Student Name ______________ **Student Class** _________

2020

HSC Open Book Test Term 1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Board approved calculators may be used.
- Write using black or blue pen.
- All necessary working should be shown in all questions.
- Write your student number at the top of every page of your answer sheets.
- You may bring in 1 page A4 paper with anything written or typed on it.

Total marks – 61

Attempt all questions

Start each question on a new sheet of paper

A Formula Sheet is provided

This paper MUST NOT be removed from the examination room

Section A Multiple Choice (5 Marks)

QUESTION 1

We can express $\sin x$ and $\cos x$ in terms of $\tan \frac{x}{2}$, for all values of x except…….

(A) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$........

(B) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$........

(C) $x = \pi, 3\pi, 5\pi$........

(D) $x = 2\pi, 6\pi, 8\pi$........

QUESTION 2

What is the value of k if $\int_0^k \frac{dx}{9+x^2} = \frac{\pi}{9}$

(A) $\sqrt{3}$

(B) 3

(C) $\frac{1}{\sqrt{3}}$

(D) $3\sqrt{3}$

QUESTION 3

What is the coefficient of x^3 in the expansion $(1-3x)^7$

(A) -945

(B) -35

(C) 35

(D) 945

QUESTION 4

An advertisement claims that '8 out of 10 prefer Winky Chocolate Bars'. If the advertisement's claim is accurate and a sample of six people are interviewed, what is the probability that at least five people prefer Winky Chocolate Bars?

(A) $1-(0.8)^4$

(B) $2(0.8)^5$

(C) $5(0.2)^5$

(D) $(0.8)^5(0.2) + (0.8)^6$

QUESTION 5

What is the derivative of $y = \cos^{-1}\left(\frac{1}{x}\right)$ with respect to x?

(A) $\dfrac{-1}{\sqrt{x^2-1}}$

(B) $\dfrac{-1}{x\sqrt{x^2-1}}$

(C) $\dfrac{1}{\sqrt{x^2-1}}$

(D) $\dfrac{1}{x\sqrt{x^2-1}}$

Section B Answer on the lined paper provided **Marks**

QUESTION 6 (8 Marks) **Start a new page**

(a) Evaluate $\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ leaving your answer in exact form. 3

(b) Show that $\int_0^{\frac{\pi}{16}} \cos^2 4x\,dx$ is $\frac{1}{16}+\frac{\pi}{32}$ 2

(c) Solve the equation $sin2x = tanx$ for $0 \le x \le \pi$ 3

QUESTION 7 (8 Marks) **Start a new page**

(a) Extensive research has shown that 1 person out of every 4 is allergic to a particular grass seed. A group of university students volunteer to try out a new treatment

(i) If 20 students are tested, what is the expected number of allergic people in the group? 1

(ii) Find how large a sample would need to be for the probability of it containing at least one allergic person, to be greater than 99.9%? 3

(iii) What assumptions have you made in your answer? 1

(b) Evaluate $\int_1^{49} \frac{1}{4(x+\sqrt{x})}$ dx using the substitution $u^2 = x, \quad u > 0$ 3

Give the answer in exact form.

QUESTION 8 (8 Marks) **Start a new page**

(a) Use the fact that for a restricted domain, $y = \sin^{-1} x$ is equivalent to 2

$x = \sin y$ to show that $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$

(b) A spherical weather balloon is being inflated from empty using a container of helium, so that its volume is increasing at a constant rate of 0.5 m^3/s. (Assume the balloon maintains a spherical shape throughout inflation.)

