Student Name: ________________

Maths class: ________________

James Ruse Agricultural High School

2020 YEAR 12 Task 3

Mathematics Extension 1

General Instructions

Reading time – 5 minutes

- Working time – 1 hour and 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided

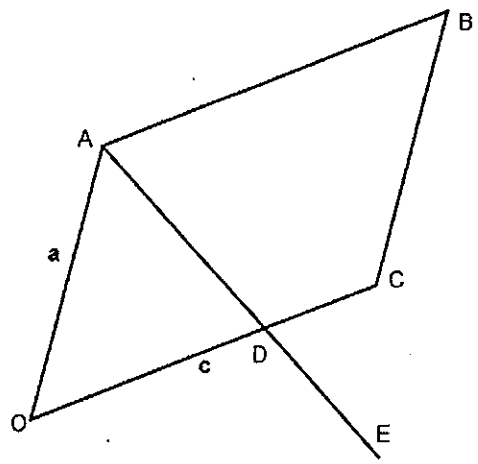
Total Marks: 63

Attempt all questions (1 – 7). Each question is worth 9 marks.

Start your answer to each question on a separate piece of paper.

Question 1 **(Start a new piece of paper)** **Marks**

(a) $OABC$ is a parallelogram with O as the origin. The position vector of A is $\underline{a}$ and the position vector of C is $\underline{c}$. D divides the segment OC such that $OD:DC = 2:1$. E is a point on AD extended so that $AD: DE = 3:2$. **4**



(Diagram not to scale)

Find the following vectors in terms of $\underline{a}$ and $\underline{c}$:

(i) $\overrightarrow{AC}$ (ii) $\overrightarrow{AD}$ (iii) $\overrightarrow{AE}$ (iv) $\overrightarrow{BE}$

(b) (i) Explain why the product $n(n+1)$ is even for any positive integer n. **1**

(ii) Prove by Mathematical Induction that $5n^3 + 7n$ is divisible by 6 for all positive integers n. **4**

Question 2 **(Start a new piece of paper)** **Marks**

(a) The area bounded by the curve $y = \ln(2x)$, the y axis and the lines $y = \ln 2$ and $y = \ln 6$ is rotated about the y axis. Find the volume of the solid so formed. **3**

(b) Consider a parallelogram $WXYZ$.

(i) Explain why $\overrightarrow{XY} = \overrightarrow{WZ}$ and (similarly) $\overrightarrow{WX} = \overrightarrow{ZY}$. **1**

(ii) If $\overrightarrow{WX} = \underline{a}$ and $\overrightarrow{XY} = \underline{b}$, express the diagonals $\overrightarrow{WY}$ and $\overrightarrow{XZ}$ in terms of $\underline{a}$ and $\underline{b}$. **2**

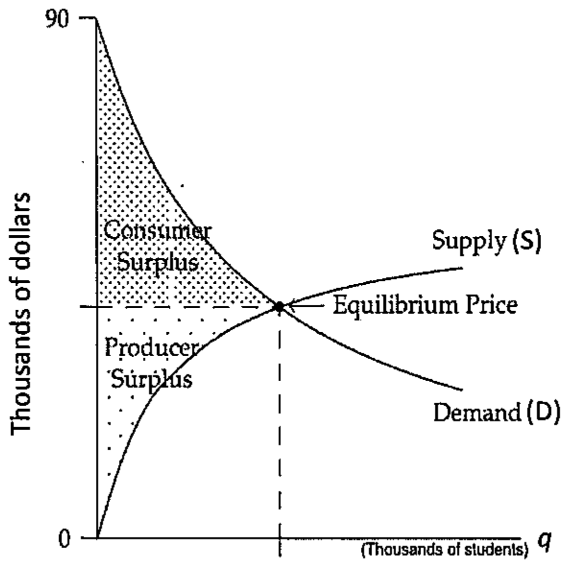
(iii) Use scalar products to find the square of the length of each diagonal in terms of $\underline{a}$ and $\underline{b}$. **2**

(iv) Hence deduce that the sum of the squares of the diagonal lengths of a parallelogram is equal to sum of the squares of the four sides. **1**

Question 3 **(Start a new piece of paper)** **Marks**

(a) $ABCD$ is a parallelogram where A, B and C have position vectors $2\mathbf{i} - 4\mathbf{j}$, $-3\mathbf{i} + 3\mathbf{j}$ and $\mathbf{i}+5\mathbf{j}$ respectively, relative to an origin O. **2**
Find the position vector of D.

(b) The Demand curve for students wanting to attend JR University each year is given by Demand Price (D), where $D = \frac{180}{q+2}$, q is measured in thousands of students and D is measured in thousands of dollars. The university is willing to accept students according to the formula for Supply Price (S), where $S = \frac{56q}{q+1}$, as shown in the diagram below.



