Student Name: ____________________

Maths class: ____________________

James Ruse Agricultural High School

2021 **YEAR 12 Task 3 Examination**

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 6–12, show relevant mathematical reasoning and/ or calculations

Total marks: 40

Section I – 5 marks (pages 2–3)

- Attempt Questions 1–5
- Allow about 7 minutes for this section

Section II – 35 marks (pages 4–7)

- Attempt Questions 6–12
- Allow about 53 minutes for this section

Section I

5 marks
Attempt Questions 1–5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5.

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1. P, Q and R are three collinear points with position vectors $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ respectively, where Q lies between P and Q. If $|\overrightarrow{QR}| = \frac{1}{2}|\overrightarrow{PQ}|$, then $\underset{\sim}{r}$ is equal to:

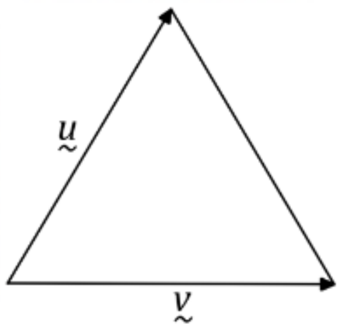
(A) $\frac{3}{2}\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p}$

(B) $\frac{3}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$

(C) $\frac{3}{2}\underset{\sim}{q} - \frac{3}{2}\underset{\sim}{p}$

(D) $\frac{1}{2}\underset{\sim}{p} - \frac{3}{2}\underset{\sim}{q}$

2. An equilateral triangle of side length 4 units is shown below.



Which of the following is the value of $\underset{\sim}{u} \cdot \underset{\sim}{v}$?

(A) 4

(B) $4\sqrt{3}$

(C) $8\sqrt{3}$

(D) 8

3. Which of the following is a well-defined expression which can be calculated involving dot-products for 2-dimensional vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$?

(A) $(\underset{\sim}{a} \cdot \underset{\sim}{b}) \cdot (\underset{\sim}{b} \cdot \underset{\sim}{a})$

(B) $(\underset{\sim}{a} \cdot \underset{\sim}{b})\underset{\sim}{c}$

(C) $\underset{\sim}{a} \cdot (\underset{\sim}{b}\underset{\sim}{c})$

(D) $(\underset{\sim}{a} \cdot \underset{\sim}{b}) \cdot \underset{\sim}{c}$

4. The coefficient of $\frac{1}{x}$ in the expansion of $(1+x)^n(1+\frac{1}{x})^n$ is

(A) $\frac{n!}{(n-1)!(n+1)!}$

(B) $\frac{(2n)!}{(n-1)!(n+1)!}$

(C) $\frac{(2n)!}{(2n-1)!(2n+1)!}$

(D) $\frac{(n+1)!}{2n!(n-1)!}$

5. Let $\underset{\sim}{p}$, $\underset{\sim}{q}$, $\underset{\sim}{r}$ and $\underset{\sim}{s}$ be four distinct vectors with $\underset{\sim}{r} \,||\, \underset{\sim}{p}$ and $\underset{\sim}{s} \,||\, \underset{\sim}{q}$. Which of the following is not an equivalent expression for the vector projection of $\underset{\sim}{p}$ onto $\underset{\sim}{q}$?

(A) $\left(\frac{\underset{\sim}{p}\cdot\underset{\sim}{q}}{|\underset{\sim}{q}|^2}\right)\underset{\sim}{q}$

(B) $\left(\frac{\underset{\sim}{p}\cdot\underset{\sim}{q}}{|\underset{\sim}{q}|}\right)\hat{\underset{\sim}{q}}$

(C) $\left(\frac{\underset{\sim}{p}\cdot\underset{\sim}{s}}{|\underset{\sim}{s}|^2}\right)\underset{\sim}{s}$

(D) $\left(\frac{\underset{\sim}{r}\cdot\underset{\sim}{s}}{|\underset{\sim}{s}|^2}\right)\underset{\sim}{s}$

Section II begins on the next page

Section II

50 marks
Attempt Questions 6-12
Allow about 53 minutes for this section

Start each question on a new page. Extra paper is available.

In Questions 6-10, your responses should include relevant mathematical reasoning and/ or calculations.

Question 6 (5 marks) **Start a new page**.

