Name:______________________

Class ___________________

HSC Assessment Task 1 2023-2024

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 65 minutes
- Board approved calculators may be used.
- Write using black or blue pen
- All necessary working should be shown in all questions
- Write your student number at the top of every page of your answer sheets.
- NESA Reference Sheet is provided

Total marks – 40

Attempt all questions

Section A – Answer on the Multiple Choice Answer Sheet

Section B - Start each question on a new sheet of paper.

NESA Reference Sheet is provided

This paper MUST NOT be removed from the examination room

Section A Multiple Choice (5 Marks)

QUESTION 1

Which of the following step is not necessary to prove a statement to be true for all positive integer values n by mathematical induction?

A. Prove that the statement is true for $n = 1$.

B. Prove that if the statement is true for some case $n = k$, then the statement is also true for $n = k + 1$.

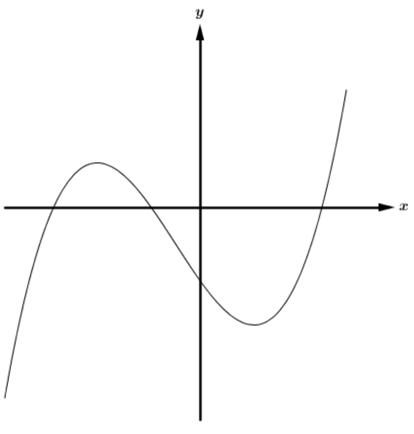
C. Check that the statement is true for a very large value of n.

D. None of the above.

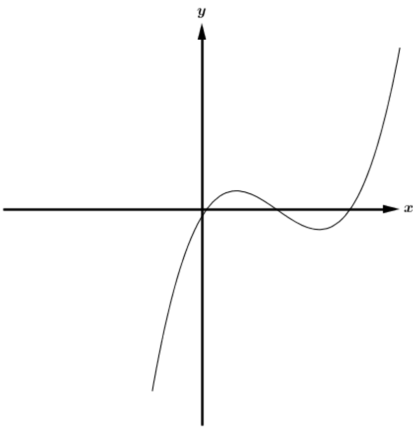
QUESTION 2

Consider the polynomial $P(x) = ax^3 + bx^2 + cx - 6$, where $a, b > 0$. Which of the following is a potential graph of $y = P(x)$?

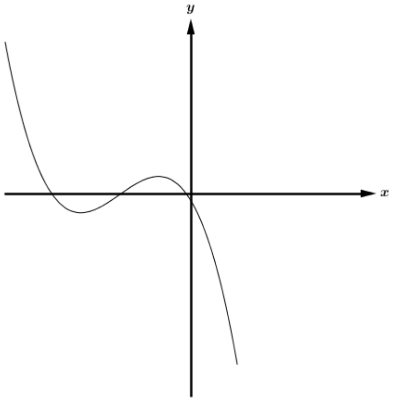
A.



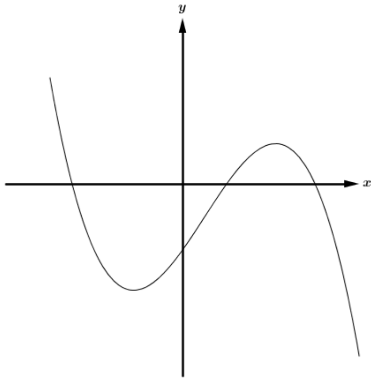
B.



C.



D.



QUESTION 3

Which of the following will have a non-zero coefficient in the expansion of $\left(a^2 + \frac{1}{a}\right)^{23}$?

A. a^{-9}

B. a^{-8}

C. a^{-7}

D. a^{-6}

QUESTION 4

Consider the expansion of $\left(2x^3 + \frac{1}{x^2}\right)^n, n \in \mathbb{Z}^+$. If it has a non-zero constant term, what is the minimum possible value of n?

