James Ruse Agricultural High School

2022 **Year 12 TASK 3**

Mathematics Extension 1

General Instructions	• Reading time – 5 minutes • Working time – 70 minutes • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided • For questions in Section II, show relevant mathematical reasoning and/ or calculations
Total marks: 55	**Section I – 5 marks** (pages 2–3) • Attempt Questions 1–5 • Allow about 7 minutes for this section **Section II – 50 marks** (pages 4–10) • Attempt Questions 6–12 • Allow about 1 hour and 3 minutes for this section

Section I

5 marks
Attempt Questions 1 – 5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5.

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1 The unit vector in the direction of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ is

A. $\dfrac{1}{40}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

B. $\dfrac{1}{\sqrt{40}}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

C. $\dfrac{1}{58}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

D. $\dfrac{1}{\sqrt{58}}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

2 Suppose $\mathbf{x} \cdot \mathbf{y} = 2$, $|\mathbf{x}| = 2$ and $|\mathbf{y}| = \sqrt{2}$. The angle between the vectors $\mathbf{x}$ and $\mathbf{y}$ is

A. 30°

B. 45°

C. 60°

D. 90°

3 The coefficient of $a^{-6}b^4$ in the expansion of $\left(\dfrac{1}{a} - \dfrac{2b}{3}\right)^{10}$ is

A. $\dfrac{1102}{27}$

B. $\dfrac{1120}{72}$

C. $\dfrac{1120}{27}$

D. $\dfrac{1120}{37}$

4 Consider the statements below:

I. If $|\underset{\sim}{u}| \geq |\underset{\sim}{v}|$ then $|\text{proj}_{\underset{\sim}{v}}(\underset{\sim}{u})| \geq |\text{proj}_{\underset{\sim}{u}}(\underset{\sim}{v})|$.

II. The angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is the same as the angle between $\text{proj}_{\underset{\sim}{v}}(\underset{\sim}{u})$ and $\text{proj}_{\underset{\sim}{u}}(\underset{\sim}{v})$.

Which of these expressions are true for all two dimensional vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$?

A. Neither I nor II are true.

B. Only I is true.

C. Only II is true.

D. Both I and II are true.

5 Suppose the vectors **u** and **v** form any two sides of a regular hexagon. How many possible values of $\mathbf{u} \cdot \mathbf{v}$ are there?

A. 2

B. 3

C. 4

D. 6

Section II

50 marks
Attempt Questions 6 – 12
Allow about 1 hour and 3 minutes for this section

Answer each question on a new page. Extra writing pages are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (7 marks) Start a new page

(a) Consider the points $P_1(3,4)$ to $P_2(5,-1)$.

(i) Write down the position vector of point P_1. Give your answer as a column vector. **1**

(ii) Find the vector $\overrightarrow{P_1P_2}$. Give your answer in component form. **2**

(iii) Find the value of $|\overrightarrow{P_1P_2}|$. **1**

(iv) If $\overrightarrow{OP_1} = \mathbf{u}$ and $\overrightarrow{OP_2} = \mathbf{v}$ find $\overrightarrow{P_1P_2}$ in terms of $\mathbf{u}$ and $\mathbf{v}$. **1**

(v) Draw a diagram showing the vectors **u**, **v** and **u+v**. **1**

(vi) Find the vector from the point A to the origin where $\overrightarrow{AP_2}$ is $2\mathbf{i} - 3\mathbf{j}$. **1**

End of Question 6

Question 7 (7 marks) Start a new page

(a) A ship S is moving along a straight line with constant velocity in kilometres per hour.
When t = 0 hours the position vector is $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$.
When t = 4 hours, the position vector is $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$.

(i) Find the average speed of S. **2**

(ii) Find the direction in which S is moving, giving your answer as a bearing. **1**

(iii) Show that $\mathbf{s} = (3t + 9)\mathbf{i} + (4t - 6)\mathbf{j}$. **2**

(b) The probability density function of the random variable X is given by **2**

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{4 - x^2}}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of the constant k.

End of Question 7

Question 8 (6 marks) Start a new page

(a) Find the constant term in the expansion of $\left(x - \dfrac{2}{x^3}\right)^{20}$. **2**

(b) Given that **4**

$$(2 + x)^5(1 + 2x)^n = 32 + ax + 27600x^2 + \cdots + 2^n x^{n+5}$$

find the values of the integers a and n.