(i) Show that the radius at a time t is given by $r = \sqrt[3]{\dfrac{3t}{8\pi}}$ 2

(ii) Show that the rate of increase of its surface area after 8 seconds is $\sqrt[3]{\dfrac{\pi}{3}}$ m^2/s. 3

(iii) If the maximum safe surface area before there is a risk that the balloon will burst is 200 m^2, what is the maximum time that the inflation should be allowed to proceed? 1

QUESTION 9 (8 Marks) **Start a new page**

(a) $X_1, X_2, \ldots\ldots\ldots, X_n$ are independent $B(1,p)$ random variables.
Let $Y = X_1 + X_2 + \ldots\ldots + X_n$ and $\hat{p} = \frac{Y}{n}$

(i) Given $var(Y) = np(1-p)$ show $var(\hat{p}) = \frac{p(1-p)}{n}$ 2

(ii) Show $0 \le var(\hat{p}) \le \frac{1}{4n}$ 2

(b) Let X be a Bernoulli random variable with parameter p, where $0 < p < 1$

(i) Explain why p cannot take the values $p = 0$ or $p = 1$. 1

(ii) How many distinct values can X take in n Bernoulli trials? 1

(iii) A biased coin is tossed where $P(\text{Head}) = \frac{1}{5}$. If you obtain Heads you win a prize of \$1.
Could this situation be modelled as a Bernoulli random variable? Explain 2

QUESTION 10 (8 Marks) **Start a new page**

(a) Use the substitution $t = tan\frac{\theta}{2}$ to prove the identity

$$\frac{1}{2}(3cos\theta + 4sin\theta + 5) = (sin\frac{\theta}{2} + 2cos\frac{\theta}{2})^2$$ **3**

(b) Show that $\binom{n+2}{3} - \binom{n}{3} = n^2$ for all integers n, where $n \geq 3$. **2**

(c) Suppose that the true, unknown proportion p of voters who will vote for a third-party candidate in the next election is 9%. What is the probability that a poll of 820 voters will find a sample proportion $\hat{p}$ that differs from the true proportion by more than 3%? You may use the normal distribution on the Reference sheet to approximate your answer to the nearest 0.1% **3**

QUESTION 11 (8 Marks) **Start a new page**

(a) The blades of a wind turbine are turning at a steady rate. The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula

$$h = 36sin(1.5t) - 15cos(1.5t) + 65$$

(i) Express $36sin(1.5t) - 15cos(1.5t)$ in the form $ksin(1.5t - a)$, where $k > 0$ and $0 < a < 2\pi$, **3**

(ii) Hence find the two values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn. **3**

(b) For the expansion $(1 - 2x - 4x^2)(1 - \frac{3}{x})^9$, find the coefficient of the term in x^{-5} **2**

QUESTION 12 (8 Marks) **Start a new page**

In the expansion of $(0.15 + 0.85)^{50}$, the terms involving $(0.15)^r$ and $(0.15)^{r+1}$ are denoted by T_r and T_{r+1}

(i) Show that $\frac{T_r}{T_{r+1}} = \frac{17(r+1)}{3(50-r)}$ **4**

The probability that Paul's train to work is late on any day is 0.15, independently of other days. The number of days on which Paul's train is late during a 50-day period is modelled by the random variable X

(ii) Find the values of r for which $P(X = r) \leq P(X = r + 1)$ **3**

(iii) Hence find the most likely number of days on which the train will be late during the 50-day period. **1**

END OF EXAMINATION

MC Answers
1C 2D 3A 4D 5D

Student Number ______

Question 6

(a) $\int_0^1 \frac{1}{\sqrt{4-2x^2}} = \frac{1}{\sqrt{2}}\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx$ ✓

$\sqrt{4-2x^2} = \sqrt{2(2-x^2)}$
$= \sqrt{2}\left(\sqrt{2-x^2}\right.$

$= \frac{1}{\sqrt{2}}\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1$

$= \frac{1}{\sqrt{2}}\left(\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(\frac{0}{\sqrt{2}}\right)\right]\right)$ ✓

$= \frac{1}{\sqrt{2}}\left[\frac{\pi}{4} - 0\right]$

$= \frac{\pi}{4\sqrt{2}}$ ✓

$= \frac{\sqrt{2}\,\pi}{8}$

(b) $\int_0^{\frac{\pi}{16}} \cos^2 4x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{16}} \cos 8x + 1\,dx$ ✓