A. 3

B. 5

C. 8

D. 10

QUESTION 5

Given $\sin\beta - \cos\beta = \frac{4}{3}$, what is the value of $\sin 2\beta$?

A. $-\frac{2}{9}$

B. $-\frac{7}{9}$

C. $\frac{2}{9}$

D. $\frac{7}{9}$

SECTION B

QUESTION 6 Start a new page (5 marks) **Marks**

a) Let $\alpha, \beta\ and\ \gamma$ be the roots of $2x^3 - 4x + 7$.

i. Find the values of $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha + \beta + \gamma$. **2**

ii. Hence or otherwise, find the value of $\alpha^3 + \beta^3 + \gamma^3$. **2**

b) How many terms are there in the expansion of $(3a + 2b)^{90209}$? **1**

QUESTION 7 Start a new page (5 marks) **Marks**

a) Prove by mathematical induction that for $n \in \mathbb{Z}^+$ **3**

$$a + ar + ar^2 + \cdots + ar^n = a\left(\frac{r^{n+1} - 1}{r - 1}\right)$$

if $r \neq 1$.

b) Given $(1 + x + x^2)^6 = a_0 + a_1x + a_2x^2 + \cdots + a_{12}x^{12}$, find the value of: **2**

$$a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12}.$$

QUESTION 8 Start a new page (6 marks) **Marks**

a) Suppose that b and d are real numbers and $d \neq 0$. Consider the polynomial $P(x) = x^4 + bx^2 + d$, $P(x) = 0$ has a double root at $x = \alpha$.

i. Find the remainder when $P(x)$ is divided by $(x - 1)$. **1**

ii. Show that $\alpha^2 = -\frac{b}{2}$. **2**

iii. Hence or otherwise, show that $\frac{b^2}{4} = d$. **2**

QUESTION 9 Start a new page (5 marks) **Marks**

a) Prove by mathematical induction that $23n^3 + 97n$ is divisible by 6 for all integers $n \geq 1$. **4**

b) Fully simplify $(2x+1)^5 - 5(2x+1)^4 + 10(2x+1)^3 - 10(2x+1)^2 + 5(2x+1) - 1$. **1**

QUESTION 10 Start a new page (5 marks) **Marks**

a) Use the t-formulae to solve the equation $2\sin\frac{x}{2} + \cos\frac{x}{2} = 1$ for $x \in [0, 8\pi]$. **3**

b) Prove that $\sin\left(\theta + \frac{\pi}{6}\right)\sin\left(\theta - \frac{\pi}{6}\right) = \sin^2\theta - \frac{1}{4}$. **2**

QUESTION 11 Start a new page (5 marks) **Marks**

a)

i. Express $\sin\theta + \sin(\theta + \frac{\pi}{3})$ in the form $R\sin(\theta + \alpha)$, where $R > 0$, and $0 < \alpha < \frac{\pi}{2}$. Show all necessary working. **3**

ii. Hence, or otherwise sketch the graph $y = \sin\theta + \sin\left(\theta + \frac{\pi}{3}\right)$, for $0 \leq \theta \leq 2\pi$, showing important features. **2**

QUESTION 12 Start a new page (5 marks) **Marks**

a) Use the binomial expansion to prove that: **2**

$$n \times 3^{n-1} = \binom{n}{1}(1) + \binom{n}{2}(2 \times 2) + \binom{n}{3}(3 \times 2^2) + \cdots \binom{n}{n}(n \times 2^{n-1}).$$

b) Given $\sin\alpha$, $\sin\beta$ and $\sin\gamma$ are the solutions to the equation to $8x^3 - 6x - \sqrt{5} = 0$, find the value of $\cot^2\alpha + \cot^2\beta + \cot^2\gamma$. **3**

End of examination

Multiple Choice:

1. (C)
2. Degree 3, positive leading coefficient therefore Option (A) or (B)

   Sum of roots $= -\frac{\mathrm{b}}{\mathrm{a}}$

   < 0

   Sum of all $x -$ intercepts should be negative value.