By showing that, when Demand Price is equal to Supply Price, $q = 2.5$ (to 2 significant figures), and that the equilibrium price is \$40 000, find the Consumer Surplus, as shaded on the diagram. Give your answer in dollars to 2 significant figures. **4**

(c) In what ratio does the x-axis divide the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$? **3**

Question 4 continues over the page

Question 4 **(Start a new piece of paper)** **Marks**

(a) Consider the quadrilateral $ABCD$ where, relative to an origin O, $\overrightarrow{OA} = 3\mathrm{i} - \mathrm{j}$, $\overrightarrow{OB} = 3\mathrm{i} + 2\mathrm{j}$, $\overrightarrow{OC} = 9\mathrm{i} + 5\mathrm{j}$ and $\overrightarrow{OD} = 6\mathrm{i} - \mathrm{j}$.

(i) Find $\overrightarrow{AC}$ and $\overrightarrow{BD}$ and verify that these vectors are perpendicular. **2**

(ii) Find $|\overrightarrow{AB}|$, $|\mathrm{BC}|$, $|\mathrm{CD}|$ *and* $|\mathrm{DA}|$ and hence determine to which family of quadrilaterals $ABCD$ belongs. **3**

(b) Prove by Mathematical Induction that, for any positive integer n, **4**

$$\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{2n-1} - \frac{1}{2n}$$

Question 5 **(Start a new piece of paper)** **Marks**

(a) Relative to an origin O, the points A, B and C have position vectors -5i +2j, 7i – 4j and 3i +pj respectively where p is a constant.

(i) Find the value of p if angle ABC is 90°. **3**

(ii) Find the value of p if A, B and C are collinear (in some order). **2**

(b) The curved surface area A of a solid of revolution generated by rotating the part of a curve between $x = a$ and $x = b$ about the x axis is given by

$$\int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx.$$

(i) The curve $y = 2\sqrt{x}$ is rotated about the x axis. Show that the curved surface area generated over the interval $0 \leq x \leq 8$ is given by **2**

$$4\pi\int_0^8 \sqrt{x+1}\,dx$$

(ii) Hence find the required curved surface area. **2**

Question 5 continues over the page

Question 6 **(Start a new piece of paper)** **Marks**

(a) (i) Find the vector projection of $15\mathbf{i} + 5\mathbf{j}$ onto $6\mathbf{i} - 8\mathbf{j}$. **2**

(ii) Hence break down $15\mathbf{i} + 5\mathbf{j}$ into components parallel and perpendicular to $6\mathbf{i} - 8\mathbf{j}$. **2**

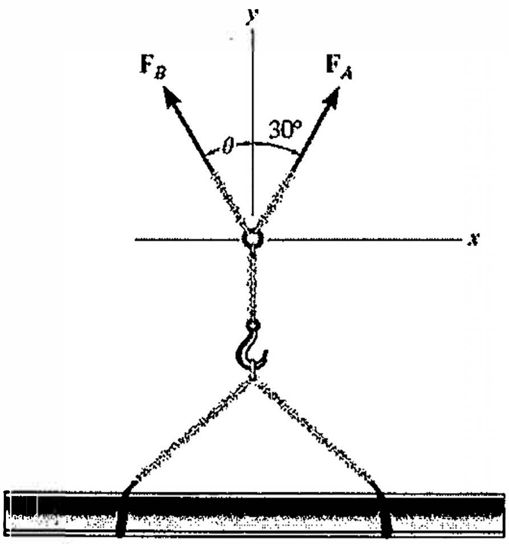
(b) (i) Show that $(n+1)^2(3n+8) = 3n^3 + 14n^2 + 19n + 8$ **1**

(ii) Prove by Mathematical Induction that, for any positive integer n, **4**

$$\frac{1}{1\text{x}3} + \frac{1}{2\text{x}4} + \frac{1}{3\text{x}5} + \ldots\ldots + \frac{1}{n(n+2)} = \frac{n(3n+5)}{4(n+1)(n+2)}$$

Question 7 **(Start a new piece of paper)** **Marks**

(a) The beam shown below is to be hoisted using two chains.