End of Question 8

Question 9 (7 marks) Start a new page

(a) Consider the parallelogram $OABC$. Let

$\mathbf{a}$ be the position vector of point A

$\mathbf{c}$ be the position vector of point C

(i) Express $\overrightarrow{OB}$ and $\overrightarrow{CA}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. **2**

(ii) Show that if $|\overrightarrow{OB}| = |\overrightarrow{CA}|$ then $\mathbf{a} \perp \mathbf{c}$. **2**

(iii) Show that if $\overrightarrow{OB} \perp \overrightarrow{CA}$ then $|\mathbf{a}| = |\mathbf{c}|$. **2**

(iv) If both assumptions of (ii) and (iii) hold, describe the result geometrically. **1**

End of Question 9

Question 10 (9 marks) Start a new page

(a) The position vectors of A, B and C referred to a point 0 are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. The point N is on AB such that $AN : NB = 2 : 1$.

(i) Show that $\overrightarrow{ON} = \frac{1}{3}(\mathbf{a} + 2\mathbf{b})$. **2**

(ii) If O is the midpoint of CN, prove that $\mathbf{a} + 2\mathbf{b} + 3\mathbf{c} = \mathbf{0}$. **1**

(iii) The point M is on CB such that $CM : MB = 2 : 3$. Show that A, O and M are collinear. **2**

(iv) Hence, find the ratio $AO : OM$. **1**

(v) If the point P is such that $\overrightarrow{NP} = \overrightarrow{AM}$, show that the ratio of **3**

$$\text{area of } PNAM : \text{area of } PNOM = 12 : 7$$

End of Question 10

Question 11 (7 marks) Start a new page

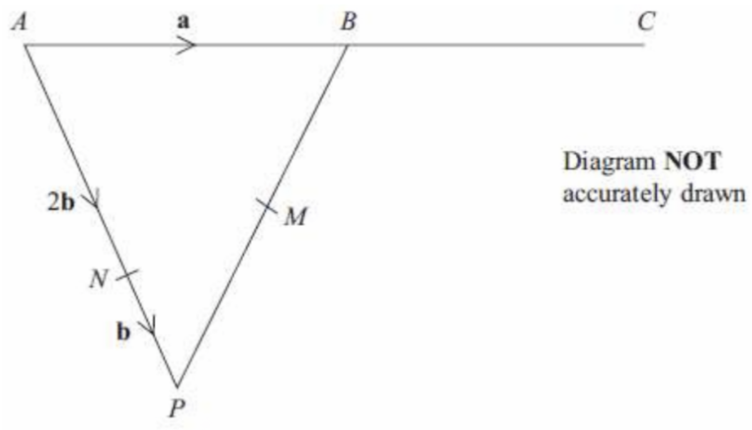
(a) Let X be a random variable with mean 25 and variance 4. If $P(Z < 1.5) = 0.9332$, then find $P(|X - 25| < 3)$. **3**

(b) Let $f(x) = a(3e^{-|x|} - 1)$ for $-\ln 3 \leq x \leq \ln 3$ be a probability density function of a random variable X where a is a constant. Find the cumulative distribution function of X in terms of x and a. **4**

End of Question 11

Question 12 (7 marks) Start a new page

(a) APB is a triangle. N is a point on AP. $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AN} = 2\mathbf{b}$, $\overrightarrow{NP} = \mathbf{b}$.



(i) Find the vector $\overrightarrow{PB}$, in terms of $\mathbf{a}$ and $\mathbf{b}$. **1**

(ii) B is the midpoint of AC. M is the midpoint of PB. Show that NMC is a straight line using vector methods. **3**

(b) By considering the expansion of $(1+x)^n$, show that **3**

$$1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + (-1)^n \frac{1}{n+1}\binom{n}{n} = \frac{1}{n+1}$$

End of Question 12

End of Examination.