   Thus, rules out Option (B)

   Therefore answer is (A)

3. Start from a^{-23}, the powers of a goes up by 3, so $-\ 23, -20, -17, -14, -11, -8,$ $-7 \therefore$ (B)

4. Constant term $\Rightarrow (x^3)^r(x^{-2})^{n-r} = x^{5r-2n} = x^0$

   $\therefore 5r - 2n = 0$

   $n = 5$ and $r = 2$ is the first possability

   (B)

5. (B)

MATHEMATICS EXT 1: Question 6

Suggested Solutions	Marks	Marker's Comments
a i, Let $p(x) = 2x^3 + 0x^2 - 4x + 7$		
$\sum \alpha = -\frac{b}{a}$		
$\therefore \alpha + \beta + \gamma = -\frac{0}{2} = 0$	1	
$\sum \alpha\beta = \frac{c}{a}$		
$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{-4}{2} = -2$	1	
ii, Since α, β, γ are roots of $p(x)$, then		
$p(\alpha) = 0 \Rightarrow 2\alpha^3 - 4\alpha + 7 = 0$ (1)		
$p(\beta) = 0 \Rightarrow 2\beta^3 - 4\beta + 7 = 0$ (2)	1	
$p(\gamma) = 0 \Rightarrow 2\gamma^3 - 4\gamma + 7 = 0$ (3)		
(1) + (2) + (3) :		
$\therefore 2(\alpha^3 + \beta^3 + \gamma^3) - 4(\alpha + \beta + \gamma) + 21 = 0$		
$2(\alpha^3 + \beta^3 + \gamma^3) = -21$		
$\alpha^3 + \beta^3 + \gamma^3 = -\frac{21}{2}$	1	
b, 90210 terms	1	

Yr 11/12 Ext 1 Task 1 DEC, 2023 Q7

a) To prove by MI $a + ar + ar^2 + \cdots + ar^n = \frac{a(r^{n+1}-1)}{r-1}$

Solution:

$n=1$, LHS $= a + ar^1 = a(r+1)$

RHS $= \frac{a(r^{1+1}-1)}{r-1} = \frac{a(r^2-1)}{r-1} = \frac{a(r+1)\cancel{(r-1)}}{\cancel{r-1}} = a(r+1)$ 1m

$\therefore$ true

Assume true for $n=k$, $k \in \mathbb{Z}^+$

$a + ar + ar^2 + \cdots + ar^k = \frac{a(r^{k+1}-1)}{r-1}$

RTP for $n=k+1$

$\underbrace{a + ar + ar^2 + \cdots + ar^k} + ar^{k+1} = \frac{a(r^{k+2}-1)}{r-1}$

LHS $\frac{a(r^{k+1}-1)}{r-1} + ar^{k+1}$ (By assumption) 1m

$= \frac{a(r^{k+1}-1) + (r-1)ar^{k+1}}{r-1}$

$= \frac{\cancel{ar^{k+1}} - a + ar^{k+2} - \cancel{ar^{k+1}}}{r-1}$ 1m

$= \frac{a(r^{k+2}-1)}{r-1} = $ RHS

Conclusion: By the principle of maths Induction, therefore true for all $n \in \mathbb{Z}^+$

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b) $(1+x+x^2)^6 = a_0 + a_1x + a_2x^2 + \cdots + a_{12}x^{12}$ find the value of $a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12}$

Solution when $x=0$, $1^6 = a_0$ $\therefore a_0 = 1$

when $x=-1$ $(\cancel{1}-\cancel{1}+1)^6 = a_0 - a_1 + a_2 - a_3 + \cdots + a_{12}$

$\cancel{1} = \cancel{1} - (a_1 + a_3 + a_5 + a_7 + a_9 + a_{11}) + (a_2 + a_4 \cdots + a_{12})$ 1m

