension 1: Question 3		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
Method III : $(1-3a)^2(1+2a)^5$		
$= \sum_{r=0}^{2} {}^2C_r(-3a)^r \sum_{k=0}^{5} {}^5C_k(2a)^k$	1	For applying Binomial theorem
$= \sum_{r=0}^{2}\sum_{k=0}^{5} {}^2C_r\ {}^5C_k(-3)^r a^r . 2^k . a^k$		
$= \sum_{r=0}^{2}\sum_{k=0}^{5} {}^2C_r\ {}^5C_k(-3)^r(2)^k a^{r+k}$		
To obtain coefficient of a^3, set $r+k=3$		
$r=0,\ k=3 \Rightarrow$ Coefficient is ${}^2C_0\ {}^5C_3(-3)^0(2)^3 = 80$		
$r=1,\ k=2 \Rightarrow$ " " ${}^2C_1\ {}^5C_2(-3)^1(2)^2 = -240$		1
$r=2,\ k=1 \Rightarrow$ " " ${}^2C_2\ {}^5C_1(-3)^2(2)^1 = 90$		
$\therefore$ Net coefficient is -70	1	For correct Coefficient

MATHEMATICS Extension 1: Question 4

Suggested Solutions	Marks	Marker's Comments
(a) (i) $f(x) = (x-3)^2 - 7$ implies $f(x) \geq -7$ for all real x. So, the range is $\{y \in \mathbf{R} \mid y \geq -7\}$.	1	No significant problems.
(ii) By inspection of the quadratic, f is increasing for all $x \in \mathbf{R}$ such that $x \geq 3$ and this is the domain containing 5.	1	Again, handled well. Candidates reasoned appropriately to restrict domain and determine appropriate branch of $\pm\sqrt{}$ to use.
Inverse: let $y = f^{-1}(x)$. Then $x = f(y)$ $= (y-3)^2 - 7$ $y = 3 \pm \sqrt{x+7}$ Now, range of f^{-1} is the domain of f $\text{rng}f^{-1} = [3, \infty) \subset \mathbf{R}$ which means we take the positive solution. So, $f^{-1}(x) = 3 + \sqrt{x+7}$	1	
(iii) The graphs of f and f^{-1} intersect on $y = x$. So, solve either $f(x) = x$ or $f^{-1}(x) = x$. Here, $x^2 - 6x + 2 = x$ $x^2 - 7x + 2 = 0$ So, $x = \dfrac{7 \pm \sqrt{41}}{2}$ And since $\text{rng}f^{-1} = \text{dom}f = \{x \in \mathbf{R} \mid x \geq 3\}$, must take	1	Two major issues: (1) candidates not realising that intersection of f and f^{-1} occurred on the line $y = x$, and so attempted to solve $f = f^{-1}$ simultaneously, leading to a quartic that was intractable given the time; (2)

<table>
<tr>
<td>$$x=\frac{7+\sqrt{41}}{2}$$
Since $y=x$ at intersection, the point is
$$\left(\frac{7+\sqrt{41}}{2},\frac{7+\sqrt{41}}{2}\right)$$</td>
<td>1</td>
<td>candidates realising that intersection occurred on $y=x$, solving correctly for x, the substituting the value back into the function to derive y...giving either a mess or an approximation of 6.7.</td>
</tr>
<tr>
<td>(b) Let T_k be the k^{th} term in the expansion of
$$\left(\frac{1}{3}+2x\right)^{18}$$
Note that $k=1,2,3,\ldots$
(i)
$$T_k=\binom{18}{k-1}\left(\frac{1}{3}\right)^{18-(k-1)}(2x)^{k-1}$$
So,
$$\frac{T_{k+1}}{T_k}=\frac{\binom{18}{k}\left(\frac{1}{3}\right)^{18-k}(2x)^k}{\binom{18}{k-1}\left(\frac{1}{3}\right)^{18-(k-1)}(2x)^{k-1}}$$
$$=\left[\frac{18!}{k!\,(18-k)!}3^{k-18}2^kx^k\right]$$
$$\div\left[\frac{18!}{(k-1)!\,(18-k+1)!}3^{k-19}2^{k-1}x^{k-1}\right]$$</td>
<td>1</td>
<td>Only significant problem with this situation was not realising that terms begin their count at $k=1$, so the k^{th} term would correspond to the coefficient aligned with $\binom{18}{k-1}$.</td>
</tr>
<tr>
<td>$$=\frac{6(19-k)}{k}x$$</td>
<td>1</td>
<td>Candidates weren't penalised for this if their subsequent reasoning was correct.</td>
</tr>
<tr>
<td>(ii) To find the greatest coefficient, we want the greatest integer k such that
$$\frac{6(19-k)}{k}\geq 1$$
$$114-6k\geq k$$
$$7k\leq 114$$
$$k\leq 16\frac{2}{7}$$</td>
<td>1</td>
<td>Important to realise that the 'term' includes variables (some students left out x).</td>
</tr>
<tr>
<td>So, $k=16$ here.
The greatest coefficient is therefore
$$\binom{18}{16}\left(\frac{1}{3}\right)^{18-16}2^{16}=1\,114\,112$$</td>
<td>1</td>
<td>Great majority of candidates were able to derive the correct coefficient.</td>
</tr>
</table>

a) 4m



1

must shade the region

X	0	0.25	0.5	0.75	1
Y	0		$\frac{\pi}{3}$	.	π

1.0472 3.1416

$x = 0.5 \quad y = 2\frac{\pi}{6} = \frac{\pi}{3} = 1.0472$

0.5054

$x = 0.25 \quad y = 2\sin^{-1}\frac{1}{4} =$

$x = 0.75 \quad y = 2\sin^{-1}\frac{3}{4} = 1.6961$

If they did not mark π on y-axis no penalty.