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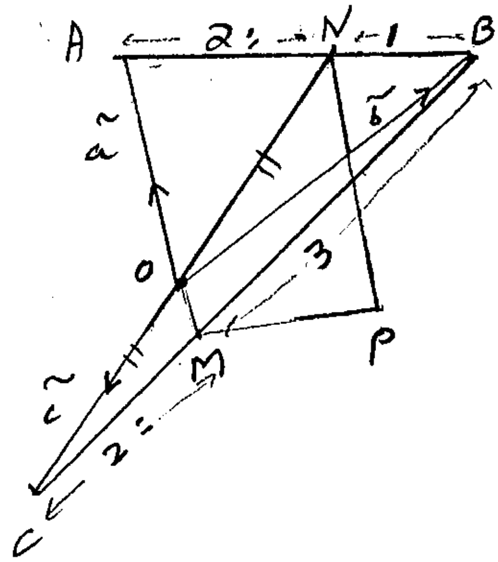
Yr 12 Ext 1 Task 3 Q10 (9 marks) P1/2



(i) $\overrightarrow{AN} = \frac{2}{3}(\vec{b} - \vec{a})$ 1

$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$

$= \tilde{a} + \frac{2}{3}(\tilde{b} - \tilde{a})$

$= \frac{1}{3}[\tilde{a} + 2\tilde{b}]$ 1

or $\overrightarrow{ON} = \tilde{b} - \frac{1}{3}(\tilde{b} - \tilde{a})$

ii) O is midpoint of CN

$\therefore \overrightarrow{CO} = \overrightarrow{ON} = -\tilde{c}$

$\frac{1}{3}(\tilde{a} + 2\tilde{b}) = -\tilde{c}$ 1

$\therefore \tilde{a} + 2\tilde{b} + 3\tilde{c} = 0$

iii) $\overrightarrow{AM} + \overrightarrow{MB} = \overrightarrow{AB} \quad \therefore \overrightarrow{AM} = \overrightarrow{AB} - \overrightarrow{MB}$

$\overrightarrow{AM} = \vec{b} - \tilde{a} - \frac{3}{5}(\vec{b} - \vec{c})$

$= \tilde{b} - \tilde{a} - \frac{3}{5}\tilde{b} + \frac{3\tilde{c}}{5}$

$= \frac{2}{5}\tilde{b} + \frac{3}{5}\tilde{c} - \tilde{a}$ 1

$= \frac{1}{5}[2\tilde{b} + 3\tilde{c}] - \tilde{a}$

$= \frac{1}{5}(-\tilde{a}) - \tilde{a}$ from (ii)

$= \frac{-6}{5}\tilde{a}$ 1

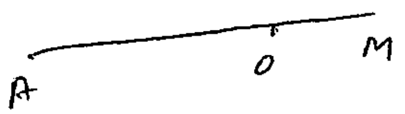
$\therefore \overrightarrow{AM} = -\frac{6}{5}\overrightarrow{OA}$

Since $\overrightarrow{AM}$ is a scalar multiple of $\overrightarrow{OA}$ and share common A

$\therefore$ A, O, M are collinear

(Alternatively prove $\overrightarrow{OM} = -\frac{1}{5}\overrightarrow{OA}$)

iv) FROM (iii) $|AM| = \frac{6}{5}|OA|$



5 : 1

-5:1 get 0 m

1 $\therefore AO : OM = 5 : 1$

v) $\vec{NP} = \vec{AM}$ given

$\therefore$ PNAM is a parallelogram

1 $|PNAM| = h|\vec{AM}|$ while h is the perpendicular from M to NP

$= h \cdot \frac{6}{5}|AO|$

PNOM is a trapezium

$|PNOM| = \frac{h}{2}\left[|NP| + |OM|\right]$

$= \frac{h}{2}\left[|AM| + \frac{1}{5}|AO|\right]$

1 $= \frac{h}{2}\left[\frac{6}{5}|AO| + \frac{1}{5}|AO|\right]$

$= \frac{h}{2}\left[\frac{7}{5}|AO|\right]$

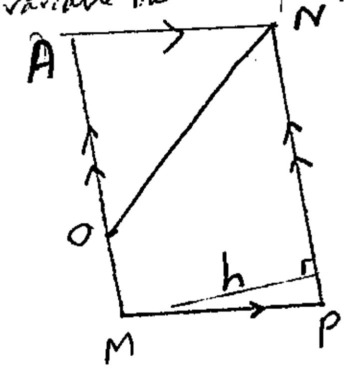
$= \frac{7}{10}|AO| \cdot h$

1 $\frac{|PNAM|}{|PNOM|} = \frac{\frac{6}{5}h\cancel{|AO|}}{\frac{7}{10}h\cancel{|AO|}} = \frac{6}{5} \times \frac{10}{7} = \frac{12}{7}$

$\therefore$ Area PNAM : Area PNOM = 12 : 7

Most student forgot to prove PNAM a parallelogram will get 2m max.