$\therefore a_1 + a_3 + a_5 + \cdots + a_{11} = a_2 + a_4 + \cdots + a_{12}$

when $x=1$ $(1+1+1)^6 = a_0 + a_1 + a_2 + \cdots + a_{11} + a_{12}$

$3^6 = 1 + 2(a_2 + a_4 + \cdots + a_{12})$

$729 - 1 = 2(a_2 + a_4 + \cdots + a_{12})$ 1m

$\underline{\underline{364}} = a_2 + a_4 + \cdots + a_{12}$

Method 2 Need to see the development of the double summation

$$(1+x+x^2)^6 = [1+x(1+x)]^6 = \sum_{r=0}^{6}\binom{6}{r}x^{6-r}(1+x)^{6-r}$$

$$= \binom{6}{0}x^6(1+x)^6 + \binom{6}{1}x^5(1+x)^5 + \binom{6}{2}x^4(1+x)^4 + \binom{6}{3}x^3(1+x)^3 + \binom{6}{4}x^2(1+x)^2 + \binom{6}{5}x(1+x) + \binom{6}{6}$$

coeff of positive even powers of x ie $x^2, x^4, x^6, x^8, x^{10}, x^{12}$

$$\binom{6}{0}\left[\binom{6}{0}+\binom{6}{2}+\binom{6}{4}+\binom{6}{6}\right] + \binom{6}{1}\left[\binom{5}{1}+\binom{5}{3}+\binom{5}{5}\right] + \binom{6}{2}\left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right]$$
$$+ \binom{6}{3}\left[\binom{3}{1}+\binom{3}{3}\right] + \binom{6}{4}\left[\binom{2}{0}+\binom{2}{2}\right] + \binom{6}{5}$$

1m

$$= 1(1+15+15+1) + 6(5+10+1) + 15(1+6+1) + 20(3+1) + 15(1+1) + 6$$
$$= 22 + 96 + 120 + 80 + 10 + 30 + 6 = \underline{\underline{364}}$$

1m

Method 3 Need to see the double summation

$$(1+x+x^2)^6 = [(1+x)+x^2]^6 = \sum_{r=0}^{6}\binom{6}{r}(1+x)^{6-r}(x^2)^r$$

$$= \sum_{r=0}^{6}\binom{6}{r}\sum_{k=0}^{6-r}\binom{6-r}{k}x^k x^{2r} = \sum_{r=0}^{6}\binom{6}{r}\sum_{k=0}^{6-r}\binom{6-r}{k}x^{2r+k}$$

General term is $\binom{6}{r}\binom{6-r}{k}x^{2r+k}$ $\quad 0\le r\le 6,\ 0\le k+r\le 6$

$2r+k=2$

k	r
0	1
2	0

$\binom{6}{1}\binom{5}{0}+\binom{6}{0}\binom{6}{2}$
$= 21$

$2r+k=4$

k	r
0	2
2	1
4	0

$\binom{6}{2}\binom{4}{0}+\binom{6}{1}\binom{5}{2}+\binom{6}{0}\binom{6}{4}$
$= 90$

$2r+4=6$

k	r
0	3
2	2
4	1
6	0

$\binom{6}{3}\binom{3}{0}+\binom{6}{2}\binom{4}{4}+\binom{6}{1}\binom{5}{4}+\binom{6}{0}\binom{6}{6}$
$= 141$

$2r+k=8$

k	r
0	4
2	3
4	2

$\binom{6}{4}\binom{2}{0}+\binom{6}{3}\binom{3}{2}+\binom{6}{2}\binom{4}{4}$
$= 90$