(a) The point A has position vector $5\underset{\sim}{i} - 4\underset{\sim}{j}$ and the point B has position vector $-5\underset{\sim}{i} - 2\underset{\sim}{j}$.

(i) Find the vector $\overrightarrow{AB}$ **1**

(ii) Find $\left|\overrightarrow{AB}\right|$. Give your answer as a simplified surd. **1**

(b) Let P be the point $(5, -8)$ in the cartesian plane.

(i) Draw the position vector $\overrightarrow{OP}$ **1**

(ii) Write $\overrightarrow{OP}$ as a column vector. **1**

(iii) Find a unit vector in the direction of $\overrightarrow{OP}$. **1**

Question 7 (5 marks) **Start a new page**.

(a) Find the angle between the vectors $3\underset{\sim}{i} + \underset{\sim}{j}$ and $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. **2**

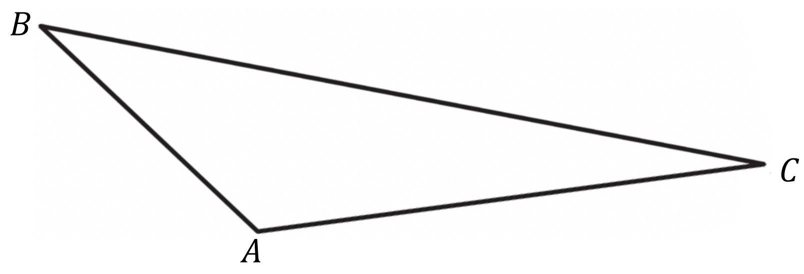
(b) Show that the vectors $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$ are perpendicular. **1**

(c) Find a vector of magnitude 15 in the direction of the vector $5\underset{\sim}{i} + 12\underset{\sim}{j}$. **2**

Please turn over

Question 8 (5 marks) **Start a new page**.

(a) In the triangle ABC below, $\overrightarrow{AB} = -2\underset{\sim}{i} + 7\underset{\sim}{j}$ and $\overrightarrow{AC} = 12\underset{\sim}{i} - \underset{\sim}{j}$. **2**

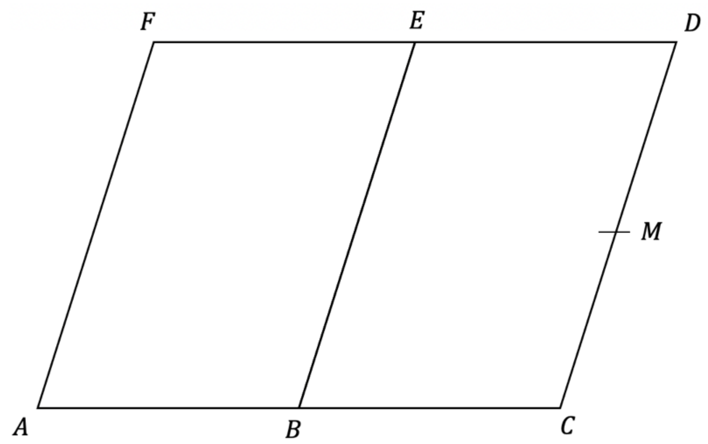


If angle BAC is 101.1°, find the area of triangle ABC?

(b) The coefficients of the $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal. Find the value of r. **3**

Question 9 (5 marks) **Start a new page**.

(a) The diagram below shows two identical parallelograms, $AFEB$ and $BEDC$. M is the midpoint of CD.



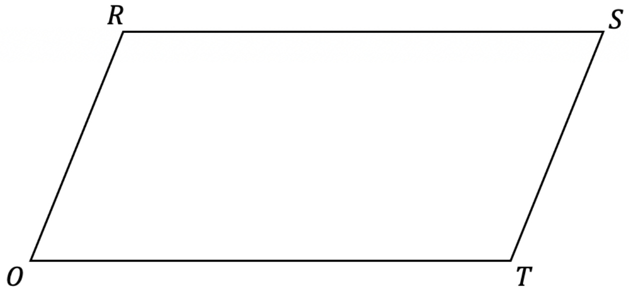
Let the vector $\overrightarrow{AF} = \underset{\sim}{q}$ and $\overrightarrow{AB} = \underset{\sim}{p}$.

