Name:____________________

Class __________________

HSC Assessment 1 Term 4 2022

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 65 minutes
- Board approved calculators may be used.
- Write using black or blue pen
- All necessary working should be shown in all questions
- Write your student number at the top of every page of your answer sheets.
- NESA Reference Sheet is provided

Total marks – 40

Attempt all questions

Section A – Answer on the Multiple Choice Answer Sheet

Section B - Start each question on a new sheet of paper.

NESA Reference Sheet is provided

This paper MUST NOT be removed from the examination room

Section A Multiple Choice (5 Marks)

QUESTION 1

What is the remainder when $x^3 - 4x^2 + x + 1$ is divided by $2x + 1$?

A. $\frac{5}{8}$

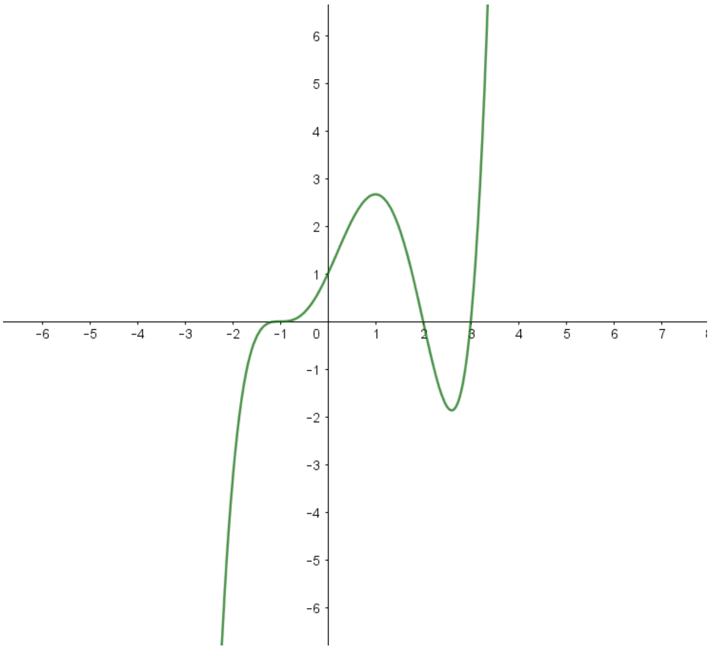
B. $-\frac{5}{8}$

C. -5

D. -25

QUESTION 2

Which of the following options best represents the graph shown below?



A. $y = (x + 1)^3(2 - x)(x - 3)$

B. $y = (x + 1)^3(2 - x)(3 - x)$

C. $y = \frac{1}{6}(x + 1)^3(2 - x)(x - 3)$

D. $y = \frac{1}{6}(x + 1)^3(2 - x)(3 - x)$

QUESTION 3

The coefficient of x^2 in the expansion of $(3-5x)^7$ is

A. 6075

B. -6075

C. 127575

D. -127575

QUESTION 4

Which of the following step is not necessary to prove a statement to be true for all positive integer values n by mathematical induction?

A. Prove that the statement is true for $n = 1$.

B. Prove that if the statement is true for some case $n = k$, then the statement is also true for $n = k + 1$.

C. Check that the statement is true for a very large value of n.

D. None of the above.

QUESTION 5

Which of the following is the largest domain for x such that the equation $\cot\left(\frac{x}{2}\right)\sin x = 0$ has two solutions?

A. $x \in [0, \pi]$

B. $x \in [-\pi, \pi]$

C. $x \in (-3\pi, 3\pi)$

D. $x \in [0, 2\pi)$

SECTION B

QUESTION 6 Start a new page (5 marks) **Marks**

a) Expand $\left(\frac{2}{x} - 3x\right)^4$, simplify fully. **2**

b) Prove by mathematical induction that $7^{2n} + 7^n + 4$ is divisible by 6 for all positive integers n. **3**

QUESTION 7 Start a new page (5 marks) **Marks**

a) The polynomial $P(x) = ax^4 + (a-b)x^3 - 2bx^2 + 12x + 8$ has a zero of multiplicity 2 at $x = 2$.

i. By finding the values of a and b, show that remaining zeroes are -2 and $-\frac{1}{2}$. **3**

ii. Sketch the graph $y = P(x)$.

