Name:	
Class:	

Year 12

Assessment Test 3
Term 2 2018

Mathematics Extension 1

Time Allowed: 90 minutes + 5 minutes reading time

General Instructions:

- All questions may be attempted
- All questions are of equal value
- Department of Education approved calculators are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- This is an open book test. You may take one A4 sheet of paper into the examination with any required notes or formulas written on the sheet.

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

MARKS:

QUESTION 1. (9 marks)

(a) A ball is projected from a point 6m above the ground at an angle of 30^0 from the horizontal with velocity of v meters per second. The equations for acceleration in the horizontal and vertical direction are given by $\ddot{x} = 0$ and $\ddot{y} = -g$ respectively.

(i) Using Calculus show that $x = \dfrac{vt\sqrt{3}}{2}$ and $y = \dfrac{-gt^2}{2} + \dfrac{vt}{2} + 6$ **3**

(ii) Hence, find the Cartesian equation of the path of the ball. **2**

(iii) Assuming that $g = 9.8$ m/s^2, will the ball clear a 50 m tall building which is 355 m away if the ball is projected with a velocity of 65 m/s? Justify your answer. **2**

(b) A particle is moving in a straight line so that its displacement x from the origin at time t, in seconds, is given by $x = \sqrt{3}\cos 2t - \sin 2t, \quad t \geq 0$.

(i) Show that the particle moves in simple harmonic motion. **2**

QUESTION 2. (9 marks) **COMMENCE A NEW PAGE**

(a) Newton's law of cooling for an object in a medium of constant temperature states $T - Ts = (T_0 - T_s)e^{-kt}$, where T is the temperature (in ^{o}C) of the object at time t (in minutes), T_s is the temperature of the surrounding medium and T_0 is the initial temperature of the object.
An egg at 96^{o}C is placed to cool in a sink of water at 15^{o}C. After 5 minutes the egg's temperature is 40^{o}C. (Assume that the temperature of the water does not change.)

(i) Find the value of k. **2**

(ii) Find the temperature of the egg when $t = 10$. **1**

(iii) How long does it take for the egg to reach a temperature of 30^{o}C? **2**

(b) A meeting room contains a round table surrounded by ten chairs. These chairs are indistinguishable and equally spaced around the table.

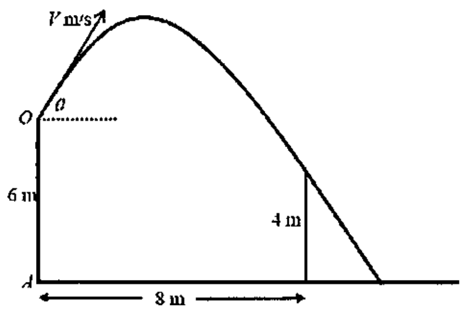
(i) A committee of ten people includes three teenagers. How many seating arrangements are there in which all three sit together? Give brief reasons for your answer. **2**

(ii) Elections are held for the positions of Chairperson and Secretary in a second committee of ten people seated around this table. What is the probability that the two people elected are sitting directly opposite each other? Give brief reasons for your answer. **2**

MARKS:

QUESTION 3. (9 marks) **COMMENCE A NEW PAGE**

(a) A projectile is fired from a point O, which is 6 metres above horizontal ground, with initial velocity V m/s, at an angle of θ to the horizontal.



There is a thin vertical post which is 4 metres high and 8 metres horizontally away from a point A, directly below O, as shown in the diagram. The equations of motion are given by:
$x = Vt\cos\theta$ and $y = Vt\sin\theta - 4.9t^2$
(Do NOT prove this.)

(i) If 2 seconds after projection, the projectile passes just above the top of the post, show that $\tan\theta = 2.2$ **2**

(ii) Show that the projectile hits the ground approximately 0.3 seconds after passing over the post. **3**

(iii) Find the angle that the projectile makes with the ground when it hits the ground, correct to the nearest degree. **2**

(b) A committee of 3 is to be elected from a club of 8 members.

(i) How many different committees can be formed? **1**

(ii) If there are 4 Queenslanders in the club, what is the probability that a randomly selected committee of 3 contains only Queenslanders? **1**

QUESTION 4. (9 marks) **COMMENCE A NEW PAGE**

(a) Lyndal hits the target on average 2 out of every 3 shots in archery competitions. During a competition she has 10 shots at the target.

(i) What is the probability that Lyndal hits the target exactly 9 times? Leave your answer in unsimplified form. **1**

(ii) What is the probability that Lyndal hits the target fewer than 9 times? Leave your answer in unsimplified form. **2**

(b) It is known that 52% of the population participates in sports on a regular basis. Five random individuals are interviewed and asked whether they participate in sports on a regular basis. Let X be the number of people who regularly participate in sports.