X	0	0.25	0.5	0.75	1
Y	0	0.5054	1.0472	1.6961	3.1416

1m

$\frac{1}{12}\left[1\times f(0) + 4f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 4f(\tfrac{3}{4}) + 1f(1)\right]$ 1m

$4\left[2\sin^{-1}\frac{1}{4} + 2\sin^{-1}\frac{3}{4}\right] + 2\times\frac{\pi}{3} + \pi$

$A = \frac{0.5}{6}\left[4\left[0.5054 + 1.6961\right] + 2\times 1.0472 + 3.1416\right]$

$= \frac{1}{12}\left[8.806 + 2.0944 + 3.1416\right]$

use Radian

$= \frac{14.042}{12}$

$= \underline{1.170}\ u^2$ 1

$= \underline{1.17}\ u^2$ (2dp)

x	y	ⓧ	ⓧy	
0	0	1	0	
$\frac{1}{4}$	$2\sin^{-1}(\frac{1}{4})$	4	$8\sin^{-1}(\frac{1}{4})$	4×0.5054
$\frac{1}{2}$	$\frac{\pi}{3}$	2	$\frac{2\pi}{3}$	2×1.0472
$3/4$	$2\sin^{-1}(\frac{3}{4})$	4	$8\sin^{-1}(\frac{3}{4})$	4×1.6961
		1	π	3.1416

b i) $y = \ln x - \frac{1}{2}\ln(1+x^2) - \frac{\tan^{-1}x}{x}$

3m

$-\left(\frac{\frac{x}{1+x^2} - \tan^{-1}x}{x^2}\right)$

$y' = \frac{1}{x} - \frac{1}{2}\cdot\frac{2x}{1+x^2} - \left(\frac{\tan^{-1}x}{x^2} + \left(-\frac{1}{x}\cdot\frac{1}{1+x^2}\right)\right)$ 1 1

must get to line to 2m

$y' = \frac{1}{x} - \frac{x}{1+x^2} + \frac{\tan^{-1}x}{x^2} + \frac{-1}{x(1+x^2)}$

$y' = \frac{1+\cancel{x^2}-\cancel{x^2}}{x\cancel{(1+x^2)}} + \frac{\tan^{-1}x}{x^2} + \frac{-1}{x\cancel{(1+x^2)}}$

Be careful of the signs in $-\frac{\tan^{-1}x}{x}$

$y' = \frac{\tan^{-1}x}{x^2}$ 1

ii) $\int_2^3 \frac{\tan^{-1}x}{x^2}\,dx = \left[\ln x - \frac{1}{2}\ln(1+x^2) - \frac{\tan^{-1}x}{x}\right]_2^3$ 1

2m

$= \left(\ln 3 - \frac{1}{2}\ln 10 - \frac{\tan^{-1}3}{3}\right) - \left(\ln 2 - \frac{1}{2}\ln 5 - \frac{\tan^{-1}2}{2}\right)$

$= \left(\ln 3 - \ln\sqrt{10} - \frac{\tan^{-1}3}{3}\right) - \left(\ln 2 - \ln\sqrt{5} - \frac{\tan^{-1}2}{2}\right)$

$= \ln\frac{3}{2} - \ln\sqrt{10} + \ln\sqrt{5} + \frac{\tan^{-1}2}{2} - \frac{\tan^{-1}3}{3}$

$= \ln\left(\frac{3}{2}\frac{\sqrt{5}}{\sqrt{10}}\right) + \frac{\tan^{-1}2}{2} - \frac{\tan^{-1}3}{3}$

$= \ln\left(\frac{3}{2\sqrt{2}}\right) + \frac{1}{2}\tan^{-1}2 - \frac{\tan^{-1}3}{3}$