$2r+k=10$

k	r
0	5
2	4

$\binom{6}{5}\binom{1}{0}+\binom{6}{4}\binom{2}{2}$
$= 21$

k	r
0	6

$\binom{6}{6}\binom{6}{0}$
$= 1$

Total $2(21+90)+141+1$
$= \underline{\underline{364}}$

Student Number ______________

QUESTION 8

i) $P(1) = 1^4 + b.1^2 + d$

$= 1+b+d$

$1+b+d$ is the remainder upon division by $(x-1)$ ✓

ii) $P(\alpha) = 0 \Rightarrow \alpha^4 + b\alpha^2 + d = 0$ ①

$P'(\alpha) = 0 \Rightarrow 4\alpha^3 + 2b\alpha = 0$ ②

✓ solving $P'(\alpha)=0$ (applying theory of multiple roots)

from ②: $4\alpha^3 + 2b\alpha = 0$

$2\alpha(2\alpha^2 + b) = 0$

$\alpha = 0$ or $2\alpha^2 = -b$

$\alpha^2 = \frac{-b}{2}$

if $\alpha = 0$ $\quad 0^4 + b.0^2 + d = 0$ from ①

$d = 0$

but $d \neq 0$ $\therefore \alpha^2 = \frac{-b}{2}$ only

✓ rejecting $\alpha = 0$

iii) Substituting this result into ①:

$\left(\frac{-b}{2}\right)^2 + b\left(-\frac{b}{2}\right) + d = 0$ ✓ substitute into $P(\alpha)$

$\frac{b^2}{4} - \frac{b^2}{2} + d = 0$ simplify.

$\frac{b^2}{4} = d$

If relationships between roots were attempted then no mark was given for just writing out $\alpha + \alpha + \beta + \gamma = \ldots$

$\alpha^2 + 2\alpha\beta + 2\alpha\gamma + \beta\gamma = \ldots$

etc

First mark was for an attempt to solve simultaneously via substitution or elimination.

Question 9

a) Test for $n = 1$

$23(1)^3 + 97(1) = 120$

$= 6 \times 20 \leftarrow$ (MUST SHOW!!!!)

$\therefore$ the statement is true for $n = 1$ (1 mark)

Assume the statement is true for $n = k, k \in \mathbb{Z}^+$

$\therefore 23k^3 + 97k = 6M_1$ where $M_1 \in \mathbb{Z}$ (1 mark for interpreting what the assumption means)

Test for $n = k + 1$

RTP: $23(k+1)^3 + 97k = 6M_2$ where $M_2 \in \mathbb{Z}^+$

$\text{LHS} = 23(k+1)^3 + 97(k+1)$

$= 23(k^3 + 3k^2 + 3k + 1) + 97(k+1)$

$= (23k^3 + 97k) + (69k^2 + 69k + 23 + 97)$

$= 6M_1 + (69k^2 + 69k + 120)$ (By Assumption)

(1 mark for using the assumption correctly, you do **NOT** get the mark if you substituted and ended up with a fraction)

$= 6M_1 + 120 + 69k(k+1)$

$= 6M_1 + 120 + 69 \times (2P)$ where $P \in \mathbb{Z}$

(the product of 2 consecutive integers must be even)

$= 6M_1 + 120 + 138P$

$= 6(M_1 + 20 + 23P)$

$= 6M_2$ (Since $M_1 + 20 + 23P \in \mathbb{Z}$)

$\therefore$ if the statement is true for $n = k$, then it is true for $n = k + 1$

$\therefore$ the statement is true for $n = \mathbb{Z}^+$ by the principle of mathematical induction. (1 mark)

(Note: There is 1 mark in this question allocated to all the small details that is required in a mathematical induction proof, such as defining the number set of any variables introduced, making clear that 120 is a multiple of 6, writing by assumption when a subsitution is made, using the write mathematical language like assume the statement is true for n=k. If you wrote words that convey an incorrect mathematical logic you will only receive 3 out of 4 as a maximum)

b) $(2x+1)^5 - 5(2x+1)^4 + 10(2x+1)^3 - 10(2x+1)^2 + 5(2x+1) - 1$
$= (2x + 1 - 1)^5$
$= (2x)^5$
$= 32x^5$ (1 mark)