$= \frac{1}{2}\left[\frac{\sin 8x}{8} + x\right]_0^{\frac{\pi}{16}}$ ✓

$= \frac{1}{2}\left[\frac{\sin\frac{\pi}{2}}{8} + \frac{\pi}{16} - \left[\left(\frac{\sin 0}{8}\right) + 0\right]\right.$

$= \frac{1}{2}\left(\frac{1}{8} + \frac{\pi}{16}\right)$

$= \frac{1}{16} + \frac{\pi}{32}$

(c) $\sin 2x = \tan x \quad 0 \le x \le \pi$

$2\sin x\cos x = \frac{\sin x}{\cos x}$

$2\sin x\cos^2 x = \sin x$ ✓

$\sin x\left(2\cos^2 x - 1\right) = 0$

$\therefore \sin x = 0$ OR $2\cos^2 x - 1 = 0$ ✓

$x = 0, \pi$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$ ✓

MATHEMATICS Extension 1: Question 7

Suggested Solutions	Marks	Marker's Comments
(a) Expected number of allergic people (i) $E(X) = 0.25 \times 20 = 5$ $\therefore$ 5 allergic people in group ✓	1	
(ii) P(at least 1 allergic person) $= 1-(0.75)^n$ $1-(0.75)^n > 0.999$ ✓ $(0.75)^n < 0.001$ $n \ln(0.75) < \ln 0.001$ ✓ $\therefore n > \dfrac{\ln 0.001}{\ln 0.75}$ $n > 24.61$ ✓ $\therefore$ 25 people are required.	3 ✓	1 mark for recognising $1-0.75^n > 0.999$ 1 mark solving for n - looking at steps 1 mark for getting $n = 24.61$ and rounding it up to get 25
(iii) accepted reasons: assumptions - Random - Independent - Unbiased - sample represents population - probability of being allergic remains constant for all ~~people~~ sample populations. - Uni students have the same chance of being allergic as all other people	1	

MATHEMATICS Extension 1: Question 7

Suggested Solutions	Marks	Marker's Comments
7(b) $\displaystyle\int_1^{49} \frac{1}{4(x+\sqrt{x})}\,dx$	3	
let $u^2 = x$ $\frac{dx}{du} = 2u$ $\therefore dx = 2u \cdot du$		✓1 mark for substituting $* \; dx = 2u \cdot du$ $* \; x = u^2$ $* \; \sqrt{x} = u$
when $x = 1$ $u = 1$ when $x = 49$ $u = 7$		
$\displaystyle\int_1^7 \frac{1}{4(u^2+u)} \cdot 2u \cdot du$		
$\displaystyle= \frac{2}{4}\int_1^7 \frac{u}{u^2+u} \cdot du$		✓1 mark for integrating with correct integral. ie $\displaystyle\frac{1}{2}\int_1^7 \frac{1}{u+1} \cdot du$
$\displaystyle= \frac{1}{2}\int_1^7 \frac{1}{u+1} \cdot du$		<u>Note</u> this is not equal to $\displaystyle\frac{1}{2}\int_1^7 \frac{1}{u}+1$
$= \frac{1}{2}\left[\ln(u+1)\right]_1^7$		
$= \frac{1}{2}\left[\ln 8 - \ln 2\right]$		✓1 substituting with correct limits $u=1$ and $u=7$
$= \frac{1}{2}\left[\ln 4\right]$		accepted $\frac{1}{2}\left[\ln 8 - \ln 2\right]$
$= \ln 2$		

Question 8b)

Monday, 20 April 2020 12:18 pm

(i) $\frac{dV}{dt} = 0.5$

$\therefore V = 0.5t + C$ when $t=0, V=0$

$\therefore C = 0$

$V = 0.5t$

$V = \frac{4}{3}\pi r^3$

$0.5t = \frac{4}{3}\pi r^3$

$t = \frac{8}{3}\pi r^3$

$r^3 = \frac{3t}{8\pi}$

$\therefore r = \sqrt[3]{\frac{3t}{8\pi}}$

① $V=0.5t$ after showing $C=0$

① expression for t

ii) Method 1: Simplest

$S = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$ $\quad \frac{dt}{dr} = 8\pi r^2$