DO NOT FIND THE COORDINATES OF TURNING POINTS. **2**

QUESTION 8 Start a new page (5 marks) **Marks**

a) In the polynomial $P(x) = (k^2 - 1)x^4 + 4x^3 - k^2x^2 + x + 3 - 3k$, solve for k such that the leading coefficient is equal to the constant term. **2**

b) Find the coefficient of x^4 in the expansion of $\left(1 - \frac{2}{x}\right)^4 (x+3)^5$. **3**

QUESTION 9 Start a new page (5 marks) **Marks**

a)

i. Express $\cos 2\theta - \sqrt{3}\sin 2\theta$ in the form $R\cos(2\theta + \alpha)$ where $R > 0$ and $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Show all necessary working. **2**

ii. Hence solve for θ in the equation $\cos 2\theta = \sqrt{3}\sin 2\theta + 1$, where $\theta \in [-\pi, \pi]$. **2**

iii. For what value(s) of c will the equation $\cos 2\theta = \sqrt{3}\sin 2\theta + c$ have exactly 4 solutions when $\theta \in [-\pi, \pi]$? **1**

QUESTION 10 Start a new page (4 marks) **Marks**

a)

i. Show that $4k^3 + 12k^2 + 11k + 3 = (k+1)(4(k+1)^2 - 1)$. **1**

ii. Hence, prove by mathematical induction that **3**

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(4n^2-1)}{3}$$

for all positive integers n.

QUESTION 11 Start a new page (6 marks) **Marks**

a) Prove that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ by considering two different ways of selecting k objects from a group of n. **3**

DO NOT PROVE THIS ALGEBRAICALLY, THIS PROOF IS TO BE COMBINATORIAL.

b)

i. By making the substitution $t = \tan\frac{\theta}{2}$, show that $\sqrt{\frac{1+\cos\theta}{1-cos\theta}} = \cot\frac{\theta}{2}$ for $\theta \in (0, \pi)$. **2**

ii. Hence or otherwise, find the exact value of $\cot\frac{\pi}{8}$ in the form $\sqrt{\frac{\sqrt{a}+b}{\sqrt{c}-d}}$. **1**

QUESTION 12 Start a new page (5 marks) **Marks**

a) $P(x)$ is a polynomial of degree 4. It is known that $P(x) \geq x$ for $x \in \mathbb{R}$ and that **2**

$$P(1) = 1$$

$$P(2) = 4$$

$$P(3) = 3$$

Find $P(4)$.

b) Solve: **3**

$$\sum_{r=0}^{2n} \binom{2n}{r} \cos^{4n-2r}\theta \sin^{2r}\theta = 16^n \cos^{4n}\theta$$

for $\theta \in [0, 2\pi]$.