(i) Find the probability that 3 people or less play sport. **2**

(ii) Find the probability that at least one person plays sport, given that no more than 3 people play sport. **2**

(iii) Find the probability that the first person interviewed plays sport but the next 2 do not. **2**

MARKS:

QUESTION 5. (9 marks) **COMMENCE A NEW PAGE**

A particle travels in a line such that its velocity, v m/s, at time t seconds is given by

$$v = 2\cos(\frac{1}{2}t - \frac{\pi}{4}), \quad 0 \leq t \leq 4\pi.$$

The initial position of the particle is $-2\sqrt{2}$ m, relative to O.

(a) Find:

(i) the first time when the particle is instantaneously at rest, **1**

(ii) the period of the motion, then **1**

(iii) using all these information sketch the graph of velocity against time. **1**

(b) Find the time when the particle passes through the origin for the second time. **2**

(c) Find the relation between the particle's velocity and position. **2**

(d) Find the relation between the particle's acceleration and position. **1**

(e) Use the information obtained in a)–d) to describe the motion of the particle. **1**

QUESTION 6. (9 marks) **COMMENCE A NEW PAGE**

(a) A rumour is spreading in a neighbourhood that contains 800 people. The rumour spreads at a rate proportional to the product of the number who have heard the rumour and the number who have not yet heard the rumour. At first only 2 people in the area have heard the rumour, but after 4 days, 15% of the residents of the neighbourhood have heard it.

(i) $N = N(t) = \dfrac{800}{1 + 399e^{-kt}}$ is the number of people who have heard the rumour after t days.
Show that $\dfrac{dN(t)}{dt} = \dfrac{k}{800}N(800 - N)$. Then find k. **3**

(ii) Determine the number of residents in the neighbourhood who have heard the rumour after 5 more days. **1**

(iii) After how many days have 50% of the residents in the area heard the rumour? **2**

(b) A particle moves along a straight line and its displacement, x centimetres, from a fixed point O at a given time t seconds is given by $x = 2 + \cos^2 t$.

(i) Show that its acceleration is given by $\ddot{x} = 10 - 4x$. **2**

(ii) Find the amplitude and period of the motion. **1**

MARKS:

QUESTION 7. (9 marks) **COMMENCE A NEW PAGE**

(a) A particle is projected from the origin with velocity v m s^{-1} at an angle α to the horizontal. The position of the particle at time t seconds is given by the parametric equations

$$x = vt\cos\alpha$$
$$y = vt\sin\alpha - \frac{1}{2}gt^2,$$

where g m s^{-2} is the acceleration due to gravity. (You are NOT required to derive these).

(i) Show that the maximum height reached, h metres is given by

$$h = \frac{v^2\sin^2\alpha}{2g}.$$

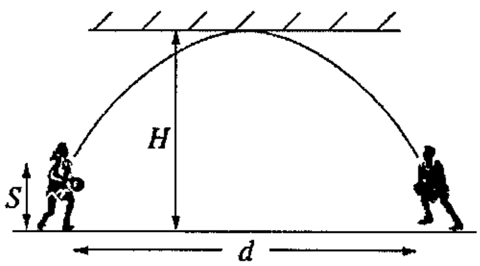
3

(ii) Show that it returns to the initial height at

$$x = \frac{v^2}{g}\sin 2\alpha.$$

2

(iii) Chris and Sandy are tossing a ball to each other in a long hallway. The ceiling height is H metres and the ball is thrown and caught at shoulder height, which is S metres for both Chris and Sandy. **4**



The ball is thrown with a velocity v m s^{-1}. Show that the maximum separation, d metres, that Chris and Sandy can have and still catch the ball is given by

$$d = 4\times\sqrt{(H-S)\left(\frac{v^2}{2g}\right) - (H-S)^2}, \qquad \text{if} \quad v^2 \geq 4g(H-S), \quad \text{and}$$
$$d = \frac{v^2}{g}, \qquad \text{if} \quad v^2 \leq 4g(H-S).$$

END OF THE TEST

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
$= -\frac{2gx^2}{3v^2} + \frac{x}{\sqrt{3}} + 6$	1	for correctly substituting & simplifying.
iii. $v = 65$, $x = 355$ $g = 9.8$		
$y = -\frac{2\times 9.8 \times 355^2}{3\times 65^2} + \frac{355}{\sqrt{3}} + 6$		
$= 16.08$ (2dp)	1	1 for correct value for y
Since $16.08 < 50$ the ball will not clear the building.	1	for correct conclusion & justification.
b) $x = \sqrt{3}\cos 2t - \sin 2t$ $t > 0$		
$\dot{x} = -2\sqrt{3}\sin 2t - 2\cos 2t$	1	for correctly getting $\dot{x}$
$\ddot{x} = -4\sqrt{3}\cos 2t + 4\sin 2t$		
$= -4(\sqrt{3}\cos 2t - \sin 2t)$		
$= -4x$		
$= -2^2 x$		
∴ The particle is moving in simple harmonic motion since		
$\ddot{x} = -n^2 x$ where $n = 2$	1	for getting $\ddot{x}$ correctly & stating it is in SHM since it is in the form $-n^2 x$