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

$= 8\pi r \times \frac{1}{8\pi r^2}$

$= \frac{1}{r}$

$= \sqrt[3]{\frac{8\pi}{3t}}$ $\quad r = \sqrt[3]{\frac{3t}{8\pi}}$

when $t = 8$

$\frac{dS}{dt} = \sqrt[3]{\frac{8\pi}{3\times 8}}$

$= \sqrt[3]{\frac{\pi}{3}}$

Method 2:

$S = 4\pi r^2$, $r = \sqrt[3]{\frac{3t}{8\pi}} = \sqrt[3]{\frac{3}{8\pi}} \times t^{1/3}$

$\frac{dS}{dr} = 8\pi r$

$\frac{dr}{dt} = \sqrt[3]{\frac{3}{8\pi}} \times \frac{1}{3} t^{-2/3}$

$= \sqrt[3]{\frac{3}{8\pi}} \times \frac{1}{3\sqrt[3]{t^2}}$

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

$= 8\pi r \times \sqrt[3]{\frac{3}{8\pi}} \times \frac{1}{3\sqrt[3]{t^2}}$

$= 8\pi \times \sqrt[3]{\frac{3t}{8\pi}} \times \sqrt[3]{\frac{3}{8\pi}} \times \frac{1}{3\sqrt[3]{t^2}}$

when $t=8$

$\frac{dS}{dt} = 8\pi \times \sqrt[3]{\frac{3}{\pi}} \times \sqrt[3]{\frac{3}{8\pi}} \times \frac{1}{3}\sqrt[3]{\frac{1}{8^2}}$

$= \frac{8\pi}{3} \times \sqrt[3]{\frac{9}{8^3\pi^2}}$

$= \sqrt[3]{\frac{8^3 \times 9\pi^3}{3^3 8^3 \pi^2}}$

$= \sqrt[3]{\frac{\pi}{3}}$

Marks:

① for $\frac{dS}{dt} = \frac{1}{r}$

① $\frac{dS}{dt}$ in terms of t

① substitution of $t=8$ and simplification.

(iii) $4\pi r^2 = 200$

$r^2 = \frac{200}{4\pi}$

$= \frac{50}{\pi}$

$r = \sqrt{\frac{50}{\pi}}$, $r>0$

$\therefore \sqrt{\frac{50}{\pi}} = \sqrt[3]{\frac{3t}{8\pi}}$

$t = \left(\sqrt{\frac{50}{\pi}}\right)^3 \times \frac{8\pi}{3}$ $\quad \frac{8 \times 50\sqrt{50}}{3\sqrt{\pi}} = \frac{400\sqrt{50}}{3\sqrt{\pi}}$

$= 531.9230405$ s

$\therefore$ max. time $= 531$ s

or 531.9 s

or 8 min 51 s

① no penalty for subsequent error of rounding up to 532s which is too long.

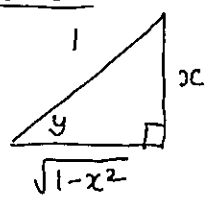
Question 8a)

Monday, 20 April 2020 12:07 pm

a) $y = \sin^{-1} x$

$x = \sin y \qquad \dfrac{dx}{dy} = \cos y$

<u>Method 1:</u>



$\cos y = \sqrt{1-x^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

<u>Method 2:</u>

$\cos^2 y + \sin^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1-\sin^2 y}$

since for $y = \sin^{-1} x$, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

and $\cos x > 0$ for this domain

$\therefore \dfrac{dx}{dy} = \sqrt{1-\sin^2 y}$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-\sin^2 y}}$