End of examination

MATHEMATICS EXT 1: Question 6

Suggested Solutions	Marks	Marker's Comments
a, $\left(\frac{2}{x}-3x\right)^4$		
$=\left(\frac{2}{x}\right)^4-4\left(\frac{2}{x}\right)^3(3x)+6\left(\frac{2}{x}\right)^2(3x)^2-4\left(\frac{2}{x}\right)(3x)^3+(3x)^4$	1	
$=\frac{16}{x^4}-\frac{96}{x^2}+216-216x^2+81x^4$	1	
b, $n=1 \rightarrow 7^{2\times1}+7^1+4 = 49+7+4$		
$=60$		
$=6\times10$		
which is div. by 6		
∴ true for $n=1$	1	Base Case
Assume that for some $k\geq1$,		
$7^{2k}+7^k+4=6M$ where $M\in\mathbb{Z}$		
$\Leftrightarrow 7^{2k}=6M-4-7^k$, $M\in\mathbb{Z}$ (*)		
Now, $7^{2(k+1)}+7^{k+1}+4$		
$=7^{2k}\cdot7^2+7^k\cdot7+4$		
$=(6M-4-7^k)\cdot7^2+7^k\cdot7+4$, by (*)	1	Correct use of assumption (reference assumption)
$=49(6M)-4(49)-49(7^k)+7(7^k)+4$		
$=6(49M)-4\cdot42-48\cdot7^k$		
$=6(49M)-4(6\times7)-(6\times8)7^k$		
$=6[49M-28+8\cdot7^k]$ (*)		
$=6N$, where $N=49M-28+8\cdot7^k$ is an integer since $\mathbb{Z}$ is closed under addition, subtraction and multiplication.		Not marked down here, but this should be included.
∴ $7^{2(k+1)}+7^{k+1}+4$ is div. by 6 if $7^{2k}+7^k+4$ is div. by 6		
∴ Statement is true for integers $n\geq1$ by Mathematical Induction.	1	Correctly completing proof.

Question 7

a)

i. $P(x) = ax^4 + (a-b)x^3 - 2bx^2 + 12x + 8$

$P'(x) = 4ax^3 + 3(a-b)x^2 - 4bx + 12$ 1 mark for differentiating

Zero of multiplicity 2 at $x = 2$.

$P(2) = 0 \rightarrow 16a + 8(a-b) - 8b + 24 + 8 = 0$

$24a - 16b = -32$

$3a - 2b = -4 - \textit{Equation 1}$

$P'(2) = 0 \rightarrow 32a + 12(a-b) - 8b + 12 = 0$

$44a - 20b = -12$

$11a - 5b = -3 - \textit{Equation 2}$

$(5 \times \textit{Equation 1}) - (2 \times \textit{Equation 2}) \rightarrow 7a = 14$

$a = 2$

$\textit{Sub } a = 2 \textit{ into Equation } 1 \rightarrow 6 - 2b = -4$

$b = 5$ 1 mark for solving for a and b

$\therefore P(x) = 2x^4 - 3x^3 - 10x^2 + 12x + 8$

$P(-2) = 2(-2)^4 - 3(-2)^3 - 10(-2)^2 + 12(-2) + 8$

$= 32 + 24 - 40 - 24 + 8$

$= 0$

$$P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^4 - 3\left(-\frac{1}{2}\right)^3 - 10\left(-\frac{1}{2}\right)^2 + 12\left(-\frac{1}{2}\right) + 8$$