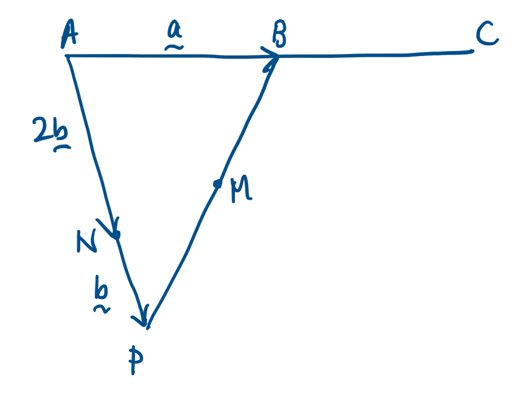
Some students introduce new variable but never define it



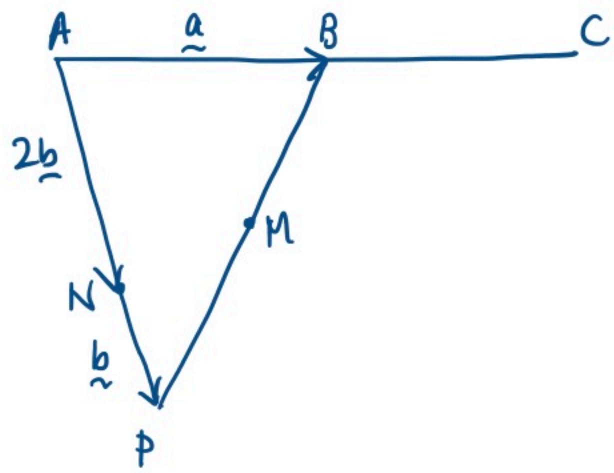
v. poorly done

Students should draw diagram to illustrate better

MATHEMATICS EXTENSION 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
		Students need to take care with vector notation. In HSC each question will be marked independently of the rest. Could lose marks multiple times.
(i) $\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB}$ $= -\underset{\sim}{b} - 2\underset{\sim}{b} + \underset{\sim}{a}$ $= \underset{\sim}{a} - 3\underset{\sim}{b}$	1	
(ii) $\overrightarrow{NM} = \overrightarrow{NP} + \overrightarrow{PM}$ $= \overrightarrow{NP} + \frac{1}{2}\overrightarrow{PB}$ $= \underset{\sim}{b} + \frac{1}{2}(\underset{\sim}{a} - 3\underset{\sim}{b})$ $= \frac{1}{2}(\underset{\sim}{a} - \underset{\sim}{b})$	1	
$\overrightarrow{MC} = \overrightarrow{MB} + \overrightarrow{BC}$ $= \frac{1}{2}\overrightarrow{PB} + \overrightarrow{AB}$ $= \frac{1}{2}(\underset{\sim}{a} - 3\underset{\sim}{b}) + \underset{\sim}{a}$ $= \frac{3}{2}(\underset{\sim}{a} - \underset{\sim}{b})$	1	correct expression for $\overrightarrow{MC}$ (or equivalent)
$= 3\overrightarrow{NM}$ $\therefore$ NMC is a straight line. ($\overrightarrow{MC} \parallel \overrightarrow{NM}$ and M is a common point)	1	Students must show that one vector is a scalar multiple of the other <u>and</u> students must state reason for collinearity
Alternative methods: eg. Show that $\overrightarrow{NM} = \frac{1}{4}\overrightarrow{NC}$		

MATHEMATICS EXTENSION 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
		Students need to take care with vector notation. In HSC each question will be marked independently of the rest. Could lose marks multiple times.
(i) $\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB}$ $= -\underset{\sim}{b} - 2\underset{\sim}{b} + \underset{\sim}{a}$ $= \underset{\sim}{a} - 3\underset{\sim}{b}$	1	
(ii) $\overrightarrow{NM} = \overrightarrow{NP} + \overrightarrow{PM}$ $= \overrightarrow{NP} + \frac{1}{2}\overrightarrow{PB}$ $= \underset{\sim}{b} + \frac{1}{2}(\underset{\sim}{a} - 3\underset{\sim}{b})$ $= \frac{1}{2}(\underset{\sim}{a} - \underset{\sim}{b})$	1	
$\overrightarrow{MC} = \overrightarrow{MB} + \overrightarrow{BC}$ $= \frac{1}{2}\overrightarrow{PB} + \overrightarrow{AB}$ $= \frac{1}{2}(\underset{\sim}{a} - 3\underset{\sim}{b}) + \underset{\sim}{a}$ $= \frac{3}{2}(\underset{\sim}{a} - \underset{\sim}{b})$	1	correct expression for $\vec{MC}$ (or equivalent)
$= 3\overrightarrow{NM}$ $\therefore$ NMC is a straight line. ($\vec{MC} \parallel \vec{NM}$ and M is a common point)	1	Students must show that one vector is a scalar multiple of the other <u>and</u> students must state reason for collinearity
Alternative methods: eg. Show that $\overrightarrow{NM} = \frac{1}{4}\overrightarrow{NC}$		