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
a) i. $\ddot{x} = 0$ $\dot{x} = C_1$ When $t = 0$, $\dot{x} = V\cos\alpha$ $= V\cos 30°$ $\therefore \dot{x} = V\cos 30$ $= \frac{\sqrt{3}}{2}V$ $x = \frac{\sqrt{3}}{2}Vt + C_2$ when $t=0$, $x=0$ $0 = \frac{\sqrt{3}}{2}V(0) + C_2$ $\therefore C_2 = 0$ $\therefore x = \frac{\sqrt{3}}{2}Vt$	1	students must show the substitution for the constants to gain full marks.
$\ddot{y} = -g$ $\dot{y} = -gt + C_3$ when $t=0$, $\dot{y} = V\sin\alpha$ $= V\sin 30$ $= \frac{V}{2}$ $\therefore \frac{V}{2} = -g(0) + C_3$ $\therefore C_3 = \frac{V}{2}$ $\therefore \dot{y} = -gt + \frac{V}{2}$ $y = \frac{-gt^2}{2} + \frac{Vt}{2} + C_4$	1	
when $t=0$, $y=0$ $\therefore 6 = 0 + 0 + C_4$ $C_4 = 6$ $\therefore y = -\frac{gt^2}{2} + \frac{Vt}{2} + 6$	1	
ii. For the Cartesian equation: $x = \frac{\sqrt{3}}{2}Vt \Rightarrow t = \frac{2x}{\sqrt{3}V}$	1	1 for rearranging to make t the subject.
$y = -\frac{g}{2}\left(\frac{2x}{\sqrt{3}V}\right)^2 + \frac{V}{2}\left(\frac{2x}{\sqrt{3}V}\right) + 6$ $= -\frac{g}{2}\left(\frac{4x^2}{3V^2}\right) + \frac{x}{\sqrt{3}} + 6$		

1/3

MATHEMATICS Extension 1 : Question...2...

<table>
<tr><th>Suggested Solutions</th><th>Marks Awarded</th><th>Marker's Comments</th></tr>
<tr><td>a i) $T - T_S = (T_0 - T_S)e^{-kt}$
$T_0 - 96^\circ$
$T_S - 15^\circ$
when $t = 0$, $T_0 = 96$, $T = 96$
$t = 5$ $T = 40$
$\therefore 40 - 15 = (96 - 15)e^{-5k}$
$^* \frac{25}{81} = e^{-5k}$</td><td>1</td><td>Error $40 - 15 = {}^* 35$</td></tr>
<tr><td>$-5k = \ln \frac{25}{81}$
$k = -\frac{1}{5} \ln \frac{25}{81}$
or $\frac{1}{5} \ln \frac{81}{25}$</td><td>1</td><td></td></tr>
<tr><td>ii) $t = 10$
$T = T_S + (T_0 - T_S)e^{-kt}$
$= 15 + (96 - 15)e^{-kt}$
$= 15 + 81 e^{{}^* -(-\frac{1}{5} \ln \frac{25}{81}) 10}$
$T = 22.72$ (2 d.p</td><td>1</td><td>forgot the -ve sign</td></tr>
</table>

MATHEMATICS Extension 1 : Question 2 — 2/3

Suggested Solutions	Marks Awarded	Marker's Comments

iii) $t = ?$ $T = 30$

$$30 = 15 + 81e^{\frac{1}{5}\ln\left(\frac{25}{81}\right)t}$$

$$* \frac{15}{81} = e^{\frac{1}{5}\ln\left(\frac{25}{81}\right)t}$$

— (1)

error $30 - 15 = 2$

$$\frac{5}{27} = e^{\frac{1}{5}\ln\left(\frac{25}{81}\right)t}$$

$$\ln\frac{5}{27} = \frac{1}{5}\ln\left(\frac{25}{81}\right)t$$

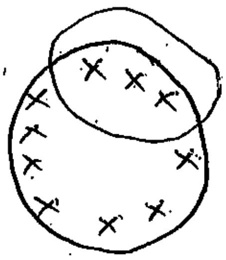
$$t = \frac{\ln\frac{5}{27}}{\frac{1}{5}\ln\left(\frac{25}{81}\right)}$$

$$= 7.17 \text{ min} \quad (2 \text{ d.p})$$

(1)

only if correct from above

b) i)



for the group of teenagers $= 3!$

for the others $= 7!$

(1)

$\therefore$ number of seating arrangements

$$= 3!\,7!$$

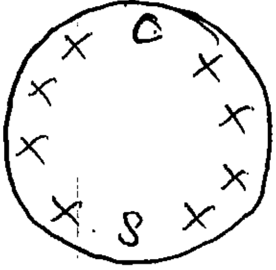
$$= 30240$$

(1)