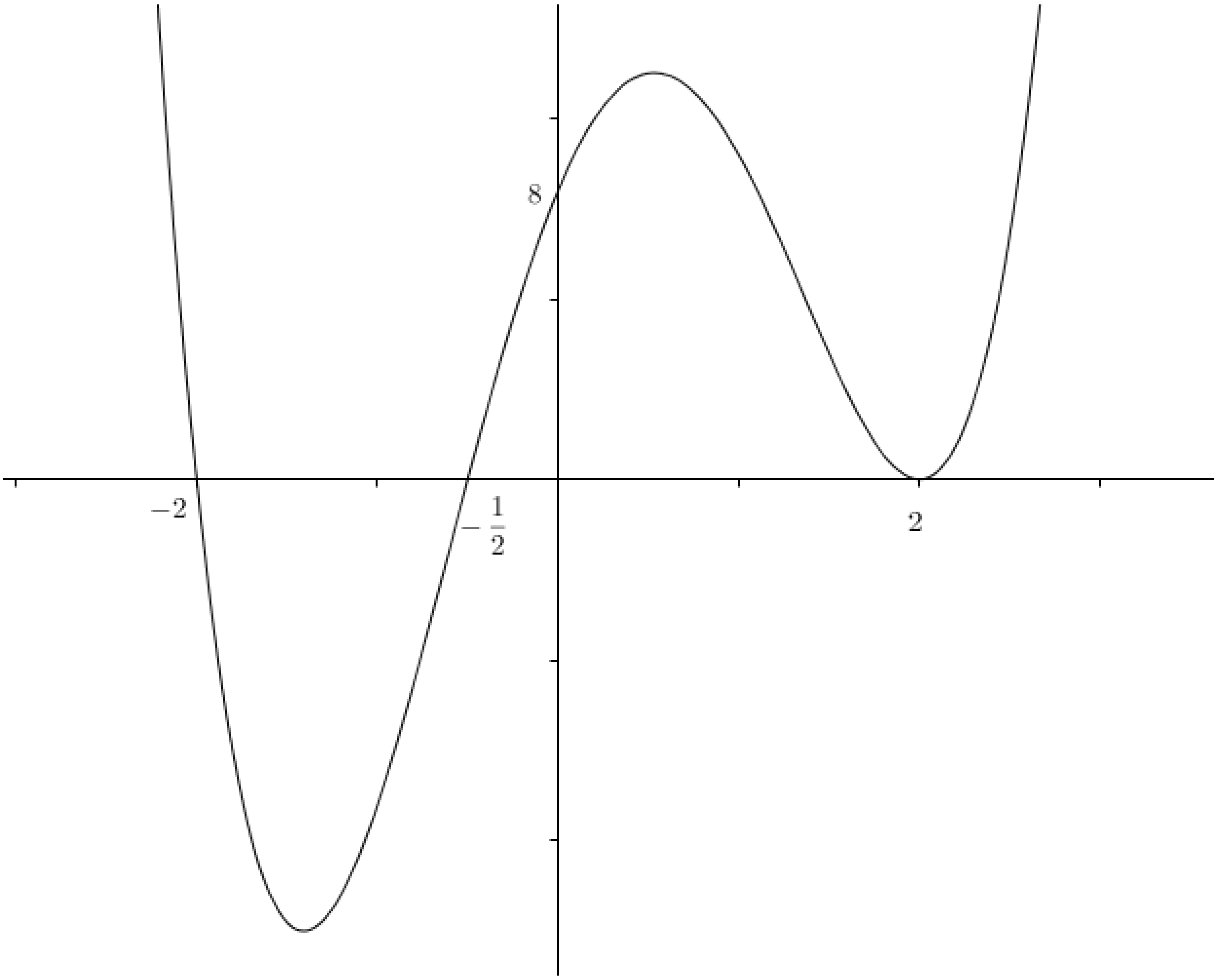
$$= \frac{1}{8} + \frac{3}{8} - \frac{10}{4} - 6 + 8$$

$$= 0$$

$\therefore x = -2$ and $x = -\frac{1}{2}$ are zeroes. 1 mark for answer showing all steps

Note: **If you chose to solve for the remaining zeroes by using sums and products of roots then, then you must solve for BOTH α and β respectively to get -2 and $-\frac{1}{2}$, and not just conclude your proof complete just because you get two possible values for α.**

ii.



1 mark for correct shape

1 mark for all other details such as intercepts and scale

(However you can’t get a mark for just drawing **<u>any</u>** graph with the correct intercepts)

8. a)

$(k^2-1) = 3-3k$

$k^2+3k-4=0$

$(k+4)(k-1)=0$

$k=-4$ or 1

but if $k=1$, then the leading term is $4x^3$.

$\therefore k \neq 1$, $\therefore k=-4$ only

b) $(1-2x^{-1})^4(x+3)^5 = \sum_{r=0}^{4}\binom{4}{r}(1)^r(-2x^{-1})^{4-r} \cdot \sum_{k=0}^{5}\binom{5}{k}x^k(3)^{5-k}$

$\therefore$ general term is $\binom{4}{r}\binom{5}{k}(-2)^{4-r}(3)^{5-k}x^{r-4+k}$

term for $x^4 \Rightarrow r-4+k=4$

$r+k=8$

$\therefore r=3, k=5$

$r=4, k=4$

$\therefore$ coefficient is given by $\left[\binom{4}{3}\binom{5}{5}(-2)^1(3)^0 + \binom{4}{4}\binom{5}{4}(-2)^0(3)^1\right]$