0.5536

$= 0.05889 + ~~3~~ - 0.4163$

$= 0.19619 \doteqdot ~~0.19~~\ 0.20$ (2dp) 1

MATHEMATICS Extension 1 : Question 6

Suggested Solutions	Marks	Marker's Comments
a) $\sin^2\theta = \frac{3}{4}$		$1 \Rightarrow 1$ solution
$\sin\theta = \pm\frac{\sqrt{3}}{2}$		$2 \Rightarrow 2$ solution plus $n \in \mathbb{Z}$
$\therefore \theta = n\pi + (-1)^n\left(\pm\frac{\pi}{3}\right), \quad n \in \mathbb{Z}$		
$= n\pi \pm \frac{\pi}{3}$		
b) $\int_0^1 \frac{2-x}{(2+x)^3}\,dx$		
let $x = u-2$		
$\frac{dx}{du} = 1$		
$\therefore dx = du$		
when $x=1, \ u=3$		
$x=0, \ u=2$		
$\therefore \int_0^1 \frac{2-x}{(2+x)^3}\,dx = \int_2^3 \frac{2-(u-2)}{(2+u-2)^3}\,du$	1	$\Rightarrow$ upper and lower limits
$= \int_2^3 \frac{4-u}{u^3}\,du$	1	correct integral in terms of u
$= \int_2^3 \left(\frac{4}{u^3} - \frac{1}{u^2}\right)du$		
$= \left[-2u^{-2} + u^{-1}\right]_2^3$	1	
$= \left(-\frac{2}{9} + \frac{1}{3}\right) - \left(-\frac{1}{2} + \frac{1}{2}\right)$		
$= \frac{1}{9}$	1	
c) $\frac{d}{dx}\left(\sin^{-1}\sqrt{1-x}\right) = \frac{-\frac{1}{2}(1-x)^{-\frac{1}{2}}}{\sqrt{1-(\sqrt{1-x})^2}}$	1 1	
$= \frac{-1}{2\sqrt{1-1+x}\,\sqrt{1-x}}$		
$= \frac{-1}{2\sqrt{x}\,\sqrt{1-x}}$	1	

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
a) (i) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right)$	1	
(ii) $\frac{d\theta}{d\alpha} = \frac{-\left(\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{2\sqrt{2}}\sin\alpha\right)}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right)^2}}$	1	derivative
$= 0$		
$\therefore \frac{1}{\sqrt{2}}\cos\alpha = \frac{1}{2\sqrt{2}}\sin\alpha$		
$\tan\alpha = 2$		
$\alpha = \tan^{-1}2$	1	value of α
b) (i) $\tan\alpha = \frac{h}{x}$, $\tan(\alpha+\theta) = \frac{h+d}{x}$	1	Students needed to show expressions for $\tan\alpha$ and $\tan(\alpha+\theta)$ for the mark.
$\alpha = \tan^{-1}\frac{h}{x}$, $\alpha+\theta = \tan^{-1}\frac{h+d}{x}$		
$\therefore \theta = \tan^{-1}\frac{h+d}{x} - \tan^{-1}\frac{h}{x}$		
(ii) $\theta = \tan^{-1}\frac{80}{x} - \tan^{-1}\frac{60}{x}$		
$\frac{d\theta}{dx} = \frac{-80/x^2}{1+\left(\frac{80}{x}\right)^2} - \frac{\frac{-60}{x^2}}{1+\left(\frac{60}{x}\right)^2}$		
$= \frac{-80}{x^2+80^2} + \frac{60}{x^2+60^2}$	1	derivative
For maximum $\frac{d\theta}{dx} = 0$		
$\frac{60}{x^2+60^2} = \frac{80}{x^2+80^2}$		

Suggested Solutions	Marks	Marker's Comments

$$3x^2 + 3\times 80^2 = 4x^2 + 4\times 60^2$$

$$x^2 = 4800$$

$$x = \sqrt{4800}, \quad x>0$$

$$= 40\sqrt{3} \qquad 1$$

x	60	$40\sqrt{3}$	80
$\frac{d\theta}{dx}$	0.0004	0	-0.0025
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$\therefore$ Since the function is continuous and there is only one stationary point for $x>0$ then $x = 40\sqrt{3}$ gives the maximum value of θ.

(iii) When $x = 40\sqrt{3}$

$$\theta = \tan^{-1}\frac{80}{x} - \tan^{-1}\frac{60}{x}$$

$$= \tan^{-1}\frac{2}{\sqrt{3}} - \tan^{-1}\frac{3}{2\sqrt{3}}$$

$$\tan\theta = \tan\left(\tan^{-1}\frac{2}{\sqrt{3}} - \tan^{-1}\frac{3}{2\sqrt{3}}\right)$$

$$= \frac{\tan\left(\tan^{-1}\frac{2}{\sqrt{3}}\right) - \tan\left(\tan^{-1}\frac{3}{2\sqrt{3}}\right)}{1 + \tan\left(\tan^{-1}\frac{2}{\sqrt{3}}\right)\tan\left(\tan^{-1}\frac{3}{2\sqrt{3}}\right)} \qquad 1$$

$$= \frac{\frac{2}{\sqrt{3}} - \frac{3}{2\sqrt{3}}}{1 + \frac{2}{\sqrt{3}}\times\frac{3}{2\sqrt{3}}}$$

MATHEMATICS Extension 1 : Question...7....		
Suggested Solutions	**Marks**	**Marker's Comments**
$= \frac{4-3}{2\sqrt{3}} \div (1+1)$ $= \frac{1}{2\sqrt{3}} \div 2$ $= \frac{1}{4\sqrt{3}}$ $\therefore \theta = \tan^{-1} \frac{1}{4\sqrt{3}}$	1	