If the resultant force is to be 600N directed along the positive y-axis,

(i) Given F_A acts at 30° from the y-axis as shown, draw a vector diagram representing the situation and explain why the magnitude of F_B is a minimum when $\theta = 60°$. Hence, **2**

(ii) determine the magnitudes of forces F_A and F_B acting on each chain.. **2**

Question 7 continues over the page

(b) (i) The area between the curve $y = x^2 + 1$, the x axis and the ordinates x=1 and x=2 is rotated about the x axis. Find the volume so formed. **2**

(ii) The area between the curve $y = x^2 + 1$, the line $y = 6$ and the ordinates x=1 and x=2 is rotated about the line $y = 6$. Explain, with the aid of a diagram, why the resulting volume is given by **3**

$$\pi \int_1^2 x^4 - 10x^2 + 25 \, dx$$

(Do not evaluate the integral)

End of Examination

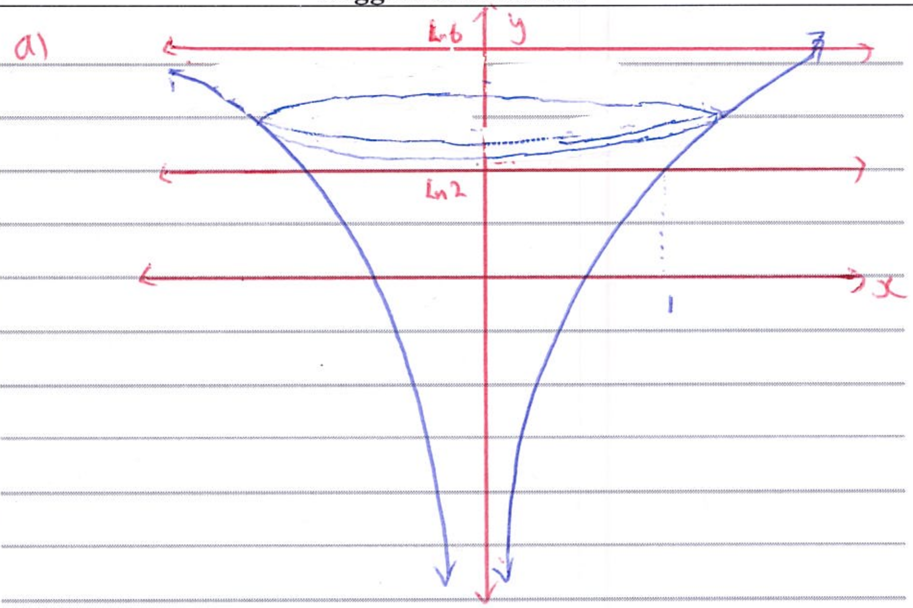
MATHEMATICS Extension 1, Question 1		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(a) (i) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{c} - \underset{\sim}{a}$ (ii) $\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = -\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{c} = \frac{2}{3}\underset{\sim}{c} - \underset{\sim}{a}$ (iii) $\overrightarrow{AE} = \frac{5}{3}\overrightarrow{AD} = \frac{5}{3}\left(\frac{2}{3}\underset{\sim}{c} - \underset{\sim}{a}\right) = \frac{10}{9}\underset{\sim}{c} - \frac{5}{3}\underset{\sim}{a}$ (iv) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE} = -\underset{\sim}{c} + \frac{5}{3}\overrightarrow{AD} = -\underset{\sim}{c} + \frac{5}{3}\left(\frac{2}{3}\underset{\sim}{c} - \underset{\sim}{a}\right)$ $= \frac{1}{9}\underset{\sim}{c} - \frac{5}{3}\underset{\sim}{a}$	1mark per correct vector. 1 mark penalty for lack of vector notation.	Question was handled well generally. Only errors involved were those where direction was not identified correctly or scalar multiples of vectors incorrectly assigned.
(b) (i) n and $(n+1)$ are successive integers. Hence one will be even while the other, odd. The product of an odd integer with an even integer yields another even integer. Hence $n(n+1)$ is even. You could prove it using the definitions of even and odd, those being: • n is even iff there is an integer k such that $n = 2k$ • n is odd iff there is an integer k such that $n = 2k+1$ Take the product: $n(n+1) = 2k(2k+1)$ which is even since $k(2k+1)$ is an integer given $k \in \mathbb{Z}$. And if $n = 2k+1$, then $n(n+1) = (2k+1)(2k+1+1)$ $= (2k+1)(2k+2)$ $= 2(2k+1)(k+1)$ which is even since $(2k+1)(k+1)$ is a combination of products and sums of integers given $k \in \mathbb{Z}$.	1 mark awarded for sufficient reasoning. No 'hard' proof required (1 mark question).	All students handled this well.