MATHEMATICS Extension 1 : Question 11

Suggested Solutions	Marks	Marker's Comments
a) $\sigma = 25 \quad sd = 2$		correct sd and recognise the meaning of the absolute value ie. $22 < x < 28$
$P(\lvert x-25 \rvert < 3) = P(-3 < x-25 < 3)$	1	
$= P\left(-\frac{3}{2} < \frac{x-25}{2} < \frac{3}{2}\right)$	1	for recognising $\frac{x-25}{2}$ is the z-score which is between $-\frac{3}{2}$ and $\frac{3}{2}$
$= 2P\left(0 < \frac{x-25}{2} < \frac{3}{2}\right)$		
$= 2 \times (0.9332 - 0.5)$		
$= 0.8664$	1	
b) $f(x) = a\left(3e^{-\lvert x \rvert} - 1\right), \quad -\ln 3 \le x \le \ln 3$		
for $-\ln 3 \le x < 0$		
$F(x) = a\int_{-\ln 3}^{x} (3e^{t} - 1)\,dt$		
$= a\left[3e^{t} - t\right]_{-\ln 3}^{x}$		
$= a\left(3e^{x} - x - 3e^{-\ln 3} - \ln 3\right)$		
$= a\left(3e^{x} - x - 1 - \ln 3\right)$	1	
$0 \le x \le \ln 3$		1 need to recognise the two functions and integrate correctly.
$F(x) = a\int_{-\ln 3}^{0} (3et - 1)\,dt + a\int_{0}^{x} (3e^{-t} - 1)\,dt$		
$= a\left[3e^{t} - t\right]_{-\ln 3}^{0} + a\left[-3e^{t} - t\right]_{0}^{x}$		
$= a\left(3 - 0 - \left(3e^{-\ln 3} - (-\ln 3)\right)\right)$		
$+ a\left(-3e^{x} - x - (-3e^{0})\right)$		

MATHEMATICS Extension 1 : Question ..11...

Suggested Solutions	Marks	Marker's Comments
$= a(3 - 1 - \ln 3)$ $+ a(-3e^{x} - x + 3)$ $= a(-3e^{-x} - x + 5 - \ln 3)$	1	
$\therefore \; F(x) = \begin{cases} a(3e^{x} - x - 1 - \ln 3), & -\ln 3 \le x < 0 \\ a(-3e^{-x} - x + 5 - \ln 3), & 0 \le x \le \ln 3 \end{cases}$	1	Also correct for $0 \le x \le \ln 3$: $a(-3e^{-x} - x) + 3a + \frac{1}{2}$ OR $a(-3e^{-x} - x) + a(1 + \ln 3) + 1$
Note: - cannot integrate absolute values, if students have done so, a mark of 0 was awarded. - There was no mark allocated for finding 'a' $a = \dfrac{1}{4 - 2\ln 3}$		

MATHEMATICS EXT 1: Question 7

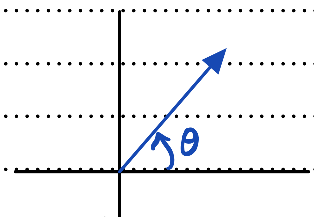
Suggested Solutions	Marks	Marker's Comments

ai) displacement vector $= (21\underset{\sim}{i} + 10\underset{\sim}{j}) - (9\underset{\sim}{i} - 6\underset{\sim}{j})$