MATHEMATICS Extension 1 : Question 2 3/3

Suggested Solutions	Marks Awarded	Marker's Comments

b ii) method 1



Fix C & S

$n(2\text{ opp}) =$ fix C & S arrange 2 groups of 4

$$= \frac{{}^8P_4 \times {}^4P_4}{2!} = 20160$$

since there are 2! ways of C & S

$$(n\ 2\text{opp})\ \text{total} = \frac{{}^8P_4 \times {}^4P_4}{2!} \times 2! = 40320.$$

$$\therefore P(2\text{ opp}) = \frac{n(2\text{ opp})}{(10-1)!} = \frac{40320}{9!} = \frac{1}{9}$$

method 2

suppose C is elected, S has to be elected

$\therefore$ total $= {}^{10}C_2 \times 2! = 90$

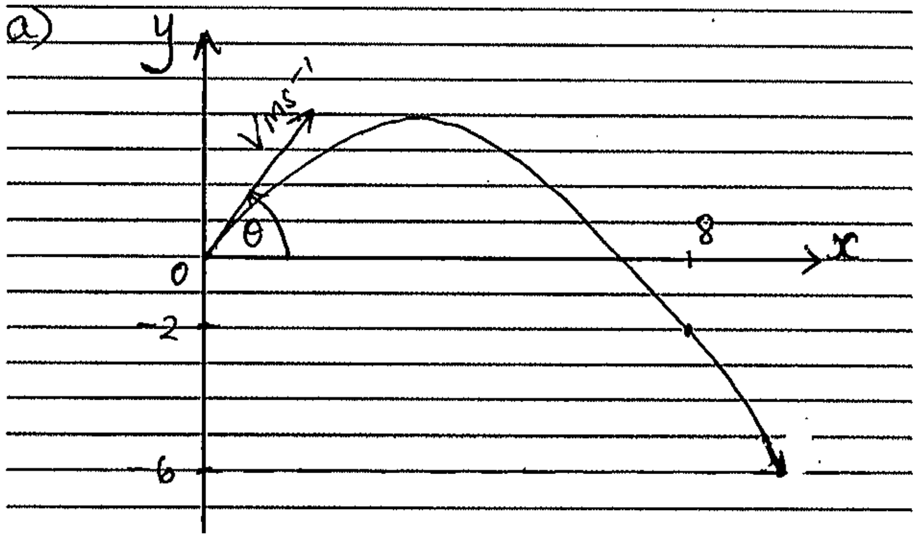
total no of opp pairs $= 5 \times 2 = 10$.

$$P(2\text{opp}) = \frac{10}{90} = \frac{1}{9}$$

1 mark for reason and of numerator value

1 mark for reason and of denominator value

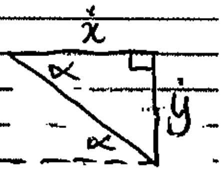
MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
a) 		A number of students changed the given equation of y to $y = Vt\sin\theta - 4.9t^2 + 6$ when the diagram clearly shows projection from $y=0$.
$x = Vt\cos\theta$, $y = Vt\sin\theta - 4.9t^2$		
i) When $t=2$, $y=-2$, $x=8$		
$2V\cos\theta = 8$ $\quad$ $2V\sin\theta - 4.9\times 4 = -2$	1	Correct substitution into given equations
$V\cos\theta = 4$ $\quad$ $2V\sin\theta = 17.6$		
$V\sin\theta = 8.8$		
$\dfrac{V\sin\theta}{V\cos\theta} = \dfrac{8.8}{4}$	1	for showing $\tan\theta = 2.2$.
$\therefore \tan\theta = 2.2$		
ii) Method 1		Poorly done - many students eliminated t from their equation when trying to find a time.
$V\cos\theta = 4$ from (i)		
$\therefore V = \dfrac{4}{\cos\theta}$		
Hits the ground when $y=-6$		
$-6 = \dfrac{4t\sin\theta}{\cos\theta} - 4.9t^2$	1	eliminating V
$-6 = 4t\tan\theta - 4.9t^2$		

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
$0 = 8.8t - 4.9t^2 + 6$ as $\tan\theta = 2.2$ from (i)	1	eliminating θ to give equation in t.
$4.9t^2 - 8.8t - 6 = 0$		
$t = \dfrac{8.8 \pm \sqrt{8.8^2 - 4\times4.9\times-6}}{2\times4.9}$	1	Solve for t and give explanation for "0.3s after".
$t = 2.323027776$, since $t>0$ which is approximately 0.3s after passing over the post at $t=2s$.		Students must arrive at $4.9t^2 - 8.8t - 6 = 0$ or an equivalent quadratic equation without fudging to get a mark for finding t.
<u>Method 2</u>		
If projectile hits ground approx 0.3s after passing over post then when $t \doteqdot 2.3s$, $y \doteqdot -6$.		
When $t = 2.3$		
$y = V(2.3)\sin\theta - 4.9(2.3)^2$	1	substitution of $t=2.3$
$= \dfrac{4}{\cos\theta} \times \sin\theta \times 2.3 - 4.9(2.3)^2$ from (ii)		
$= 4\times2.3\times\tan\theta - 4.9(2.3)^2$	1	eliminating V
$= 4\times2.3\times2.2 - 4.9(2.3)^2$ from (i)		
$= -5.681$		
$\doteqdot -6$		