QUESTION 10 Start a new page (5 marks) **Marks**

a) Use the t-formulae to solve the equation $2\sin\frac{x}{2} + \cos\frac{x}{2} = 1$ for $x \in [0, 8\pi]$. 3

b) Prove that $\sin\left(\theta + \frac{\pi}{6}\right)\sin\left(\theta - \frac{\pi}{6}\right) = \sin^2\theta - \frac{1}{4}$. 2

10.

a)

$$\text{Let } t = \tan\frac{x}{4}$$
$$\therefore \frac{4t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$$
$$4t + 1 - t^2 = 1 + t^2$$
$$2t^2 - 4t = 0$$
$$2t(t-2) = 0$$
$$t = 0 \text{ or } t = 2$$
$$\therefore \tan\frac{x}{4} = 0 \text{ or } \tan\frac{x}{4} = 2$$
$$\therefore \frac{x}{4} = 0, \pi, 2\pi, 1.11 \text{ or } 4.25$$
$$\therefore x = 0, 4\pi, 8\pi, 4.44 \text{ or } 17.0$$

$$0 \le x \le 8\pi$$
$$0 \le \frac{x}{4} \le 2\pi$$

b)

$$\text{LHS} = \left(\sin\theta\cos\frac{\pi}{6} + \cos\theta\sin\frac{\pi}{6}\right)\left(\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6}\right)$$
$$= \sin^2\cos^2\frac{\pi}{6} - \cos^2\theta\sin^2\frac{\pi}{6}$$
$$= \frac{3}{4}\sin^2\theta - \frac{1}{4}\cos^2\theta$$
$$= \frac{3}{4}\sin^2\theta - \frac{1}{4}(1 - \sin^2\theta)$$
$$= \frac{3}{4}\sin^2\theta + \frac{1}{4}\sin^2\theta - \frac{1}{4}$$
$$= \sin^2\theta - \frac{1}{4}$$

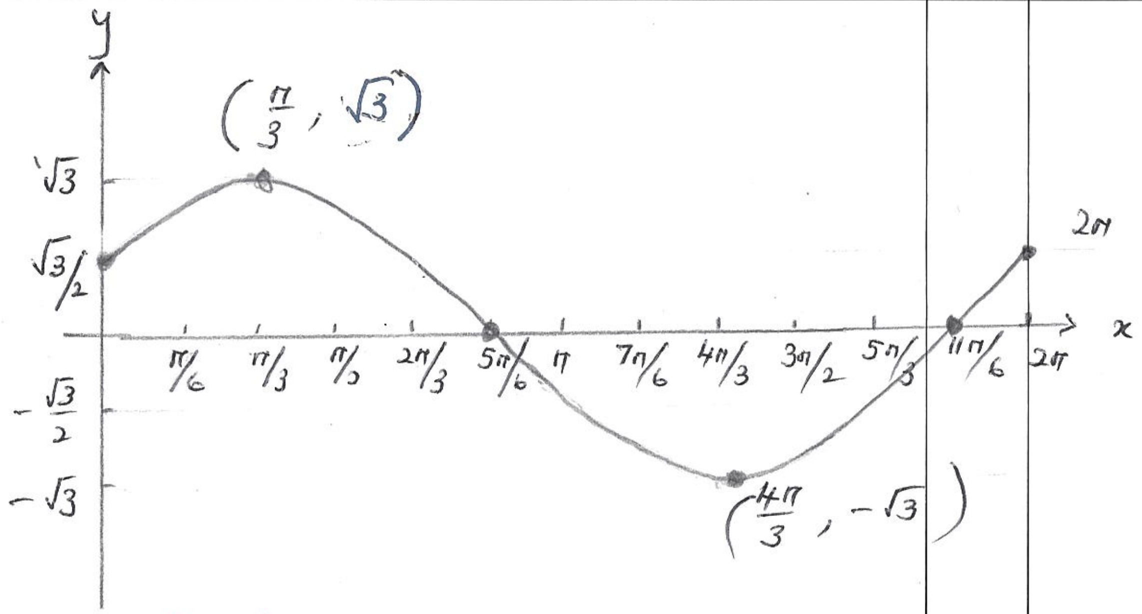
RHS $=(\sin\theta)^2 - \frac{1}{4} = \left(\sin\theta + \frac{1}{2}\right)\left(\sin\theta - \frac{1}{2}\right) = \left(\sin\theta + \sin\frac{\pi}{6}\right)\left(\sin\theta - \sin\frac{\pi}{6}\right) =$
$2\sin\left(\frac{\theta+\frac{\pi}{6}}{2}\right)\cos\left(\frac{\theta-\frac{\pi}{6}}{2}\right) \times 2\sin\left(\frac{\theta-\frac{\pi}{6}}{2}\right)\cos\left(\frac{\theta+\frac{\pi}{6}}{2}\right) = 2\sin\left(\frac{\theta+\frac{\pi}{6}}{2}\right)\cos\left(\frac{\theta+\frac{\pi}{6}}{2}\right) \times 2\sin\left(\frac{\theta-\frac{\pi}{6}}{2}\right)\cos\left(\frac{\theta-\frac{\pi}{6}}{2}\right) =$
$\sin\left(\theta + \frac{\pi}{6}\right)\sin\left(\theta - \frac{\pi}{6}\right)$