$= 12\underset{\sim}{i} + 16\underset{\sim}{j}$ — (1)

$$\begin{aligned} \text{speed} &= \lvert \text{velocity} \rvert \\ &= \left| \frac{\text{displacement}}{\text{time}} \right| \\ &= \left| \frac{12\underset{\sim}{i} + 16\underset{\sim}{j}}{4} \right| \\ &= \lvert 3\underset{\sim}{i} + 4\underset{\sim}{j} \rvert \\ &= 5 \text{ km/h} \end{aligned}$$ — (1)

ii) We want the direction of $12\underset{\sim}{i} + 16\underset{\sim}{j}$



$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{16}{12}\right) \\ &= 53°8' \end{aligned}$$

$\therefore$ direction of S is 037°T or N 37° E — (1)

Marker's Comments: Many lost the mark from not having bearing in correct form.

iii) In 4 hours, the displacement is $12\underset{\sim}{i} + 16\underset{\sim}{j}$

$\therefore$ In 1 hour, the displacement is $3\underset{\sim}{i} + 4\underset{\sim}{j}$ — (1)

$\therefore$ In t hours, the displacement is $t(3\underset{\sim}{i} + 4\underset{\sim}{j})$

MATHEMATICS EXT 1: Question 7		
Suggested Solutions	**Marks**	**Marker's Comments**

So the position vector of S is

$$\underset{\sim}{s} = (9\underset{\sim}{i} - 6\underset{\sim}{j}) + t(3\underset{\sim}{i} + 4\underset{\sim}{j})$$

$$= (3t+9)\underset{\sim}{i} + (4t-6)\underset{\sim}{j} \qquad 1$$

Comments:

* Students who subbed $t=0$ & 4 into $\underset{\sim}{s} = 21\underset{\sim}{i} + 10\underset{\sim}{j}$ receive only one mark, unless they explicitly state that "since the equation satisfied two known points, it must be the equation of the line."

* The following solution only received one mark as well:

$$\underset{\sim}{s} = \text{starting point} + t(\text{direction})$$

$$= \begin{pmatrix} 9 \\ -6 \end{pmatrix} + t\begin{pmatrix} 12 \\ 16 \end{pmatrix}, \quad t \in \mathbb{R}$$

$$= \begin{pmatrix} 9 \\ -6 \end{pmatrix} + 4t\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad t \in \mathbb{R}$$

$$= \begin{pmatrix} 9 \\ -6 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad t \in \mathbb{R}$$

$$= (3t+9)\underset{\sim}{i} + (4t-6)\underset{\sim}{j}$$

→ t is NOT an arbitrary constant in this scenario. It represents time, so your direction vector must be the displacement for one change in t (i.e. one hour)

MATHEMATICS EXT 1: Question 7

Suggested Solutions	Marks	Marker's Comments
b) Since $f(x)$ is a PDF:		
$\int_{-\infty}^{\infty} f(x)\,dx = 1$		Too many errors or "overkill" methods for this integral. It's a STANDARD FORM! Know your reference sheet & don't do unnecessary trig subs.
$\therefore \int_0^1 \frac{k}{\sqrt{4-x^2}}\,dx = 1$		
$\left[k \sin^{-1}\left(\frac{x}{2}\right)\right]_0^1 = 1$	1	
$k\left(\frac{\pi}{6} - 0\right) = 1$		
$k = \frac{6}{\pi}$	1	

Question 6: $P_1(3,4)$, $P_2(5,-1)$

(i) $\overrightarrow{OP_1} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(ii) $\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$