(i) Use vector methods to show that BM is parallel to AD. **2**

(ii) Write $\overrightarrow{BM}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$. **1**

Question 9 continues on the next page

(b) In parallelogram $ORST$, it is known that $\overrightarrow{OR} = \underset{\sim}{a} + 2\underset{\sim}{b}$ and that $\overrightarrow{OT} = \underset{\sim}{a} - 2\underset{\sim}{b}$, where $\underset{\sim}{a}$ is the position vector of the point A and $\underset{\sim}{b}$ is the position vector of the point B. **2**



Copy the diagram onto your answer sheet and show the positions of the points A and B.

Question 10 (5 marks) **Start a new page**.

Let X be a continuous random variable and $\lambda > 0$ be a positive real number. X is said to have a Sine Square distribution, if the probability density function of X is expressed as:

$$f_X(x) = \begin{cases} \dfrac{2}{\lambda\pi}\sin^2\dfrac{x}{2\lambda} & \text{for } 0 < x < \lambda\pi, \text{ and } \lambda > 0; \\ 0 & \text{otherwise} \end{cases}$$

Note that $f_X(x) \geq 0$ for all values of X in the domain $(0, \lambda\pi)$.

(a) Find the cumulative distribution function $F_X(x)$ of the Sine Square distribution. **4**

(b) Hence, or otherwise, find $P(X \leq \pi)$ when $\lambda = 2$. Give your answer to 3 decimal places. **1**

Please turn over

Question 11 (5 marks) **Start a new page**.

(a) Find λ when the magnitude of the vector projection of $\underset{\sim}{a} = \lambda\underset{\sim}{i} + \underset{\sim}{j}$ onto $\underset{\sim}{b} = 8\underset{\sim}{i} + 6\underset{\sim}{j}$ equals 4. **2**

(b) By considering the expansion of $(1 + x)^n$, show that **3**

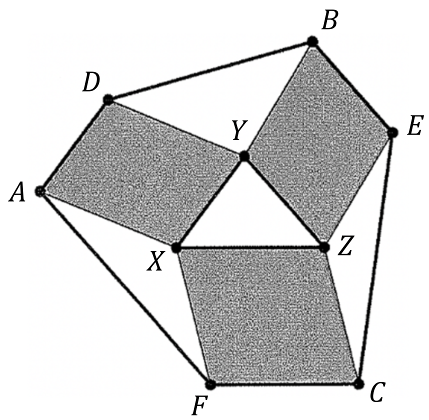
$$2\binom{n}{2} + 6\binom{n}{3} + 12\binom{n}{4} + \cdots + n(n-1)\binom{n}{n} = n(n-1)2^{n-2}$$

Question 12 (5 marks) **Start a new page**.

(a) Suppose that A, B, C, D, E, F are any points in the plane. Use vector arithmetic to verify that **2**

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \overrightarrow{BD} + \overrightarrow{CE} + \overrightarrow{AF}$$

(b) Suppose now that A, B, C, D, E, F are arranged to form the vertices of a hexagon surrounding a triangle XYZ, as shown in the diagram below. It is given that the shaded regions are parallelograms. **3**



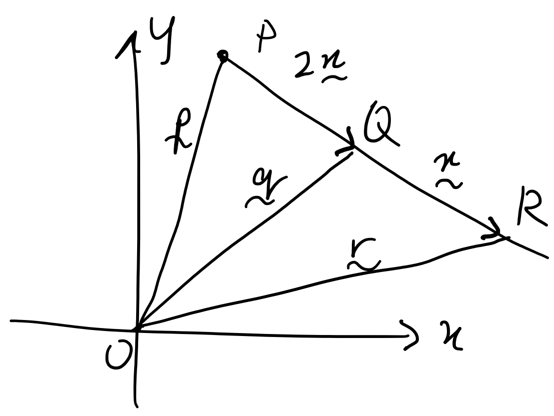
Show that the line segments DB, EC and FA can be moved parallel to themselves to assemble a triangle.