$= -8+15$

$= 7$

Question 9

i) $Let\ R\cos(2\theta+\alpha)=\cos 2\theta-\sqrt{3}\sin 2\theta$

$\therefore R\cos 2\theta\cos\alpha - R\sin 2\theta\sin\alpha = \cos 2\theta - \sqrt{3}\sin 2\theta$

$\therefore R\cos\alpha = 1 \quad - Equation\ 1$

$R\sin\alpha = \sqrt{3} - Equation\ 2$

$Since\ R>0,$

$\therefore \alpha \in \left(0,\frac{\pi}{2}\right)$

$Equation\ 2 \div Equation\ 1 \rightarrow \tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3},\left(\alpha\in\left(0,\frac{\pi}{2}\right)\right)$

$Sub\ \alpha=\frac{\pi}{3}\ into\ Equation\ 1:$

$R\cos\frac{\pi}{3}=1$

$R = 1\div\frac{1}{2}$

$= 2$

$\therefore \cos 2\theta - \sqrt{3}\sin 2\theta = 2\cos\left(2\theta+\frac{\pi}{3}\right)$

(1st mark for solving one of R or α, 2nd mark for the other variable AND answering the question)

ii) $\cos 2\theta = \sqrt{3}\sin 2\theta + 1$

$\cos 2\theta - \sqrt{3}\sin 2\theta = 1$

$2\cos\left(2\theta+\frac{\pi}{3}\right)=1$

$\cos\left(2\theta+\frac{\pi}{3}\right)=\frac{1}{2}$

$2\theta+\frac{\pi}{3}=\pm\frac{\pi}{3},\pm\frac{5\pi}{3},\pm\frac{7\pi}{3}\ldots$

$2\theta = 0,-\frac{2\pi}{3},\frac{4\pi}{3},-2\pi,2\pi$ $\qquad If\ \theta\in[-\pi,\pi], then\ 2\theta\in[-2\pi,2\pi]$

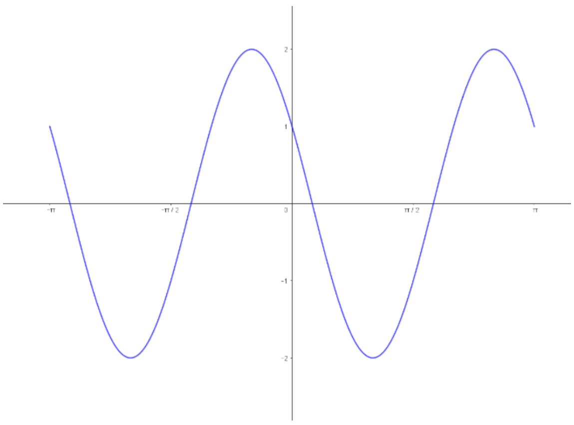
$\theta = 0,-\frac{\pi}{3},\frac{2\pi}{3},-\pi,\pi$

(1st mark for solving some correct values of θ , 2nd mark for all correct solutions of θ)

iii) $\cos 2\theta = \sqrt{3}\sin 2\theta + c$

$\cos 2\theta - \sqrt{3}\sin 2\theta = c$

$2\cos\left(2\theta+\frac{\pi}{3}\right)=c$



As the period of the function is π, there will always be 4 solutions when $x\in[-\pi,\pi]$ except for when

$c=\pm 2, 1.$

$\therefore c\in(-2,1)\cup(1,2)$ or

$-2<c<2, c\neq 1.$

1 mark for correct answer

Maths Ext 1 Question 10.

a) i) $RHS = (k+1)(4(k+1)^2-1)$

$= (k+1)(4k^2+8k+4-1)$

$= (k+1)(4k^2+8k+3)$

$= 4k^3+8k^2+3k+4k^2+8k+3$ ①

$= 4k^3+12k^2+11k+3$

$= LHS$

For 1 mark long division is time wasting.