$= 5\vec{i} - \vec{j} - (3\vec{i} + 4\vec{j})$

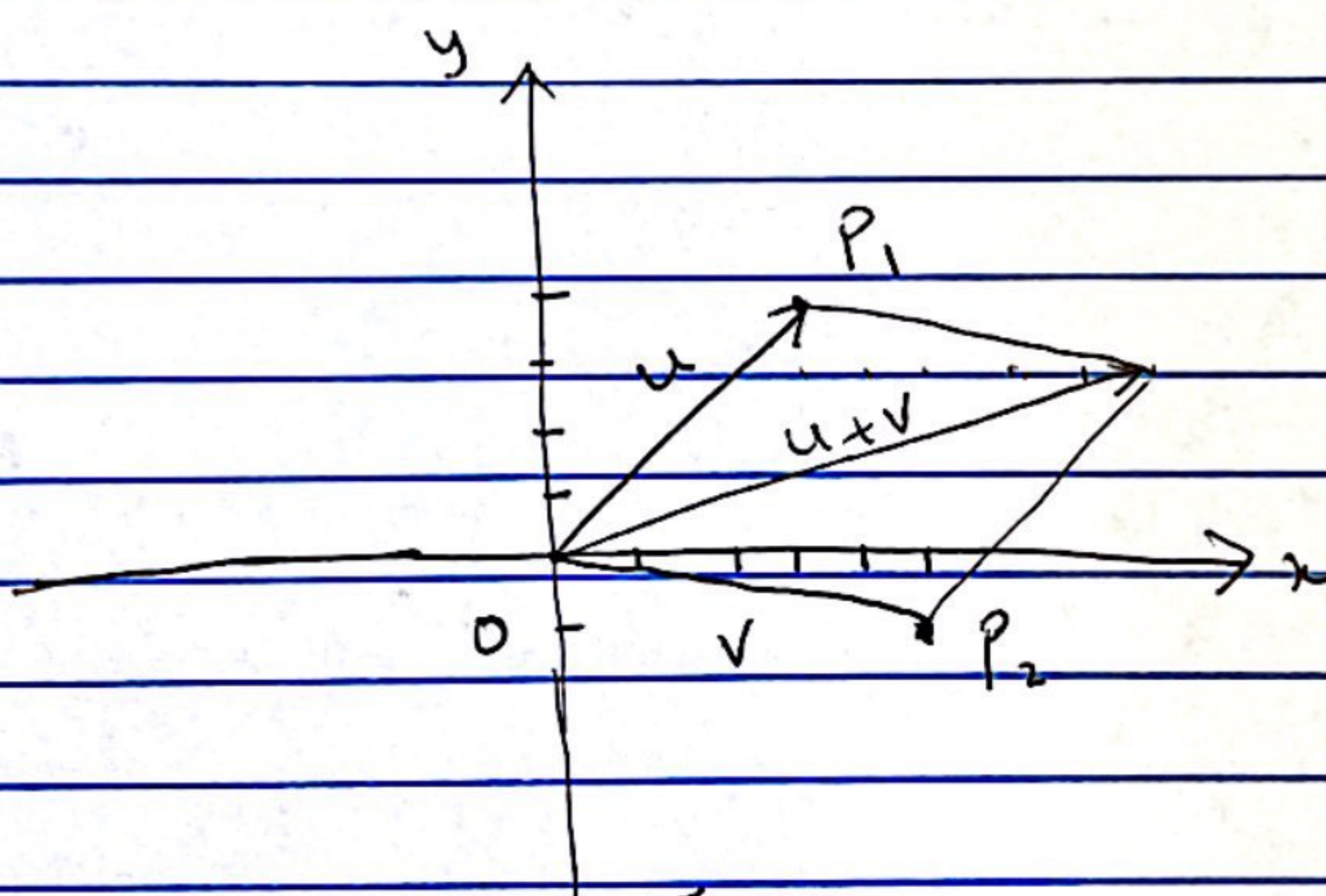
$= 2\vec{i} - 5\vec{j}$

(iii) $|\overrightarrow{P_1P_2}| = \sqrt{2^2+(-5)^2} = \sqrt{29}$

(iv) $u = \overrightarrow{OP_1}$, $v = \overrightarrow{OP_2}$

$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = v - u$

(v)



(vi) $\overrightarrow{AP_2} = 2\vec{i} - 3\vec{j}$

$\overrightarrow{OP_2} - \overrightarrow{OA} = 2\vec{i} - 3\vec{j}$

$\overrightarrow{AO} = 2\vec{i} - 3\vec{j} - \overrightarrow{OP_2}$

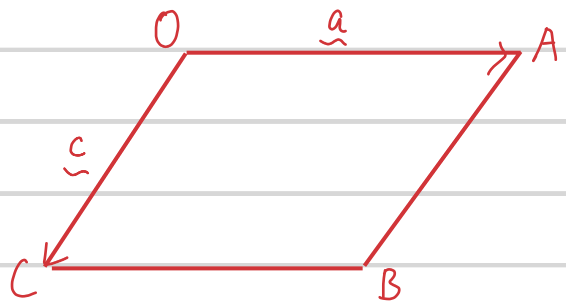
$= 2\vec{i} - 3\vec{j} - (5\vec{i} - \vec{j})$