(ii) Let $S(n)$ be the statement that 6 divides $5n^3 + 7n$; i.e. $$6\|5n^3 + 7n \leftrightarrow 5n^3 + 7n = 6m$$ for some $m \in \mathbb{Z}$. We test the base case at $n = 1$: $$5(1)^3 + 7(1) = 12 = 6 \times 2$$ The result is a multiple of 6, hence the statement is true for $n = 1$.	1 mark awarded for setup of correct mathemat-ical form and reasonable working from there to the point of the quadratic in $S(n+1)$. 1 mark for base case.	Great majority of students established this correctly. All students established the base case.
Now assume the statement $S(n)$ to be true. Then we consider $S(n+1)$: $$5(n+1)^3 + 7(n+1) = 5n^3 + 15n^2 + 15n + 5 + 7n + 7$$ $$= (5n^3 + 7n) + 15n^2 + 15n + 12$$ $$= 6m + 15n(n+1) + 12$$ (as $5n^3 + 7n = 6m$ by hypothesis). Now, by (i), $n(n+1)$ is even, hence $n(n+1) = 2K$ for some $K \in \mathbb{Z}$. Hence we may continue as $$6m + 15n(n+1) + 12 = 6m + 15(2K) + 12$$	1 mark for dealing appropria-tely with this section where use of part (i) was made, or a separate induction.	Most students satisfied this part using the result of part (i). Those who didn't use part (i) either used a second induction (fine, but more work) or stopped – can't proceed from this point without showing the quadratic to be divisible by 6.
$$= 6(m + 5K + 2)$$ Since $m, K \in \mathbb{Z}$, we have that $(m + 5K + 2) \in \mathbb{Z}$, meaning 6 divides $5(n+1)^3 + 7(n+1)$ if 6 divides $5n^3 + 7n$. Hence if $S(n)$ is true, then $S(n+1)$ is true. As $S(1)$ is true, it follows by the principle of mathematical induction that $S(n)$ is true for all $n \in \mathbb{N}$.	1 mark for correct finish, identifying correctly that the LHS of $S(n+1)$ is a multiple of 6.	Those who were able to continue needed to then finish appropriately by identifying $S(n+1)$ as being of form $6K$ for K some integer. This part was finished well.

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments

a)



$$V = \pi \int_{\ln 2}^{\ln 6} x^2 \, dy$$

$$y = \ln(2x)$$

$$e^y = 2x$$

$$x = \frac{e^y}{2}$$ Write x in terms of y ①

$$= \pi \int_{\ln 2}^{\ln 6} \frac{e^{2y}}{4} \, dy$$ ① To Integral

$$= \pi \left[\frac{e^{2y}}{8} \right]_{\ln 2}^{\ln 6}$$

$$= \pi \left[\frac{36}{8} - \frac{4}{8} \right]$$

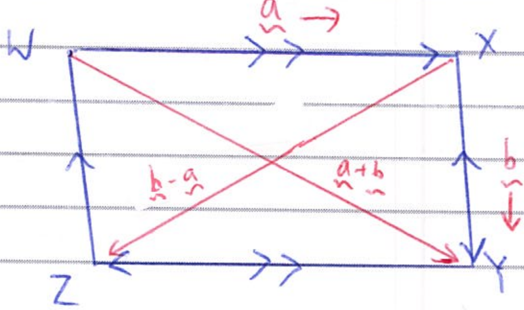
$$= \frac{32}{8} \pi$$

$$= 4\pi \ u^3$$ ① Answer

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments

b) i)



$\overrightarrow{XY} = \overrightarrow{WZ}$ because opposite sides of a parallelogram are equal and parallel, and the 2 vectors are in the same direction

$\therefore$ Equal magnitude and direction.

Similarly true for $\overrightarrow{WX}$ and $\overrightarrow{ZY}$, $\therefore \overrightarrow{WX} = \overrightarrow{ZY}$

Marker's comments: Must require either:
- diagram or
- worded explanation about the vertex order so that the vectors that are parallel are not opposites

ii) $\overrightarrow{WY} = \underset{\sim}{a} + \underset{\sim}{b}$ — 1

$\overrightarrow{XZ} = \underset{\sim}{b} - \underset{\sim}{a}$ — 1

iii) $|\overrightarrow{WY}|^2 = |\underset{\sim}{a}+\underset{\sim}{b}|^2$

$= (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b})$

$= |\underset{\sim}{a}|^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + |\underset{\sim}{b}|^2$ — 1

$|\overrightarrow{XZ}|^2 = |\underset{\sim}{b}-\underset{\sim}{a}|^2$

$= (\underset{\sim}{b}-\underset{\sim}{a})\cdot(\underset{\sim}{b}-\underset{\sim}{a})$

$= |\underset{\sim}{b}|^2 - 2\underset{\sim}{a}\cdot\underset{\sim}{b} + |\underset{\sim}{a}|^2$ — 1

Marker's comments: Students lost marks if you wrote things such as:
- $a^2+2ab+b^2$
- $|a|^2+2|a||b|+|b|^2$

iv) $\therefore |\overrightarrow{WY}|^2 + |\overrightarrow{XZ}|^2 = (|\underset{\sim}{a}|^2+2\underset{\sim}{a}\cdot\underset{\sim}{b}+|\underset{\sim}{b}|^2) + (|\underset{\sim}{a}|^2-2\underset{\sim}{a}\cdot\underset{\sim}{b}+|\underset{\sim}{b}|^2)$

$= 2(|\underset{\sim}{a}|^2+|\underset{\sim}{b}|^2)$ — 1

Which is equal to the sum of the squares of the four sides.