$= \dfrac{1}{\sqrt{1-x^2}}$

① for finding $\dfrac{dx}{dy} = \cos y$

<u>Method 1:</u>

① for using triangle to show $\cos y = \sqrt{1-x^2}$ and $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

care that $\dfrac{dy}{dx} \neq \dfrac{dx}{dy}$

① for using Pythagorean identity to show $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

Suggested Solutions	Marks	Marker's Comments
a) i) $X_1, X_2, \ldots, X_n$, $B(1, p)$ $Y = X_1 + X_2 + \cdots + X_n$, $\hat{p} = \frac{Y}{n}$ $Var(Y) = np(1-p)$ given. Show $var(\hat{p}) = \frac{p(1-p)}{n}$. we know $Var(Y) = npq$, $q = (1-p)$ M1 $var(\hat{p}) = Var\left(\frac{Y}{n}\right)$ $= E\left(\left(\frac{Y}{n}\right)^2\right) - E\left(\frac{Y}{n}\right)^2$ $= \frac{1}{n^2}E(Y^2) - \frac{1}{n^2}[E(Y)]^2$ $= \frac{1}{n^2}\left[E(Y^2) - (E(Y))^2\right]$		1
$= \frac{1}{n^2} Var(Y)$ $= \frac{1}{n^2}(np(1-p))$ from above		1
$= \frac{p(1-p)}{n}$ shown. M2 Given $Var(Y) = np(1-p)$ $Var(\hat{p}) = Var\left(\frac{Y}{n}\right)$ $= \frac{1}{n^2} Var(Y)$ $\rightarrow$ (since variance is the square of std dev) $= \frac{1}{n^2}(np(1-p))$ $= \frac{p(1-p)}{n}$ (shown) example from maths in Focus 12 (Ext 1) pg 692.		

MATHEMATICS: Question 9 ... 2/3

Suggested Solutions	Marks	Marker's Comments
a ii) Show $0 \le var(\hat{p}) \le \frac{1}{4n}$		
$y = p(1-p)$ $\frac{dy}{dp} = 1 - 2p$ when $\frac{dy}{dp} = 0$ $p = \frac{1}{2}$.		(1) - mark to show understand of stationary points and concave down
y_{max} t.p at $p = \frac{1}{2}$ $= \frac{1}{2}(1 - \frac{1}{2})$ $= \frac{1}{4}$		(1) - mark for stating max tp at $p = \frac{1}{2}$ and y_{max} at $\frac{1}{4}$ or have shown a graphical interpretation.
- greatest value of $p(1-p)$ occurs when $p = \frac{1}{2}$; $p(1-p) = \frac{1}{4}$ - smallest value of $p(1-p)$ occurs when $p = 0$ or 1; $p(1-p) = 0$ - smallest value of $var(\hat{p}) = \frac{0}{n} = 0$ - greatest value of $var(\hat{p}) = \frac{\frac{1}{4}}{n} = \frac{1}{4n}$ $\therefore 0 \le var(\hat{p}) \le \frac{1}{4n}$		(1) for why $var(\hat{p}) \ge 0$.
b) i) - If $p = 0$ or $p = 1$, there is a 100% chance that the trial will be a failure or success. - A Bernoulli trial is an experiment that has 2 outcomes, either failure or success. - If there is a 100% chance of either occuring, that means that there is only one outcome. $\therefore$ The experiment will not be a Bernoulli trial bcoz p cannot be 0 or 1		(1) - mark. Have to mention something along the line why/how Bernoulli trial happens.