MATHEMATICS Extension 1: Question 11 Task 1 2024		
Suggested Solutions	**Marks**	**Marker's Comments**
11ai) $\sin\theta + \sin(\theta + \frac{\pi}{3})$ in the form $R\sin(\theta+\alpha)$ $R>0$ $0<\alpha<\pi/2$		
$\sin\theta + \sin(\theta + \frac{\pi}{3}) = R[\sin(\theta+\alpha)]$		
LHS $\sin\theta + \sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3}$		working
$= \frac{3}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$		equate
RHS $R[\sin\theta\cos\alpha + \cos\theta\sin\alpha]$		1
$R\cos\alpha = \frac{3}{2}$ — i $R\sin\alpha = \frac{\sqrt{3}}{2}$ — ii square both sides		
$R^2\cos^2\alpha = \frac{9}{4}$ $R^2\sin^2\alpha = \frac{3}{4}$ add		
$R^2(\cos^2\alpha + \sin^2\alpha) = 3$ $R = \sqrt{3}$ $(R>0)$		
sub R into ii $\sin\alpha = \frac{\sqrt{3}}{\sqrt{3}(2)}$		1
$\alpha = \frac{\pi}{6}$ $0<\alpha<\frac{\pi}{2}$		
$\therefore R\sin(\theta+\alpha) = \sqrt{3}\sin(\theta+\frac{\pi}{6})$		1

11a ii)

MATHEMATICS Extension 1. Question 11 Task 1 2024

Suggested Solutions	Marks	Marker's Comments



* - min pt
- max pt
- x int
- end points
- scale - consistent for x & y
- shape
- domain
- end points not curved (not max t.p)

> 4 mistakes - 0

< 4 mistakes - 1

$\sqrt{3}\sin\left(\theta + \frac{\pi}{6}\right) = 0$ (to find x int)

$\sin\left(\theta + \frac{\pi}{6}\right) = 0$

$\theta + \frac{\pi}{6} = 0, \pi, 2\pi$

$\theta = -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

$\sqrt{3}\sin\left(\theta + \frac{\pi}{6}\right) = \sqrt{3}$

$\sin\left(\theta + \frac{\pi}{6}\right) = 1$

$\theta + \frac{\pi}{6} = \frac{\pi}{2}, \frac{5\pi}{2}$

(max pt) $\theta = \pi/3$

$\sqrt{3}\sin\left(\theta + \frac{\pi}{6}\right) = -\sqrt{3}$

$\sin\left(\theta + \frac{\pi}{6}\right) = -1$

$\theta = \frac{3\pi}{2} - \frac{\pi}{6}$

$= \frac{4\pi}{3}$ (min pt)