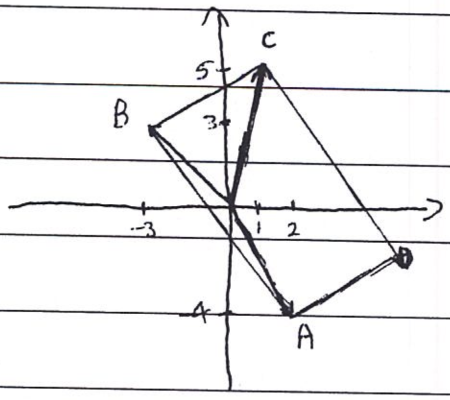
Marker's comments: Students must either:
i) write a worded conclusion
or
ii) Connect the expression to $\overrightarrow{WX}$, $\overrightarrow{XY}$, $\overrightarrow{YZ}$ and $\overrightarrow{ZW}$

Question 3

Solutions

Student Number [redacted]

(a)



$\vec{OD} = \vec{OA} + \vec{AD}$

$= \vec{OA} + \vec{BC}$ ($\vec{AD} = \vec{BC}$, opposite sides of parallelogram)

$\vec{BC} = \vec{OC} - \vec{OB}$

$\vec{BC} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ 1

$\vec{OD} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

The position vector of D is $6\underset{\sim}{i} - 2\underset{\sim}{j}$. 1

(b) when Demand Price = Supply Price

$\frac{180}{q+2} = \frac{56q}{q+1}$

$180q + 180 = 56q^2 + 112q$

$56q^2 - 68q - 180 = 0$

$14q^2 - 17q - 45 = 0$

$q = \frac{17 \pm \sqrt{17^2 - 4(14)(-45)}}{28}$ $q > 0$

$\therefore q = \frac{5}{2} = 2.5$

when $q = 2.5$ $D = \frac{180}{4.5} = 40$

$\therefore$ Equilibrium Price is \$40,000. 1

Solutions.

Student Number [redacted]

Consumer surplus $= \int_0^{2.5} \left(\frac{180}{q+2} - 40\right) dq$

$= \left[180 \ln|q+2| - 40q\right]_0^{2.5}$ (1)

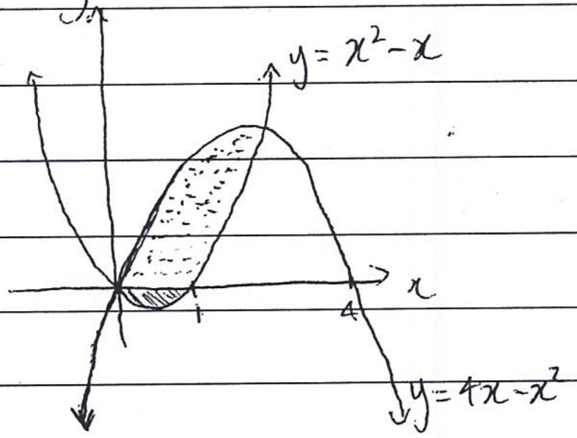
$= 180 \ln 4.5 - 100 - 180 \ln 2 + 0$

$= 45.967...$ (1)

But q & D are in thousands so,

consumer surplus $= \$46{,}000{,}000$ to 2 significant figures. (1)

(c)



Points of intersection:

$x^2 - x = 4x - x^2$

$2x^2 - 5x = 0$

$x(2x - 5) = 0$

$x = 0,\ 2.5$

Area below x-axis $= \left|\int_0^1 (x^2 - x)\,dx\right|$

$= \left|\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1\right|$

$= \left|\frac{1}{3} - \frac{1}{2}\right|$

$= \frac{1}{6}\,u^2$ (1)

Area between curves $=$

$\int_0^{2.5} (4x - x^2) - (x^2 - x)\,dx$

$= \int_0^{2.5} 5x - 2x^2\,dx$

$= \left[\frac{5x^2}{2} - \frac{2x^3}{3}\right]_0^{2.5}$

$= \frac{125}{24}\,u^2$ (1)