MATHEMATICS Extension 1: Question 3

Suggested Solutions	Marks	Marker's Comments
$\therefore$ Projectile hits ground approx 0.3s after passing over post	1	showing $y \doteqdot -6$ and conclusion
(iii) $\dot{x} = V\cos\theta$ $= 4$ from (i)		many students evaluated x and y rather than $\dot{x}$ and $\dot{y}$
When $t = 2.3$ $\dot{y} = V\sin\theta - 9.8t$ $= 8.8 - 9.8 \times 2.3$ from (i) $= -13.74$	1	evaluating $\dot{x}$ and $\dot{y}$
 $\tan\alpha = \frac{-13.74}{4}$ $\alpha = 73.7686$	1	finding α
$\therefore$ ie. an angle of $74°$ when it strikes the ground.		many students did not use values of $V\cos\theta$ and $V\sin\theta$ found in (i)
b) i) ${}^8C_3 = 56$ committees	1	Well done!
ii) P(only Queenslanders) $= \frac{{}^4C_3}{{}^8C_3}$ $= \frac{1}{14}$	1	Most students gained both marks.

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
a) i) $P(\text{hit}) = \frac{2}{3}$ $\quad P(\text{miss}) = \frac{1}{3}$		
$P(9 \text{ hits}) = {}^{10}C_9 \left(\frac{1}{3}\right)\left(\frac{2}{9}\right)^9$	1	
ii) $P(\text{fewer than } 9 \text{ times})$		
$= 1 - P(9 \text{ hits}) - P(10 \text{ hits})$	1	using compliment
$= 1 - {}^{10}C_9\left(\frac{1}{3}\right)\left(\frac{2}{9}\right)^9 - {}^{10}C_{10}\left(\frac{2}{9}\right)^{10}$	1	P(10 hits) and P(9 hits)
b) $P(\text{participates in sport}) = 0.52$		
i) $P(3 \text{ or less play sport})$		
$= 1 - {}^5C_4(0.48)(0.52)^4 - {}^5C_5(0.52)^5$	1	
$= 0.7865008128\ldots$		
$= 0.7865$ (4dp)	1	
ii) $P = \dfrac{{}^5C_1(0.48)^4(0.52) + {}^5C_2(0.48)^3(0.52)^2 + {}^5C_3(0.48)^2(0.52)^3}{1 - {}^5C_4(0.48)(0.52)^4 - {}^5C_5(0.52)^5}$		
$= \dfrac{0.761020416\ldots}{0.7865008128\ldots}$	1	for numerator
$= 0.9676028348\ldots$		
$= 0.9676$ (4dp)	1	
OR $P = 1 - \dfrac{{}^5C_0(0.48)^5}{1 - {}^5C_4(0.48)(0.52)^4 - {}^5C_5(0.52)^5}$		1 − P(none play sport) 0 marks awarded.
iii) $P = 0.52 \times 0.48 \times 0.48 \times 1 \times 1$	1	
$= 0.119808$	1	

MATHEMATICS Extension 1 : Question 5

<table>
<tr><th>Suggested Solution</th><th>Marks</th><th>Marker's Comments</th></tr>
<tr><td>

(a) Given $v = 2\cos\left(\frac{1}{2}t - \frac{\pi}{2}\right)$ m/s and $x(0) = -2\sqrt{2}$ m:

(i) Particle at rest when $v = 0$. For $t \geq 0$, we have

$$2\cos\left(\frac{1}{2}t - \frac{\pi}{4}\right) = 0$$

implies

$$\frac{t}{2} - \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

Hence the first time for which the particle is instantaneously at rest is

$$t = \frac{\pi}{4} + \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

seconds.

</td><td>1</td><td>

Handled well.

The unit, *seconds*, was missing in a lot of responses.

</td></tr>
<tr><td>

(ii) Technically, 'period of motion' refers to the time taken for a particle to complete one cycle of its motion, assuming the motion is cyclical. We will justify this by stating that the anti-derivative of velocity gives displacement, and the anti-derivative of a purely sinusoidal function is another sinusoidal function. As such, the displacement will be periodic, and since the argument of the displacement function's sinusoidal component will be that of the sinusoidal involved in velocity, we may find the period of motion using the argument of the velocity sinusoid. So,

$$T = \frac{2\pi}{n} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

seconds.

</td><td>1</td><td>

Candidates were awarded the mark without the reasoning provided here.

Again, units of time were omitted in a lot of responses. The mark was still awarded, but it may not have been the case if additional marks were available to award.