Suggested Solutions	Marks Awarded	Marker's Comments
b) ii) How many distinct values can X take in 'n' Bernoulli trials? The answer is 2 or 0 or 1	1	Many students interpreted it incorrectly. The question said X = Bernoulli random variable NOT Binomial random variable ∴ This was the issue → For Bernoulli random variable – there are only 2 distinct val X can take. ie: 2 or 0 or 1
b) iii) $P(H) = \frac{1}{5}$ if heads win $1 Is this Bernoulli random variable, (BRV) ie from my comment, BRV has 2 outcomes; if $P(H) = \frac{1}{5}$ win (1) $P(\bar{H}) = \frac{4}{5}$ lose (0) as per definition. Therefore the answer is Yes and the second mark is awarded if student explains this correctly.		1 1

Q10.

a) Let $t = \tan\frac{\theta}{2}$ (Method 1)

$$\therefore \frac{1}{2}(3\cos\theta + 4\sin\theta + 5)$$

$$= \frac{1}{2}\left(3 \times \frac{1-t^2}{1+t^2} + 4 \times \frac{2t}{1+t^2} + 5\right)$$

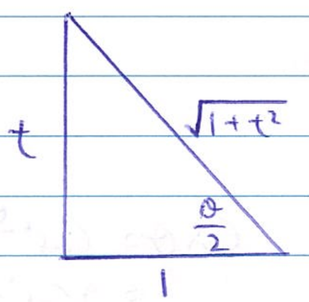
$$= \frac{1}{2}\left(\frac{3-3t^2}{1+t^2} + \frac{8t}{1+t^2} + \frac{5+5t^2}{1+t^2}\right)$$

$$= \frac{1}{2}\left(\frac{3-3t^2+8t+5+5t^2}{1+t^2}\right)$$

$$= \frac{1}{2}\left(\frac{2t^2+8t+8}{1+t^2}\right)$$

$$= \frac{t^2+4t+4}{1+t^2}$$ ① Arriving at this expression from LHS

$$= \frac{(t+2)^2}{1+t^2}$$



$$\sin\frac{\theta}{2} = \frac{t}{\sqrt{1+t^2}}$$

$$\cos\frac{\theta}{2} = \frac{1}{\sqrt{1+t^2}}$$

① This step is a <u>must</u> (including the triangle) as students are not allowed to quote $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$ in a "prove" question

$$\therefore \text{RHS} = \left(\frac{t}{\sqrt{1+t^2}} + 2 \times \frac{1}{\sqrt{1+t^2}}\right)^2$$

$$= \left(\frac{t+2}{\sqrt{1+t^2}}\right)^2$$

$$= \frac{(t+2)^2}{1+t^2}$$ ① Arriving at this expression from RHS.

$= \text{LHS}$

or After arriving $LHS = \frac{(t+2)^2}{1+t^2}$ like in method 1
(Method 2)

$$LHS = \frac{(t+2)^2}{1+t^2} \quad \text{——— } 1$$

$$= \frac{\left(\tan\frac{\theta}{2}+2\right)^2}{1+\tan^2\frac{\theta}{2}}$$

$$= \frac{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+2\right)^2}{\sec^2\frac{\theta}{2}} \quad \text{——— } 1$$

Getting rid of t or $\tan\frac{\theta}{2}$ in the expression

$$= \cos^2\frac{\theta}{2}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+2\right)^2$$

$$= \left[\cos\frac{\theta}{2}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+2\right)\right]^2$$

$$= \left(\sin\frac{\theta}{2}+2\cos\frac{\theta}{2}\right)^2 \quad \text{——— } 1$$

Arriving to RHS.

$\boxed{\text{What NOT to do}}$

Intention of Qn

$$RHS = \left(\sin\frac{\theta}{2}+2\cos\frac{\theta}{2}\right)^2$$

$$= \sin^2\frac{\theta}{2}+4\sin\frac{\theta}{2}\cos\frac{\theta}{2}+4\cos^2\frac{\theta}{2}$$

$$= 1+3\cos^2\frac{\theta}{2}+4\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$= 1+\frac{3\cos\theta+3}{2}+2\sin\theta \; *$$

Sub $\cos\theta = \frac{1-t^2}{1+t^2}$ and $\sin\theta = \frac{2t}{1+t^2}$

$$\therefore RHS = \vdots$$

$$\frac{(t+2)^2}{1+t^2}$$

$$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$
$$= 2\cos^2\frac{\theta}{2} - 1$$
$$\therefore \cos^2\frac{\theta}{2} = \frac{\cos\theta+1}{2}$$