MATHEMATICS Extension 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
a) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n$		
Differentiate with respect to x:		
$n(1+x)^{n-1} = \binom{n}{1} + \binom{n}{2}2x + \binom{n}{3}3x^2 + \dots + \binom{n}{n}nx^{n-1}$	1	
Sub. $x=2$		
$n(1+2)^{n-1} = \binom{n}{1} + \binom{n}{2}2(2) + \binom{n}{3}3(2)^2 + \dots + \binom{n}{n}n2^{n-1}$	1	
$n \times 3^{n-1} = \binom{n}{1}(1) + \binom{n}{2}(2\times 2) + \binom{n}{3}(3\times 2^2) + \dots + \binom{n}{n}(n\times 2^{n-1})$		
b) $\cot^2\alpha + \cot^2\beta + \cot^2\gamma$		
$= \operatorname{cosec}^2\alpha - 1 + \operatorname{cosec}^2\beta - 1 + \operatorname{cosec}^2\gamma - 1$		
$= \frac{1}{\sin^2\alpha} + \frac{1}{\sin^2\beta} + \frac{1}{\sin^2\gamma} - 3$		
$= \frac{\sin^2\beta\sin^2\gamma + \sin^2\alpha\sin^2\gamma + \sin^2\alpha\sin^2\beta}{\sin^2\alpha\sin^2\beta\sin^2\gamma} - 3$	1	
$(\sin\alpha\sin\beta + \sin\alpha\sin\gamma + \sin\gamma\sin\beta)^2 =$		
$\sin^2\alpha\sin^2\beta + \sin^2\alpha\sin\beta\sin\gamma + \sin\alpha\sin^2\beta\sin\gamma$		
$+ \sin^2\alpha\sin\beta\sin\gamma + \sin^2\alpha\sin^2\gamma + \sin\alpha\sin\beta\sin^2\gamma$		
$+ \sin\alpha\sin^2\beta\sin\gamma + \sin\alpha\sin\beta\sin^2\gamma + \sin^2\gamma\sin^2\beta$		
$= \sin^2\alpha\sin^2\beta + \sin^2\alpha\sin^2\gamma + \sin^2\gamma\sin^2\beta$		
$+ 2\sin^2\alpha\sin\beta\sin\gamma + 2\sin\alpha\sin^2\beta\sin\gamma + 2\sin\alpha\sin\beta\sin^2\gamma$		

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
$= \sin^2\alpha\sin^2\beta + \sin^2\alpha\sin^2\gamma + \sin^2\gamma\sin^2\beta + 2\sin\alpha\sin\beta\sin\gamma(\sin\alpha + \sin\beta + \sin\gamma)$		
$\therefore \sin^2\alpha\sin^2\beta + \sin^2\alpha\sin^2\gamma + \sin^2\gamma\sin^2\beta = (\sin\alpha\sin\beta + \sin\alpha\sin\gamma + \sin\beta\sin\gamma)^2 - 2\sin\alpha\sin\beta\sin\gamma(\sin\alpha + \sin\beta + \sin\gamma)$	1	
$\therefore \cot^2\alpha + \cot^2\beta + \cot^2\gamma = \dfrac{(\sin\alpha\sin\beta + \sin\alpha\sin\gamma + \sin\beta\sin\gamma)^2 - 2\sin\alpha\sin\beta\sin\gamma(\sin\alpha + \sin\beta + \sin\gamma)}{(\sin\alpha\sin\beta\sin\gamma)^2} - 3$		
$= \dfrac{\left(-\frac{6}{8}\right)^2 - 2\left(\frac{\sqrt{5}}{8}\right)(0)}{\left(\frac{\sqrt{5}}{8}\right)^2} - 3$		
$= \dfrac{21}{5}$	1	