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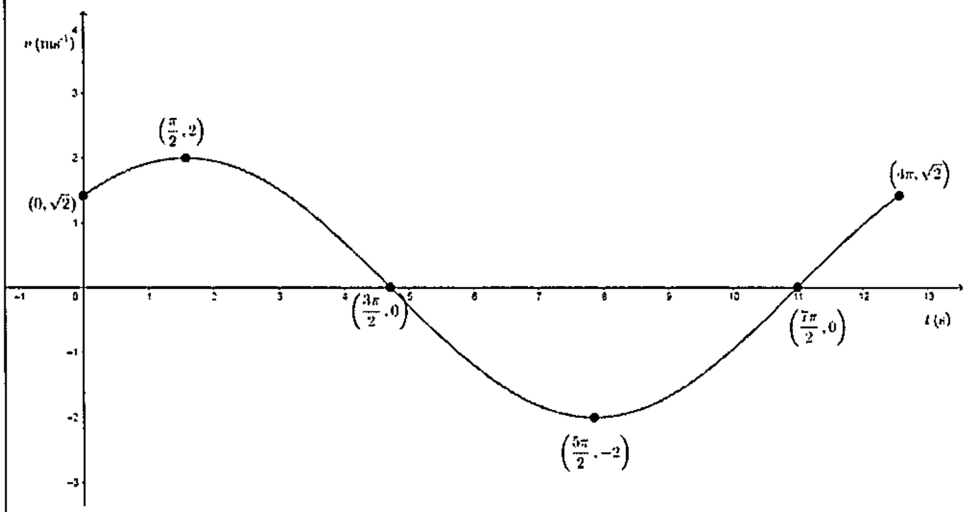
</table>

(iii) 	1	Since only a single mark available, mark awarded for appropriate shape of graph, and indication of maximum and minimum. **But,** all such graphs should indicate, **explicitly**: endpoints, intercepts on both axes (wherever possible), turning points, and appropriate labelling of axes with units.
(b) To find the time when the particle passes through the origin for the *second* time, find the displacement function: $\frac{dx}{dt} = 2\cos\left(\frac{t}{2} - \frac{\pi}{4}\right)$ $\therefore x = 4\sin\left(\frac{t}{2} - \frac{\pi}{4}\right) + C$ Now, $x(0) = -2\sqrt{2}$ m, so $-2\sqrt{2} = 4\sin\left(-\frac{\pi}{4}\right) + C$ $-2\sqrt{2} = 4 \cdot \left(-\frac{1}{\sqrt{2}}\right) + C$ $\therefore C = 0$ So, the displacement function is $x = 4\sin\left(\frac{t}{2} - \frac{\pi}{4}\right)$ **OR** $\int_{-2\sqrt{2}}^{x} dx = \int_0^t 2\cos\left(\frac{t}{2} - \frac{\pi}{4}\right)\, dt$	1	Problems encountered: **integration,** such as $2\sin\left(\frac{t}{2} - \frac{\pi}{4}\right)$ (failing to take into account division by ½ from argument of function), **no constant of integration** (in which case, obtained correct displacement function by accident), **solving for $v = 0$** instead of $x = 0$,

$$x\big\|_{-2\sqrt{2}}^{x} = 4\sin\left(\frac{t}{2}-\frac{\pi}{4}\right)\Big\|_{0}^{t}$$ $$x-\left(-2\sqrt{2}\right) = 4\sin\left(\frac{t}{2}-\frac{\pi}{4}\right) - 4\sin\left(-\frac{\pi}{4}\right)$$ $$x = 4\sin\left(\frac{t}{2}-\frac{\pi}{4}\right)$$ The times $t \geq 0$ for which the particle is at the origin are those times when $x = 0$: $$4\sin\left(\frac{t}{2}-\frac{\pi}{4}\right) = 0$$ $$\therefore \frac{t}{2}-\frac{\pi}{4} = n\pi \qquad \text{for integral } n$$ $$t = 2n\pi + \frac{\pi}{2}$$ $$\geq 0$$ which is the case here for $n = 0, 1, 2, \ldots$. We want the *second* time, so, for $n = 1$, we have $$t = 2\pi + \frac{\pi}{2}$$ $$= \boldsymbol{\frac{5\pi}{2}}$$ seconds.	1	**finding the first time** instead of the second.
(c) For the relation between the particle's velocity and position, note: $$x = 4\sin\left(\frac{t}{2}-\frac{\pi}{4}\right) \to \frac{x}{4} = \sin\left(\frac{t}{2}-\frac{\pi}{4}\right) \quad \ldots(1)$$ $$v = 2\cos\left(\frac{t}{2}-\frac{\pi}{4}\right) \to \frac{v}{2} = \cos\left(\frac{t}{2}-\frac{\pi}{4}\right) \quad \ldots(2)$$ Squaring (1) and (2), and adding, $$\left(\frac{x}{4}\right)^2 + \left(\frac{v}{2}\right)^2 = \sin^2\left(\frac{t}{2}-\frac{\pi}{4}\right) + \cos^2\left(\frac{t}{2}-\frac{\pi}{4}\right) = 1$$ Hence $$\frac{x^2}{16} + \frac{v^2}{4} = 1$$ (an elliptical relationship).	1	Many students gave a verbal description of the relationship. By *relation*, we mean *mathematical relation*/an expression relating variables using mathematical operators. Many other errors encountered, most