So area above x-axis

$= \frac{125}{24} - \frac{1}{6} = \frac{121}{24}$

$\therefore$ Ratio is $\frac{1}{6} : \frac{121}{24}$

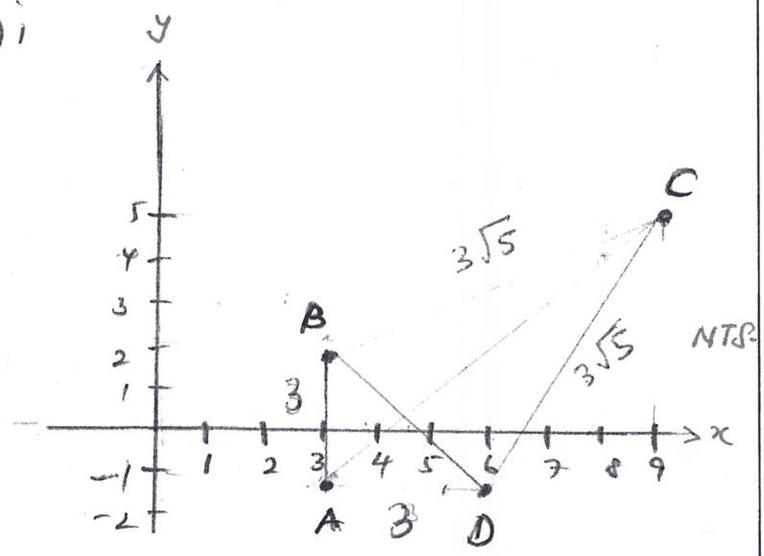
$= 4 : 121$ (1)

or $121 : 4$

Yr12 Task 3 2020

MATHEMATICS: Ext 1 Question 4 1/3

Suggested Solutions	Marks Awarded	Marker's Comments
a) i		* Always draw



$\vec{AC} = \vec{OC} - \vec{OA}$

$= (9\underset{\sim}{i} + 5\underset{\sim}{j}) - (3\underset{\sim}{i} - \underset{\sim}{j})$

$= (6\underset{\sim}{i} + 6\underset{\sim}{j})$

$\vec{BD} = \vec{OD} - \vec{OB}$

$= (6\underset{\sim}{i} - \underset{\sim}{j}) - (3\underset{\sim}{i} + 2\underset{\sim}{j})$

$= 3\underset{\sim}{i} - 3\underset{\sim}{j}$ 1

$\vec{AC} \cdot \vec{BD} = 18 - 18$

$= 0$ (angle is $90°$)

$\therefore \vec{AC} \perp \vec{BD}$ 1

Marker's Comments:
- most students were competent to find $\vec{AC}$ & $\vec{BD}$ and perform dot product which resulted in 0 $\therefore$ is perpendicular

Ext 1

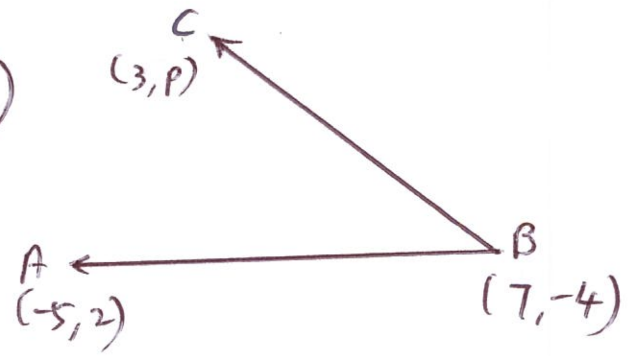
MATHEMATICS: Question 4 .... 2/3

Suggested Solutions	Marks Awarded	Marker's Comments
aii) $\lvert\vec{AB}\rvert = \lvert 3j \rvert$ $= 3$ $\lvert\vec{BC}\rvert = \lvert 6i + 3j\rvert$ $= \lvert 45 \rvert = 3\sqrt{5}$ $\lvert\vec{CD}\rvert = \lvert -3i - 6j\rvert$ $= \lvert 45\rvert = 3\sqrt{5}$ $\lvert\vec{DA}\rvert = \lvert -3i\rvert$ $= 3$ adjacent sides equal and diagonals bisects at right angle from ai) therefore kite	– 1 mark for each pair of correct sides – 1 mark for correct quadrilateral	if students drew the diag and labelled, would have b easy to visu + most stude found 2 pai of equal sid but failed to determine to which family of quadrilat ABCD belonged
b) Prove true for $n=1$ $LHS = \frac{1}{1+1} = \frac{1}{2}$ $RHS = 1 - \frac{1}{2} = \frac{1}{2}$ (1) $\therefore LHS = RHS$ (statement is true for $n=1$) Assume true for $n=k$, where $k \in Z^+, k \geq 1$ $\boxed{\frac{1}{k+1} + \frac{1}{k+2} + \dots + \frac{1}{2k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2k-1} - \frac{1}{2k}}$ (2) Prove true for $n = k+1$ $\frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} + \frac{1}{2k+1} + \frac{1}{2k+2} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2k-1} -$		– most were able to set correctly. You cannot (1) $LHS = \frac{1}{2} = 1 - \frac{1}{2} = R$ (2) $LHS = \frac{1}{2}$ $= 1 - \frac{1}{2}$ $= RHS$