OR

$LHS = 4k^3+12k^2+11k+3$

$= 4k^3+4k^2+8k^2+8k+3k+3$

$= 4k^2(k+1)+8k(k+1)+3(k+1)$

$= (k+1)(4k^2+8k+3)$

$= (k+1)(4k^2+8k+4-1)$

$= (k+1)(4(k^2+2k+1)-1)$

$= (k+1)(4(k+1)^2-1)$

a) ii) For $n=1$

$LHS = 1^2 = 1$ $\quad$ $RHS = \dfrac{(1)(4\times1^2-1)}{3} = 1$ ①

$\therefore$ LHS = RHS, true for $n=1$.

Assume statement true for $n=k$

i.e. $1^2+3^2+5^2+\dots+(2k-1)^2 = \dfrac{k(4k^2-1)}{3}$

Prove statement true for $n=k+1$

You need to show some substitution. LHS = 1 RHS = 1 is insufficient.

State assumption clearly.

$1^2+3^2+5^2+\dots+(2k-1)^2+(2(k+1)-1)^2$

$= \dfrac{k(4k^2-1)}{3} + (2k+1)^2$ By assumption ①

make it clear where you used assumption.

$= \dfrac{k(4k^2-1)+3(2k+1)^2}{3}$

$= \dfrac{4k^3-k+12k^2+12k+3}{3}$

show steps clearly

$= \dfrac{4k^3+12k^2+11k+3}{3}$

$= \dfrac{(k+1)(4(k+1)^2-1)}{3}$ from part i). ①

"Hence ..." means you must use part i) and clearly state where you used it.

$\therefore$ statement is true for $n=k+1$, given it is true for $n=k$.

$\therefore$ Since statement is true for $n=1$, from above it must be true for $n=1+1=2$, $2+1=3$ and so on for all positive integers n.

QUESTION 11 Start a new page (6 marks) **Marks**

a) Prove that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ by considering two different ways of selecting k objects from a group of n. **3**

DO NOT PROVE THIS ALGEBRAICALLY, THIS PROOF IS TO BE COMBINATORIAL.

b)

i. By making the substitution $t = \tan\frac{\theta}{2}$, show that $\frac{1+\cos\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$ for $\theta \in (0, \pi)$. **2**

ii. Hence or otherwise, find the exact value of $\cot\frac{\pi}{8}$ in the form $\sqrt{\frac{\sqrt{a}+b}{\sqrt{c}-d}}$. **1**

11. a) To choose k objects from n:

1st way: ${}^{n}C_{k}$ (By definition)

2nd way: Consider 1 object from the list, if it is included, then there are n-1 objects left to choose k-1, $\therefore {}^{n-1}C_{k-1}$

If it is not included, then we need to choose k from n-1, which is ${}^{n-1}C_{k}$.

$\therefore$ Total is ${}^{n-1}C_{k-1} + {}^{n-1}C_{k}$

$$\therefore {}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$$

1 mark for recognising the case where a specific object is **NOT** included and obtaining the correct expression from it.

1 mark for doing the same with the case where a specific object **MUST** be inclued

1 mark for connecting that the sum of the 2 cases must be the same as n choose k.

b)

Given $t = \tan \theta/2$

So $\cos \theta = \dfrac{1-t^2}{1+t^2}$

$\therefore$ L.H.S $= \sqrt{\dfrac{1+\frac{1-t^2}{1+t^2}}{1-\frac{1-t^2}{1+t^2}}}$

$= \sqrt{\dfrac{1+\cancel{t^2}+1-\cancel{t^2}}{\cancel{1}+t^2-\cancel{1}+t^2}}$

$= \sqrt{\dfrac{2}{2t^2}}$ 1

$= \dfrac{1}{t}$ as $t > 0$ for $\theta \in (0, \pi)$

$= \cot \theta/2$

$=$ R.H.S 2nd mark

(ii) from above

$\cot \pi/8 = \sqrt{\dfrac{1+\cos \pi/4}{1-\cos \pi/4}}$

$= \sqrt{\dfrac{1+1/\sqrt{2}}{1-1/\sqrt{2}}} = \sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}$ 1

Task 1 T4 2022

MATHEMATICS Extension ~~1~~: Question 12

Suggested Solutions	Marks	Marker's Comments

a) $P(x)$ - polynomial degree 4.