		stemming from not realising that x and v involve sine and cosine functions respectively, and hence failing to arrange expressions so that the sum of the squares of those trig. functions could be exploited.
(d) We take the derivative of the velocity function to find the acceleration, $\ddot{x}$, $$\ddot{x} = \frac{dv}{dt} = -\sin\left(\frac{t}{2} - \frac{\pi}{4}\right) = -\frac{x}{4}$$ given $x = 4\sin\left(\frac{t}{2} - \frac{\pi}{4}\right)$; that is, $$\ddot{x} = -\left(\frac{1}{2}\right)^2 x$$ is the required relationship.	1	Handled well by the majority.
Note, technically, there are units involved (s^{-2}), so the full, correct expression would be $$\ddot{x} = -\left(\frac{1}{2}\right)^2 x \quad s^{-2}$$		Units were not necessary, but it should be understood that if you were to replace x with a quantity in metres, then the units on the right-hand side should give units of ms^{-2}.

(e) From the result in part (d), we see that **the acceleration is always proportional to the displacement and oppositely directed to it; that is, the equation of motion is that of *simple harmonic motion*.** The displacement function shows that the particle oscillates with period 4π seconds, about a centre of motion $x = 0$, reaching upper and lower bounds of ± 4 m respectively. The velocity function indicates maximal *speed* to be 2 ms^{-1} and the particle experiences a maximal magnitude in acceleration of 1 ms^{-2}. The particle is initially at $x = -2\sqrt{2}$ m, and given $v(0) > 0$, moves first in the positive direction of displacement, coming to rest momentarily at times $t = \frac{3\pi}{2} + 2n\pi$, for $n \geq 0$ an integer.	**1**	Since one mark available only, it was enough to state *simple harmonic motion* with some other result, such as bounds of oscillation. Candidates who described the motion *accurately*, but without indicating simple harmonic, were also awarded the mark.

MATHEMATICS Extension 1 : Question ...6...		
Suggested Solutions	**Marks**	**Marker's Comments**
a) $N = 800(1+399e^{-kt})^{-1}$		
$\frac{dN}{dt} = -800(1+399e^{-kt})^{-2} \times -k \times 399e^{-kt}$	1	students using quotient rule forget $\frac{d}{dt}(800) = 0$
$= \frac{399 \times 800ke^{-kt}}{(1+399e^{-kt})^2}$		
$= \frac{800}{1+399e^{-kt}} \times \frac{399ke^{-kt}}{1+399e^{-kt}}$		
$= kN\left(\frac{1+399e^{-kt}-1}{1+399e^{-kt}}\right)$		other methods but it had to be clear how each step was obtained.
$= kN\left(1 - \frac{1}{1+399e^{-kt}}\right)$		
Now $1+399e^{-kt} = \frac{800}{N}$		
$= kN\left(1 - \frac{N}{800}\right)$		
$= \frac{kN}{800}(800 - N)$	1	

MATHEMATICS: Question 6 ×1

Suggested Solutions	Marks	Marker's Comments
$120 = \dfrac{800}{1+399e^{-k4}}$		
$1+399e^{-4k} = \dfrac{800}{120}$		
$399e^{-4k} = \dfrac{20}{3} - 1$		
$e^{-4k} = \dfrac{\frac{17}{3}}{399}$		
$= \dfrac{17}{1197}$		
$-4k = \ln\left(\dfrac{17}{1197}\right)$	1	
$k = -\dfrac{1}{4} \ln \dfrac{17}{1197}$		
(ii) $N = \dfrac{800}{1+399^{\left(\frac{1}{4}\ln\frac{17}{1197}\right)\times 9}}$		Many students did not read question which said 5 <u>more</u> days
$= 778.77$ people	1	
or 778 people.		

MATHEMATICS X1: Question 6		
Suggested Solutions	Marks Awarded	Marker's Comments
(iii) $400 = \dfrac{800}{1+399e^{\frac{1}{4}\ln\frac{17}{1197}t}}$	1	It is best to use exact value rather than rounded value.
$1+399e^{t\frac{1}{4}\ln\frac{17}{1197}}=2$		
$\therefore t = \dfrac{4\ln\left(\frac{1}{399}\right)}{\ln\frac{17}{1197}}$		
$=5.63$ days.	1	
(h) $\dot{x} = -2\cos t\sin t$		
$\ddot{x} = -2\cos^2 t + 2\sin^2 t$	1	
Now $x = 2+\cos^2 t \quad \therefore \cos^2 t = x-2$		
$\therefore \ddot{x} = -2\cos^2 t + 2(1-\cos^2 t)$		Students missed critical steps out.
$= -2(x-2)+2(1-(x-2))$		
$= \underline{-4x+10}$	1	