END OF EXAMINATION

Multiple choice

Sunday, 6 June 2021 9:58 pm

1.



$$\underset{\sim}{r} = \vec{OR} = \vec{OP} + \vec{PQ} + \vec{QR}$$
$$= \underset{\sim}{p} + \vec{PO} + \vec{OQ} + \vec{QR}$$
$$= \underset{\sim}{p} + -\underset{\sim}{p} + \underset{\sim}{q} + \tfrac{1}{2}(-\underset{\sim}{p}+\underset{\sim}{q})$$
$$= -\tfrac{1}{2}\underset{\sim}{p} + \tfrac{3}{2}\underset{\sim}{q}$$

A

2. $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$
$$= 4 \times 4 \times \cos 60^\circ$$
$$= 16 \times \tfrac{1}{2}$$
$$= 8$$

D

3. ~~A~~

B ✓

~~C~~

~~D~~

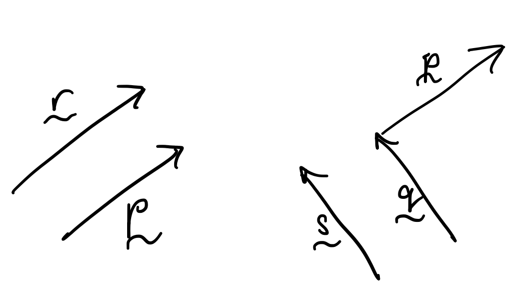
4.

$$(1+x)^n\left(1+\frac{1}{x}\right)^n = \frac{(1+x)^n(1+x)^n}{x^n}$$
$$= \frac{(1+x)^{2n}}{x^n}$$

$\therefore$ coeff of $\frac{1}{x}$ in $(1+x)^n\left(1+\frac{1}{x}\right)^n$ = coeff of x^{n-1} in $(1+x)^{2n}$
$$= \binom{2n}{n-1} = \frac{(2n)!}{(n-1)!(n+1)!}$$

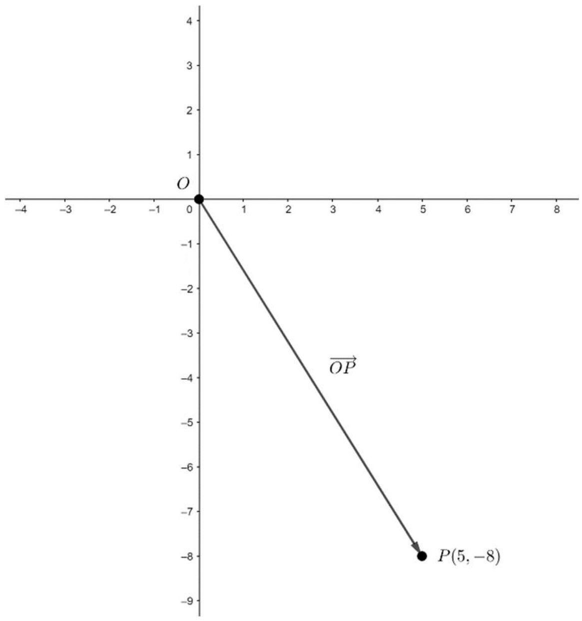
B

5.



$$\frac{\underset{\sim}{p} \cdot \underset{\sim}{q}}{|\underset{\sim}{q}|^2}\underset{\sim}{q}$$

D

Year 12 Extension 1: Q6		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(a) Subtracting $\overrightarrow{OB}$ from $\overrightarrow{OA}$: (i) $\overrightarrow{AB} = (-5-5)\underset{\sim}{i} + \left(-2-(-4)\right)\underset{\sim}{j} = -10\,\underset{\sim}{i} + 2\underset{\sim}{j}$	1	Handled well.
(ii) $\lvert\overrightarrow{AB}\rvert = \sqrt{(-10)^2 + 2^2}$ $= \sqrt{104}$ $= 2\sqrt{26}$	1	Handled well.
(b) (i) 	1	Some students failed to give reference/place the vector in the Cartesian plane.
(ii) $\overrightarrow{OP} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$	1	Handled well, except for notation. Vectors are not points, so rounded brackets shouldn't be used. For example, $(5, -8)$ is notation for a point. Why is this important? Because in the context of vectors, points AND vectors are often mixing together as part of a problem. The two must be separated in notation to avoid confusion.
(iii) Multiply by the reciprocal of the magnitude of $\overrightarrow{OP}$ to obtain a vector in the direction of $\overrightarrow{OP}$ with magnitude 1: $\lvert\overrightarrow{OP}\rvert = \sqrt{5^2 + (-8)^2} = \sqrt{25+64} = \sqrt{89}$ Hence the required unit vector is $\frac{1}{\sqrt{89}}\begin{bmatrix} 5 \\ -8 \end{bmatrix}$	1	