$= -3\vec{i} - 2\vec{j}$

Year 12 **Task** 3 **MATHEMATICS** Ext 1 **Question** 9

Suggested Solutions	Marks	Marker's Comments

a)



i) $\overrightarrow{OB} = \underset{\sim}{a} + \underset{\sim}{c}$ — 1

$\overrightarrow{CA} = \underset{\sim}{a} - \underset{\sim}{c}$ — 1

ii) If $|\overrightarrow{OB}| = |\overrightarrow{CA}|$, then $|\overrightarrow{OB}|^2 = |\overrightarrow{CA}|^2$

$$\overrightarrow{OB} \cdot \overrightarrow{OB} = \overrightarrow{CA} \cdot \overrightarrow{CA}$$

$$(\underset{\sim}{a} + \underset{\sim}{c}) \cdot (\underset{\sim}{a} + \underset{\sim}{c}) = (\underset{\sim}{a} - \underset{\sim}{c}) \cdot (\underset{\sim}{a} - \underset{\sim}{c})$$ — 1

$$|\underset{\sim}{a}|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{c} + |\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 - 2\underset{\sim}{a} \cdot \underset{\sim}{c} + |\underset{\sim}{c}|^2$$

$$2\underset{\sim}{a} \cdot \underset{\sim}{c} = -2\underset{\sim}{a} \cdot \underset{\sim}{c}$$

$$4\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$$

$$\therefore \underset{\sim}{a} \cdot \underset{\sim}{c} = 0$$

$$\therefore \underset{\sim}{a} \perp \underset{\sim}{c}$$ — 1

iii) If $\overrightarrow{OB} \perp \overrightarrow{CA}$, $\overrightarrow{OB} \cdot \overrightarrow{CA} = 0$

$$(\underset{\sim}{a} + \underset{\sim}{c}) \cdot (\underset{\sim}{a} - \underset{\sim}{c}) = 0$$ — 1

$$\underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{c} \cdot \underset{\sim}{c} = 0$$

$$|\underset{\sim}{a}|^2 - |\underset{\sim}{c}|^2 = 0$$

$$|\underset{\sim}{a}|^2 = |\underset{\sim}{c}|^2$$

$|\underset{\sim}{a}| = |\underset{\sim}{c}|$ (both positive) — 1

iv) OABC is a square — 1

(parallelogram with one pair of adjacent perpendicular sides)

Year _____ Task _______ MATHEMATICS ______ Question______

Suggested Solutions	Marks	Marker's Comments

Year _____ Task _______ MATHEMATICS ______ Question______

Suggested Solutions	Marks	Marker's Comments

Year _____ Task _______ MATHEMATICS ______ Question______

Suggested Solutions	Marks	Marker's Comments

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Year _____ Task _______ MATHEMATICS ______ Question______

Suggested Solutions	Marks	Marker's Comments

Year _____ Task _______ MATHEMATICS ______ Question______

Suggested Solutions	Marks	Marker's Comments

Year 12 Extension 1: Q8		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(a) General term of $(x-2x^{-3})^{20}$ is $\binom{20}{k}x^{20-k}(-2x^{-3})^k = \binom{20}{k}(-2)^k x^{20-4k}$ Constant term occurs when $20-4k=0 \to k=5$: $\binom{20}{5}(-2)^5 = -496128$	1 First mark for appropriate working leading to a general term. 1 Final mark for correct result.	
(b) $(2+x)^5(1+2x)^n = \left(\sum_{k=0}^{5}\binom{5}{k}2^k x^{5-k}\right)\left(\sum_{j=0}^{n}\binom{n}{j}2^j x^j\right)$ $= \sum_{k=0}^{5}\sum_{j=0}^{n}\binom{5}{k}\binom{n}{j}2^{k+j}x^{5-k+j}$ We first need to extract the coefficient of x^2. The simplest way to do so is to take expansions to x^2 in each binomial and multiply judiciously: $(32+80x+80x^2+\cdots)(1+2nx+2n(n-1)x^2+\cdots)$ $= 32+(64n+80)x+(64n(n-1)+160n+80)x^2+\cdots$ We have then $64n(n-1)+160n+80 = 27600$ so $64n^2+96n-27520=0$ which gives, using quadratic formula, $n=20$ as the only positive, integral solution. Hence, the coefficient of x is $a = 64n+80$ $= 64\cdot 20+80$ $= 1360$	1 First mark for work leading to expansion. 1 Next mark for acquiring correct equation in n. 1 Mark for extracting correct n. 1 Final mark for correct value of a.	Many errors made, primarily since candidates did not multiply expansions correctly. ECF marks not necessarily awarded if errors simplified the problem.