- Those who approached RHS this way gets 1/3 MAX from for the entire question as you've essentially done the prove up to * without t formula.

b) $LHS = \frac{(n+2)!}{[(n+2)-3]!\,3!} - \frac{n!}{(n-3)!\,3!}$

$= \frac{(n+2)!}{(n-1)!\,3!} - \frac{n!}{(n-3)!\,3!}$ 1

$= \frac{n(n+1)(n+2)}{3!} - \frac{n(n-1)(n-2)}{3!}$

$= \frac{n\left[(n^2+3n+2)-(n^2-3n+2)\right]}{6}$

$= \frac{n \times 6n}{6}$ 1

$= n^2$

c) $p = 0.09 \quad q = 0.91, \quad n = 820$

$\sigma = \sqrt{npq} \doteqdot 8.195$ 1

using formula for S.D

3% of 820 = 24.6.

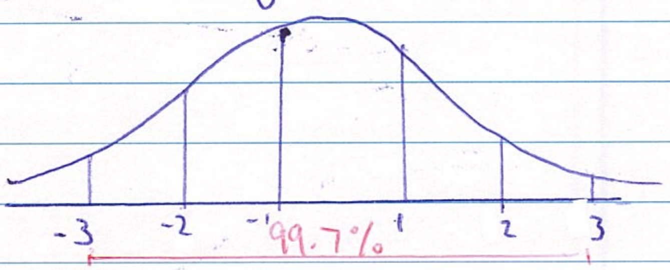
9% of 820 = 73.8

$\therefore$ looking for $P(49.2 \le X \le 98.4)$

$Z_{49.2} = \frac{49.2 - 73.8}{\sigma} \doteqdot -3.002$ 1

One of the z-scores

$Z_{98.4} = \frac{98.4 - 73.8}{\sigma} \doteqdot 3.002$



$\therefore P(|X - 73.8| > 24.6) = 0.03\ \%$

1

$\sigma = \sqrt{\frac{pq}{n}} \doteqdot 9.994 \times 10^{-3}$ 1

using formula for S.D

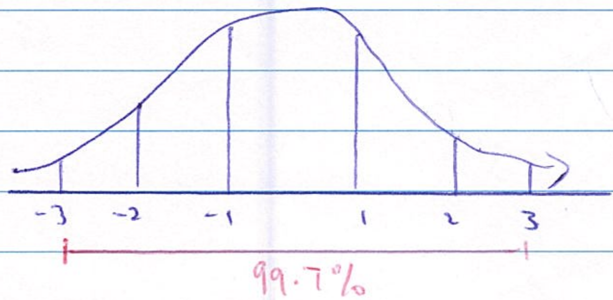
$P(0.06 \le \hat{p} \le 0.12)$:

$Z_{0.06} = \frac{0.06 - 0.09}{\sigma} \doteqdot -3.002$

1

$Z_{0.12} = \frac{0.12 - 0.06}{\sigma} \doteqdot 3.002.$

$\therefore$



$\therefore P(|\hat{p} - 0.09| > 0.03) = 0.03\ \%$

1

Q11 – marking standards

a) i) $h = 36\sin(1.5t) - 15\cos(1.5t) + 65$

→ ① $k = \sqrt{36^2+15^2} = \sqrt{1521} = 39$ — either $\sqrt{1521}$ or 39 – 1 mark.

→ ② $\tan\alpha = \frac{15}{36} = \frac{5}{12} \Rightarrow \alpha = \tan^{-1}(\frac{5}{12})$

or $\sin\alpha = \frac{5}{13} \Rightarrow \alpha = \sin^{-1}(\frac{5}{13})$ – either – 1 mark.

or $\cos\alpha = \frac{12}{13} \Rightarrow \alpha = \cos^{-1}(\frac{12}{13})$ — α can have only one value.

or $\alpha = 0.39479\ldots$ (α can't be in degree form).