Question 7 (5 marks)

a) Find the angle between the vectors $3\underset{\sim}{i}+\underset{\sim}{j}$ and $\binom{2}{-1}$. (2 marks)

b) Show that the vector $\binom{1}{4}$ and $\binom{8}{-2}$ are perpendicular. (1 mark)

c) Find a vector of magnitude 15 in the direction of the vector $5\underset{\sim}{i} +12\underset{\sim}{j}$. (2 marks)

Part (a) Solution

Find the angle between the vectors $3\underset{\sim}{i}+\underset{\sim}{j}$ and $\binom{2}{-1}$.

$$\begin{pmatrix}3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\-1\end{pmatrix} = \left|\begin{pmatrix}3\\1\end{pmatrix}\right|\left|\begin{pmatrix}2\\-1\end{pmatrix}\right|\cos\theta$$

(1 mark)

$$5 = \sqrt{10}\times\sqrt{5}\cos\theta$$

$$\cos\theta = \frac{\sqrt{5}}{\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$$\theta = 45°$$

(1 mark)

Part (b) Solution

Show that the vector $\binom{1}{4}$ and $\binom{8}{-2}$ are perpendicular.

$$\begin{pmatrix}1\\4\end{pmatrix}\cdot\begin{pmatrix}8\\-2\end{pmatrix} = 8-8$$

$$= 0$$

Since the dot product is 0, the vectors are perpendicular. (1 mark)

Part (c) Solution

Find a vector of magnitude 15 in the direction of the vector $5\underset{\sim}{i} + 12\underset{\sim}{j}$.

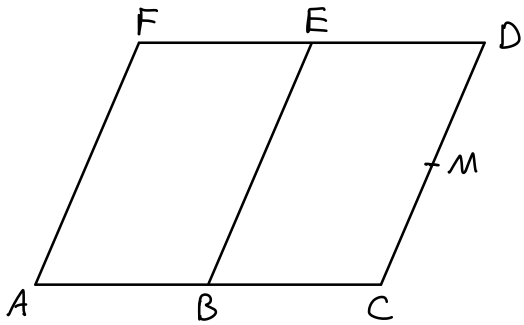
$\lvert 5\underset{\sim}{i}+12\underset{\sim}{j} \rvert$	=	13	(1 mark)
The vector of magnitude 15 and in the direction of vector $5\underset{\sim}{\hat{i}}+12\underset{\sim}{\hat{j}}$	=	$\frac{15}{13}(5\underset{\sim}{i}+12\underset{\sim}{j})$	(1 mark)

MATHEMATICS Extension 1 : Question 8

Suggested Solutions	Marks	Marker's Comments
a) $\lvert\vec{AB}\rvert = \sqrt{(-2)^2+(7)^2} = \sqrt{53}$		
$\lvert\vec{AC}\rvert = \sqrt{12^2+(-1)^2} = \sqrt{145}$	1	
$\therefore \text{Area} = \dfrac{\sqrt{53}\times\sqrt{145}\times\sin 101.1^\circ}{2}$		
$= 43.01\,u^2$	1	
b) $(1+x)^{34} = {}^{34}C_0 + {}^{34}C_1 x + \dots {}^{34}C_r x^r + \dots {}^{34}C_{34} x^{34}$	1	
1st term 2nd term		Note: If your answer is $\frac{40}{3}$, you will receive 1/3 MAX!
$\therefore$ Coefficient of $(r-5)^{th}$ term $= {}^{34}C_{r-6}$		
Coefficient of $(2r-1)^{th}$ term $= {}^{34}C_{2r-2}$	1	
$\therefore r-6 = 2r-2$ or $(r-6)+(2r-2)=34$		- Getting $r=14$ is only worth 1 mark on its own without appropriate working
$r=-4$ $3r-8=34$		
but $r \geq 0$. $3r=42$		
$r=14$	1	
$\therefore r=14$ only.		- You will also not receive full marks if your final step was trial and error but you did not sub the value to check that it is correct.