MATHEMATICS: Question…….. 3/3

Suggested Solutions	Marks Awarded	Marker's Comments
LHS $\underbrace{\left(\frac{1}{k+1}+\frac{1}{k+2}+\dots+\frac{1}{2k}\right)}-\frac{1}{k+1}+\frac{1}{2k+1}+\frac{1}{2k+2}$		1 (by assumption
$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots+\frac{1}{2k-1}-\frac{1}{2k}+\frac{1}{k+1}+\frac{1}{2k+1}+\frac{1}{2k+2}$		
$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots+\frac{1}{2k+1}+\frac{1}{2(k+1)}-\frac{1}{k+1}$		1
$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\dots+\frac{1}{2k+1}+\frac{1-2}{(k+1)}$		
$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\dots+\frac{1}{2k+1}\ \ast-\frac{1}{2k+2}$		what is needed.
$=$ RHS		
True for $n=k$, true for $n=k+1$ $k=$	Z^+ $k>0$	(
By the process of mathematical	induction, it	
is true for all positive integer		
$-\frac{1}{k+1}+\frac{1}{2k+2}$		many student couldn't simpl the algebraic part to prov - they knew what to get to but could do basic alge shown
$\frac{-(2k+2)+k+1}{(2k+2)(k+1)}$		
$=\frac{-2k-2+k+1}{}$		
$=\frac{-k-1}{(2k+2)(k+1)}\ \ast=\frac{-1\cancel{(k+1)}}{(2k+2)\cancel{(k+1)}}$		

a i)



FOR $\angle ABC = 90^\circ$, $\vec{BA} \cdot \vec{BC} = 0$

$(-12\underset{\sim}{i} + 6\underset{\sim}{j}) \cdot (-4\underset{\sim}{i} + (p+4)\underset{\sim}{j}) = 0$ 1 m

$48 + 6(p+4) = 0$

$48 + 6p + 24 = 0$ 1 m

$6p = -72$

$\underline{p = -12}$ 1 m

Well done

Wrong vectors max 1 m

ii) FOR A, B, C collinear

$\vec{BA} = k\vec{BC}$

$(-12\underset{\sim}{i} + 6\underset{\sim}{j}) = k(-4\underset{\sim}{i} + (p+4)\underset{\sim}{j})$ 1 m

$-12 = -4k$

$\therefore k = 3$

$6 = 3(p+4)$

$6 = 3p + 12$

$-6 = 3p$

$\underline{p = -2}$ 1 m

or Similarly $\vec{BA} = k\vec{AC}$ etc

Some students use co-ordinate Geom

$m(AC) = m(AB)$

does not get any mark.

MUST show vectors !!

If use dot produ
$\theta = 0$ or 180°

Q5

b i) $y = 2\sqrt{x}$

$\frac{dy}{dx} = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$ 1 m

$SA = \int_0^8 2\pi \, 2\sqrt{x}\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\, dx$

or

$= 4\pi\int_0^8 \sqrt{x}\sqrt{1+\frac{1}{x}}\, dx$

plus one more step 1m

$= 4\pi\int_0^8 \sqrt{x+\frac{x}{x}}\, dx$

$= 4\pi\int_0^8 \sqrt{x+1}\, dx$

without clearly showing $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

$2\pi \, 2\sqrt{x}\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}$

max only 1m

b ii) $SA = 4\pi\int_0^8 \sqrt{x+1}\, dx$

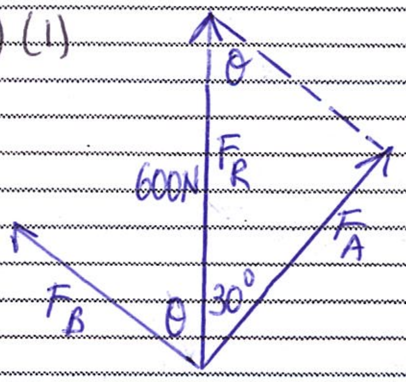
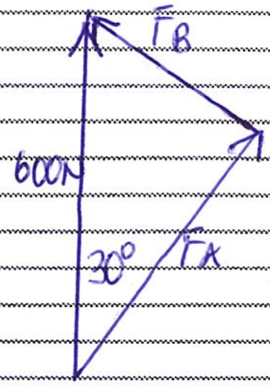
$= 4\pi\frac{2}{3}\left[(x+1)^{3/2}\right]_0^8$ 1m