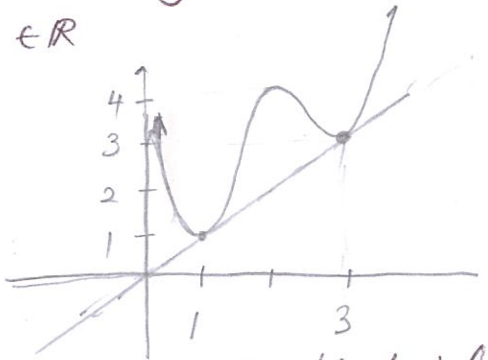
$P(x) \geq x$, $x \in \mathbb{R}$

$P(1) = 1$

$P(2) = 4$

$P(3) = 3$

Find $P(4)$.



Method 1

diagram reveals that if ($P(x) \geq x$) & $P(1) = 1$ and $P(3) = 3$, then $P(x)$ must be tangential to $y = x$ at $x = 1$ & $x = 3$. ie double roots to the eqn $P(x) - x = 0$ or $P(x) = x$

Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$\therefore ax^4 + bx^3 + cx^2 + (d-1)x + e = 0$ has 4 roots

$\boxed{1, 1, 3, 3}$ ($\alpha, \beta, \gamma, \delta$)

* $x^4 - (\alpha+\beta+\gamma+\delta)x^3$ (1) $+ (\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)x^2$ (2) $-$ (3) $(\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)x + \alpha\beta\gamma\delta$ (4) $= 0$

using sum and product of roots identity

$-\dfrac{b}{a} = 8$ (sum of roots) (1)

$b = -8a$ —— i

$(1+3+3) + (3+3) + (3\times 3)$

$\dfrac{c}{a} = 7 + 6 + 9$ (sum of products of each pair of roots) (2)

$c = 22a$ —— ii

$-\dfrac{(d-1)}{a} = 3 + 3 + 9 + 9$ (sum of products of each triplet of roots) (3)

$d = 1 - 24a$ —— iii

$\dfrac{e}{a} = 9$ (product of roots) (4)

$e = 9a$ —— iv

Marks: (1) — understanding double roots and forming an eqn. — finding the polynomial eqn.

Marks: (1) — evaluating $P(4)$

Poorly done! students who attempted SE, didnt get anywhere because they didnt apply the identity rules.
- students didnt understand $P(x) \geq x$ thus the mistake

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Possibly only <10 students were able to do this question

MATHEMATICS: Question……..

Suggested Solutions	Marks Awarded	Marker's Comments
if $P(2)=4$, then $16a+8b+4c+2d+e=4$ — v sub i, ii, iii & v into v.. $16a+8(-8a)+4(22a)+2(1-24a)+9a=4$ $a=2$ $b=-16$ $c=44$ $d=-47$ $e=18$. all 5 values ① mark $\therefore p(x)=2x^4-16x^3+44x^2-47x+18$ $\therefore p(4)=22$ # — ① Method 2. Let $Q(x)=p(x)-x$ and $p(1)=1 \Rightarrow Q(1)=0$ $p(3)=3 \Rightarrow Q(3)=0$ $x=1$ & 3 are zeros of $Q(x)$ with m2, since $Q(x)\geq 0$. $Q(x)=A(x-1)^2(x-3)^2$ $p(2)=4 \Rightarrow Q(2)+2=4$ ← $A(2-1)^2(2-3)^2=2 \therefore A=2$ $Q(x)=2(x-1)^2(x-3)^2$ $p(4)=Q(4)+4$ $=2(4-1)^2(4-3)^2+4$ $=22.$ — ①	① $Q(2)=p(2)-2$	1 mark for 3 correct value. No marks awarded if answer was correct and no working out – to show relevant working out to find value of A.