Question 9:

ai,



$\vec{AF} = \underset{\sim}{q}$

$\vec{AB} = \underset{\sim}{p}$

RTP: $AD \parallel BM$

$$\vec{AD} = \vec{AC} + \vec{CD}$$

$$= 2\vec{AB} + \vec{CD}$$

$$\vec{AD} = 2\underset{\sim}{p} + \underset{\sim}{q} \qquad \text{——— } 1$$

$$\vec{BM} = \vec{BC} + \vec{CM}$$

$$\vec{BM} = \underset{\sim}{p} + \tfrac{1}{2}\underset{\sim}{q}$$

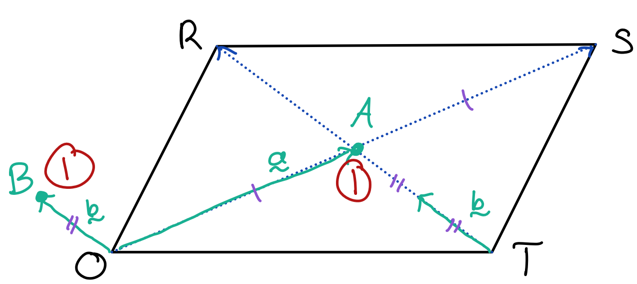
$$= \tfrac{1}{2}(2\underset{\sim}{p} + \underset{\sim}{q})$$

1 $\Big\{$ $\vec{BM} = \frac{1}{2}\vec{AD}$

$\because \vec{BM}$ is a scalar multiple of $\vec{AD}$, then $BM \parallel AD$.

ii, From above, $\vec{BM} = \underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$ ——— 1

b,



$\vec{OR} = \underset{\sim}{a} + 2\underset{\sim}{b}$ ①

$\vec{OT} = \underset{\sim}{a} - 2\underset{\sim}{b}$ ②

①+②: $2\underset{\sim}{a} = \vec{OR} + \vec{OT}$

$\underset{\sim}{a} = \frac{1}{2}(\vec{OR} + \vec{OT})$

①−②: $4\underset{\sim}{b} = \vec{OR} - \vec{OT}$

$\underset{\sim}{b} = \frac{1}{4}(\vec{OR} - \vec{OT})$

MATHEMATICS Extension 1 : Question 10

Suggested Solutions	Marks	Marker's Comments
a) $\int_0^x \frac{2}{\lambda\pi}\sin^2\frac{x}{2\lambda}\,dx$	1	setting up the integral with boundaries. If not using boundaries, first mark will go towards working out the + c.
$= \frac{2}{\lambda\pi}\int_0^x \frac{1}{2}\left(1-\cos\frac{x}{\lambda}\right)dx$	1	
$= \frac{1}{\lambda\pi}\int_0^x \left(1-\cos\frac{x}{\lambda}\right)dx$		
$= \frac{1}{\lambda\pi}\left[x-\lambda\sin\frac{x}{\lambda}\right]_0^x$	1	
$= \frac{1}{\lambda\pi}\left[x-\lambda\sin\frac{x}{\lambda}\right] - 0$		
$\therefore\ F_x(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{\lambda\pi}\left(x-\lambda\sin\frac{x}{\lambda}\right), & 0 < x < \lambda\pi \\ 1, & x \geq \lambda\pi \end{cases}$	1	must show all three parts. Not many students did this.
b) when $\lambda = 2$, $x = \pi$		
$P(X \le \pi) = \frac{1}{2\pi}\left(\pi - 2\sin\frac{\pi}{2}\right)$		$\frac{1}{2}-\frac{1}{\pi}$ was also accepted. The sin value must be evaluated. A lot of students used degrees rathar than radians!
$= \frac{\pi}{2\pi} - \frac{2}{2\pi}$		
$= \frac{1}{2} - \frac{1}{\pi}$		
$= 0.182$ (3dp)	1	

Question 11

Sunday, 8 August 2021 10:15 pm

(a) Find λ when the magnitude of the vector projection of $\underset{\sim}{a} = \lambda\underset{\sim}{i} + \underset{\sim}{j}$ onto $\underset{\sim}{b} = 8\underset{\sim}{i} + 6\underset{\sim}{j}$ equals 4. **2**