$= \frac{8\pi}{3}(27-1)$

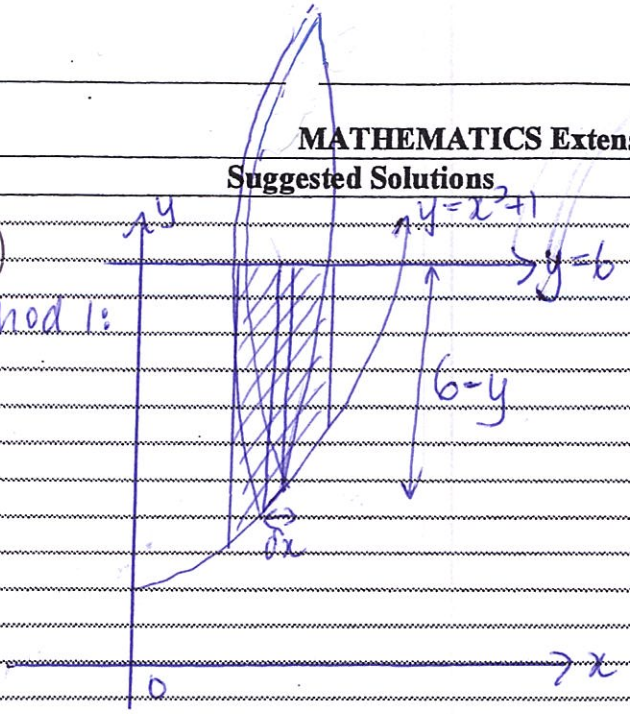
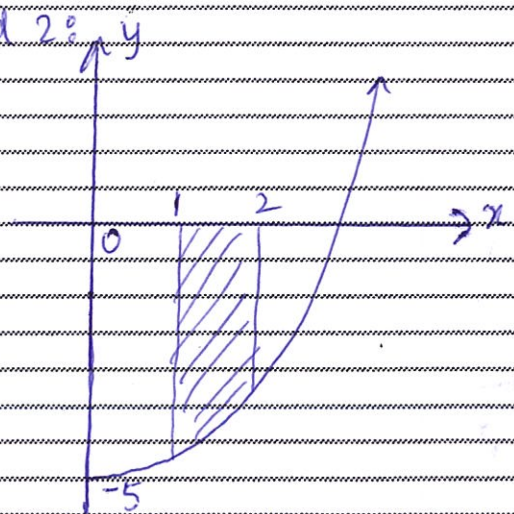
$= \frac{8\pi}{3} \times 26$

$= \frac{208\pi}{3}$ units² 1m

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
a) (i)  OR 	1	Diagrams were poorly drawn, most students did not show 600N along y-axis. Students need to understand that the resultant force is the sum of F_A and F_B
F_B is minimum when the angle between F_A and F_B is 90° as the shortest distance is the perpendicular distance. $\therefore \theta = 60°$	1	Students did not explain that the shortest distance is perpendicular distance.
(ii) $F_A = 600\cos 30°$ $= 600 \times \sqrt{3}/2$ $= 300\sqrt{3}$ N $F_B = 600 \sin 30°$ $= 300$N		
b) (i) Volume $= \pi \int_1^2 y^2\,dx$ $= \pi \int_1^2 x^4 + 2x^2 + 1\,dx$		Most errors were made in the substitution. Students who showed the substitution were most successful.
$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_1^2$	1	
$= \pi \left[\frac{32}{5} + \frac{16}{3} + 2 - \frac{1}{5} - \frac{2}{3} - 1 \right]$		
$= \frac{178\pi}{15}$ cubic units	1	

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
b) ii) Method 1: 	1	→ for diagram showing radius
Rather than rotate the curve about $y=0$ with radius of the disc being y, the radius of the disc is $6-y$.	1	→ for explanation
We then sum up the disc volumes to give $V=\pi\int_1^2 (6-y)^2\,dx = \pi\int_1^2 (5-x^2)^2\,dx$ $= \pi\int_1^2 x^4-10x^2+25\,dx$	1	→ Volume expression which follows from diagram and explanation
Method 2: 	1	→ diagram showing shift
Volume is equivalent to the volume of $y=x^2+1$ shifted down 6 units and rotated about $y=0$. ie. $y=(x^2+1)-6 = x^2-5$	1	→ explanation of shift, $y=x^2-5$ and $y=0$ as axis of rotation
$V=\pi\int_1^2 (x^2-5)^2\,dx = \pi\int_1^2 x^4-10x^2+25\,dx$	1	→ expression which follows from explanation