MATHEMATICS Extension 2: Question ___

Method 1 Suggested Solutions	Marks	Marker's Comments

b) Solve $\sum_{r=0}^{2n} \binom{2n}{r} \cos^{4n-2r}\theta \sin^{2r}\theta = 16^n \cos^{4n}\theta$

for $\theta \in [0, 2\pi]$

poorly done !

$$\sum_{r=0}^{2n} \binom{2n}{r} (\cos^2\theta)^{2n-r} (\sin^2\theta)^r = 16^n \cos^{4n}\theta$$

$$(\cos^2\theta + \sin^2\theta)^{2n} = 16^n \cos^{4n}\theta$$

$$1^{2n} = 16^n \cos^{4n}\theta$$

1

or $\rightarrow$ * $1^{2n} = 2^{4n}\cos^{4n}\theta$

$\frac{1}{4^n} = \cos^{4n}\theta$

$$\frac{1^{2n}}{\cos^{4n}\theta} = 16^n$$

$$(\cos^4\theta)^n = \left(\frac{1^2}{16}\right)^n$$

$$\cos^4\theta = \frac{1}{16}$$

$$\cos\theta = \pm\frac{1}{2}$$

$\therefore \theta = 60°$

120°	60°
240°	300°

acute angle = 60

$$\therefore \theta = \underbrace{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

2 1 mark for each pair of correct answer.

Method 2

LHS

$$\binom{2n}{0}\cos^{4n}\theta + \binom{2n}{1}\cos^{4n-2}\sin^2\theta + \binom{2n}{2}\cos^{4n-4}\theta\sin^4\theta + \dots + \binom{2n}{2n-1}\cos^{4n-2(2n-1)}\theta\sin^{2(2n-1)}\theta + \binom{2n}{2n}\sin^{4n}\theta$$

$$= \cos^{4n}\theta + \binom{2n}{1}\cos^{4n-2}\sin^2\theta + \binom{2n}{2}\cos^{4n-4}\theta\sin^4\theta + \dots + \binom{2n}{2n-1}\cos^2\theta\sin^{4n-2}\theta + \binom{2n}{2n}\sin^{4n}\theta$$

($\div$ by $\cos^{4n}\theta$)

$$= 1 + \binom{2n}{1}\frac{\sin^2\theta}{\cos^2\theta} + \binom{2n}{2}\frac{\sin^4\theta}{\cos^4\theta} + \dots + \binom{2n}{2n-1}\frac{\sin^{4n-2}\theta}{\cos^{4n-2}\theta} + \binom{2n}{2n}\frac{\sin^{4n}\theta}{\cos^{4n}\theta}$$

$$= 1 + \binom{2n}{1}\tan^2\theta + \binom{2n}{n}\tan^4\theta + \dots + \binom{2n}{2n-1}\tan^{4n-2}\theta + \binom{2n}{2n}\tan^{4n}\theta$$

$$= (1 + \tan^2\theta)^{2n}$$

1 – for working

MATHEMATICS Extension 2: Question ___		
Suggested Solutions	**Marks**	**Marker's Comments**
RHS $(1+\tan^2\theta)^{2n} = \dfrac{16^n \cos^{4n}\theta}{\cos^{4n}\theta}$ (as done on LHS) $(1+\tan^2\theta)^{2n} = 16^n$ $1+\tan^2\theta = 16^{n/2n}$ $\tan^2\theta = \sqrt{16} - 1$ $= 3$ $\tan\theta = \pm\sqrt{3}$ $\therefore \theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$ — 2 1 mark for each correct pair of answers		