Solution:

$$\left|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}\right| = \left|\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{b}|}\right|$$

$$= \left|\frac{\begin{pmatrix}\lambda\\1\end{pmatrix}\cdot\begin{pmatrix}8\\6\end{pmatrix}}{\sqrt{8^2+6^2}}\right|$$

$$= \left|\frac{8\lambda+6}{10}\right| = 4$$

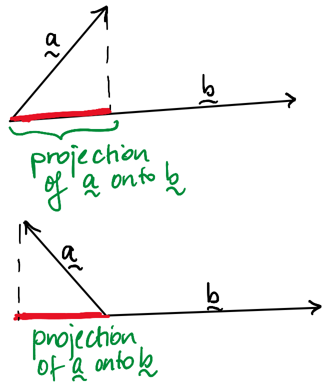
$8\lambda+6=40$ or $8\lambda+6=-40$

$8\lambda = 34$ $\qquad 8\lambda = -46$

$\lambda = 4.5$ or -5.75

1 $\qquad$ 1

Many student did not consider the magnitude of the vector projection. They omitted the absolute value and so only found one value of λ.



1 mark was awarded for only one value of λ.
2 marks were awarded for two correct values of λ.

(b) By considering the expansion of $(1+x)^n$, show that **3**

$$2\binom{n}{2} + 6\binom{n}{3} + 12\binom{n}{4} + \cdots + n(n-1)\binom{n}{n} = n(n-1)2^{n-2}$$

Solution:

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \binom{n}{4}x^4 + \ldots + \binom{n}{n}x^n$$

Differentiating both sides:

$$n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + 4\binom{n}{4}x^3 + \cdots + n\binom{n}{n}x^{n-1}$$

Differentiating both sides again:

$$n(n-1)(1+x)^{n-2} = 2\binom{n}{2} + 6\binom{n}{3}x + 12\binom{n}{4}x^2 + \ldots + n(n-1)\binom{n}{n}x^{n-2}$$

Substituting $x=1$ in both sides

$$n(n-1)2^{n-2} = 2\binom{n}{2} + 6\binom{n}{3} + 12\binom{n}{4} + \cdots + n(n-1)\binom{n}{n}$$

1st mark was awarded for differentiating both sides once with explanation
2nd mark was awarded for differentiating both sides again with explanation
3rd mark was awarded for substitution of x = 1 into the expression with explanation

Notes:

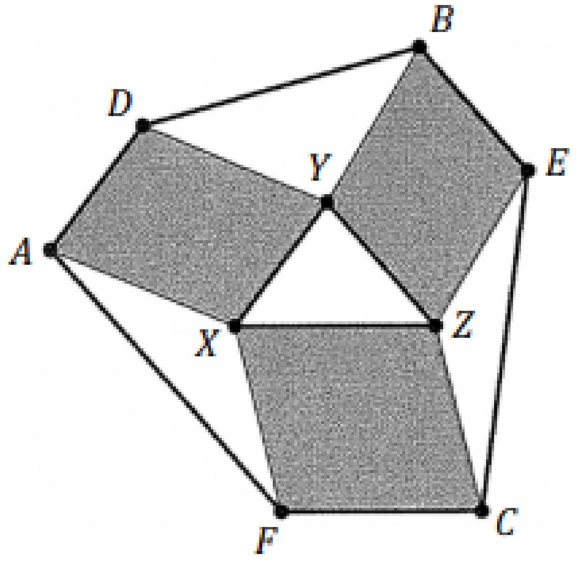
- Many students did not write out enough terms in their initial statement so were unable to show the final statement given in the question.
- Students wrote the extra terms in subsequent lines of working without showing where they initially came from.
- **Students who did not show enough terms to arrive at the given expression were awarded maximum 2 marks**
- Many students wrote lines of working without explaining how they arrived at each line.
- Students who did not explain they were differentiating each line were awarded only the mark for substitution.
- Many students who used unacceptable notation and explanations were NOT awarded marks. These included: LHS', RHS'', $\frac{d(1)}{dx}$, $((1+x)^n)'$, "diff"

Question 12 (5 marks) **Start a new page.**

(a) Suppose that A, B, C, D, E, F are any points in the plane. Use vector arithmetic to verify that **2**

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \overrightarrow{BD} + \overrightarrow{CE} + \overrightarrow{AF}$$

(b) Suppose now that A, B, C, D, E, F are arranged to form the vertices of a hexagon surrounding a triangle XYZ, as shown in the diagram below. It is given that the shaded regions are parallelograms. **3**



Show that the line segments DB, EC and FA can be moved parallel to themselves to assemble a triangle.