$= 22°37'$ or $22.62°$ (×)

→ ③ $39\sin(1.5t - \tan^{-1}(\frac{5}{12}))$ — expression – 1 mark.

↳ or any of the 4 angle forms above

ii) $h = 39\sin(1.5t - \tan^{-1}(\frac{5}{12})) + 65 = 100$ — no mark if sin is missing from the expression

→ ① $39\sin(1.5t - \tan^{-1}(\frac{5}{12})) = 35$ — correct sin ratio – 1 mark.

$\sin(1.5t - \tan^{-1}(\frac{5}{12})) = \frac{35}{39}$

radian form — degree form (ECF)

→ ② $1.5t - \tan^{-1}(\frac{5}{12}) = \sin^{-1}(\frac{35}{39})$

$1.5t = \tan^{-1}(\frac{5}{12}) + \sin^{-1}(\frac{35}{39})$ — $\sin^{-1}(\frac{35}{39}) \doteqdot 63°49'$

$t = \frac{2}{3}(\tan^{-1}(\frac{5}{12}) + \underbrace{\sin^{-1}(\frac{35}{39})}_{1.1139\ldots})$ — $t = 57°37'42.99''$

$= 1.0058089\ldots$ — $= 57.6286°$ — correct t-value ① – 1 mark.

or ③ $1.5t - \tan^{-1}(\frac{5}{12}) = \pi - \sin^{-1}(\frac{35}{39})$

$t = \frac{2}{3}(\pi - \sin^{-1}(\frac{35}{39}) + \tan^{-1}(\frac{5}{12}))$ — $t = 92°31'52.36''$

$= 1.614974\ldots$ — $= 92.53121°$ — correct t-value ② – 1 mark.

b) $(1-2x-4x^2)(1-\frac{3}{x})^9$

$= (1-2x-4x^2)\left({}^9C_0 + {}^9C_1(-\frac{3}{x})^1 + {}^9C_2(\frac{-3}{x})^2 + \dots + {}^9C_5(\frac{-3}{x})^5 + {}^9C_6(\frac{-3}{x})^6 + {}^9C_7(\frac{-3}{x}\right.$

Terms in x^{-5} ${}^9C_5(-3)^5 - 2\,{}^9C_6(-3)^6 - 4\,{}^9C_7(-3)^7$ — 1 mark

$= -30618 \quad -2\times 61236 \quad -4\times(-78732)$

$= -30618 - 122472 + 314928$

$= 161838$ — 1 mark

or 3 correct expressions but wrong calculation — 1 mark/2

or $\left\{\begin{array}{l}\text{2 correct expressions + correct calculation} \\ \text{1 wrong expression + correct ECF calculation}\end{array}\right.$ — 1 mark/2

or in the expressions, forget to include "−" from "(-3)" — 1 mark/2

or in the expressions, forget to consider "( )" from "(-3)" — 1 mark/2

12) $$\frac{T_r}{T_{r+1}} = \frac{\dfrac{50!}{r!(50-r)!} \times 0.15^r \times (0.85)^{50-r}}{\dfrac{50!}{(r+1)!(50-(r+1))!} \times 0.15^{r+1} \times 0.85^{50-(r+1)}}$$

$$= \frac{r+1}{50-r} \quad \frac{0.85}{0.15}$$

$$= \frac{85(r+1)}{15(50-r)}$$

$$= \frac{17(r+1)}{3(50-r)}$$

(ii) $P(X=r) \leqslant P(X=r+1)$

$\frac{P(X=r)}{P(X=r+1)} \leqslant 1$ ie $\frac{T_r}{T_{r+1}} \leqslant 1$

$\therefore \frac{17(r+1)}{3(50-r)} \leqslant 1$

$17r + 17 \leqslant 150 - 3r$

$20r \leqslant 133$

$r \leqslant 6.65$

$\therefore$ Values will be $r = 0, 1, 2, \ldots 6$.

(iii) most likely number of days late is 7.