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks Awarded	Marker's Comments
Alternative method,		
$x = 2 + \frac{1}{2} + \frac{1}{2}\cos 2t.$		This made this part b easier.
$\dot{x} = -\sin 2t$		
$\ddot{x} = -2\cos 2t$	1	
$\cos 2t = 2x - 5$		
$\ddot{x} = -2(2x-5)$	1	
$= -4x + 10$		
$= -4(x - 10/4)$		
(ii) Period $= \frac{2\pi}{n}$		
$= \frac{2\pi}{2}$		Had to get both correct to get the mark.
$= \pi.$		
amplitude from $x = \frac{5}{2} + \frac{1}{2}\cos 2t$		• Most people got the period but too many students did not think about the fact the centre was not origin
is $\frac{1}{2}$	1	• Students who changed x to $\frac{5}{2} + \frac{1}{2}\cos 2t$ were more successful.

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
a)		
i) $\dot{y} = v\sin\alpha - gt$	1	
$\dot{y} = 0 \to v\sin\alpha - gt = 0$		
$gt = v\sin\alpha$		
$t = \dfrac{v\sin\alpha}{g}$	1	
Sub $t = \dfrac{v\sin\alpha}{g}$ into y		
$\therefore y = v\sin\alpha\left(\dfrac{v\sin\alpha}{g}\right) - \dfrac{1}{2}g\left(\dfrac{v\sin\alpha}{g}\right)^2$		
$= \dfrac{v^2\sin^2\alpha}{g} - \dfrac{v^2\sin^2\alpha}{2g}$		
$= \dfrac{v^2\sin^2\alpha}{2g}$	1	
ii) $y = 0 \to vt\sin\alpha - \frac{1}{2}gt^2 = 0$		
$t(v\sin\alpha - \frac{1}{2}gt) = 0$		
$\therefore t = 0$ or $v\sin\alpha - \frac{1}{2}gt = 0$	1	
$\therefore \frac{1}{2}gt = v\sin\alpha$		
$t = \dfrac{2v\sin\alpha}{g}$		

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments

Sub $t = \dfrac{2v\sin\alpha}{g}$ into x,

$$\therefore x = v\left(\frac{2v\sin\alpha}{g}\right)\cos\alpha$$

$$= \frac{v^2\,2\sin\alpha\cos\alpha}{g}$$

$$x = \frac{v^2\sin 2\alpha}{g}$$ (1)

iii) Let the shoulder height be initial y.

If $h \leq H - S$, max range can be achieved without restrictions. i.e $\alpha = 45°$

$$\therefore \frac{v^2\sin^2 45}{2g} \leq H - S$$

$$\frac{v^2\,\frac{1}{2}}{2g} \leq H - S$$

$$\frac{v^2}{4g} \leq H - S$$

$$v^2 \leq 4g(H - S)$$ (1)

and $x = \dfrac{v^2\sin(2\times 45)}{g}$

$x = \dfrac{v^2}{g}$ when $v^2 \leq 4g(H-S)$ (1)

MATHEMATICS Extension 1 : Question........		
Suggested Solutions	**Marks**	**Marker's Comments**

If $h \geq H-S$,

$$\frac{v^2\sin^2\alpha}{2g} \geq H-S,$$

max $\frac{v^2\sin^2 45}{2g} \geq H-S$

$$\frac{v^2\frac{1}{2}}{2g} \geq H-S$$

$$\frac{v^2}{4g} \geq H-S$$

$$v^2 \geq 4g(H-S)$$

However, $\alpha < 45°$ as it will be restricted.

when $h = H-S$,

$$\frac{v^2\sin^2\alpha}{2g} = H-S$$

$$\sin^2\alpha = \frac{2g(H-S)}{v^2} \quad (1)$$

$$1-\cos^2\alpha = 1-\frac{2g(H-S)}{v^2}$$

$$\cos^2\alpha = \frac{v^2-2g(H-S)}{v^2}$$

$\therefore x = \frac{v^2\sin 2\alpha}{g}$

$= \frac{v^2}{g}2\sin\alpha\cos\alpha$

$= \frac{2v^2}{g}\cdot\frac{\sqrt{2g(H-S)}}{v}\cdot\frac{\sqrt{v^2-2g(H-S)}}{v}$

MATHEMATICS Extension 2: Question……..		
Suggested Solutions	**Marks**	**Marker's Comments**
$= \frac{2}{g}\sqrt{2v^2g(H-S) - 4g^2(H-S)^2}$		
$= \frac{2}{g}\sqrt{\frac{4v^2g^2}{2g}(H-S) - 4g^2(H-S)^2}$		
$= 4\sqrt{(H-S)\left(\frac{v^2}{2g}\right) - (H-S)^2}$	1	
when $v^2 \geq 4g(H-S)$		