(a) Suppose that A, B, C, D, E, F are any points in the plane. Use vector arithmetic to verify that **2**

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \overrightarrow{BD} + \overrightarrow{CE} + \overrightarrow{AF}$$

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}$$

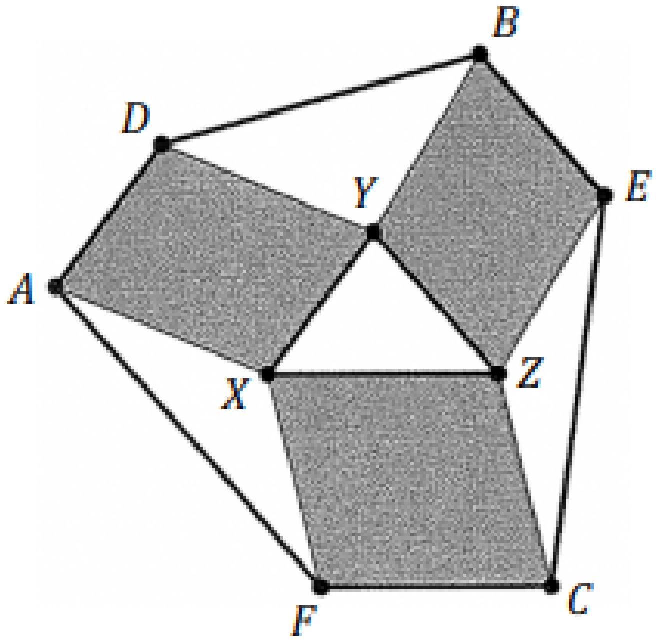
$$= \overrightarrow{AF} + \overrightarrow{FD} + \overrightarrow{BD} + \overrightarrow{DE} + \overrightarrow{CE} + \overrightarrow{EF} \qquad 1$$

$$= \overrightarrow{AF} + \overrightarrow{BD} + \overrightarrow{CE} + \overrightarrow{EF} + \overrightarrow{FD} + \overrightarrow{DE}$$

$$= \overrightarrow{AF} + \overrightarrow{BD} + \overrightarrow{CE} + \overrightarrow{EE} \qquad 1$$

$$= \overrightarrow{AF} + \overrightarrow{BD} + \overrightarrow{CE}$$

(b) Suppose now that A, B, C, D, E, F are arranged to form the vertices of a hexagon surrounding a triangle XYZ, as shown in the diagram below. It is given that the shaded regions are parallelograms. **3**



Show that the line segments DB, EC and FA can be moved parallel to themselves to assemble a triangle.

$$\overrightarrow{BD}+\overrightarrow{AF}+\overrightarrow{CE}$$

$$=\overrightarrow{BY}+\overrightarrow{YD}+\overrightarrow{AX}+\overrightarrow{XF}+\overrightarrow{CZ}+\overrightarrow{ZE} \qquad 1$$

$\overrightarrow{BY}=\overrightarrow{EZ}$ Since BYZE is a parallelogram, and opposite sides are parallel and equal.

$\overrightarrow{YD}=\overrightarrow{XA}$ Since ADYX is a parallelogram, and opposite sides are parallel and equal 1

$\overrightarrow{XF}=\overrightarrow{ZC}$ Since XFCZ is a parallelogram, and opposite sides are parallel and equal

Now using The above informations, we obtain:

$$\overrightarrow{BY}+\overrightarrow{YD}+\overrightarrow{AX}+\overrightarrow{XF}+\overrightarrow{CZ}+\overrightarrow{ZE}$$

$$=\overrightarrow{BY}+\overrightarrow{YD}+\overrightarrow{XF}-\overrightarrow{YD}-\overrightarrow{XF}-\overrightarrow{BY}$$

$$=\overrightarrow{BY}-\overrightarrow{BY}+\overrightarrow{YD}-\overrightarrow{YD}+\overrightarrow{XF}-\overrightarrow{XF}$$

$$=\vec{0} \qquad 1$$

Therefore the three vectors $\overrightarrow{DB}$, $\overrightarrow{EC}$ and $\overrightarrow{FA}$ are the sides of a triangle.