Student Name: ____________________

Maths class: ____________________

James Ruse Agricultural High School

2023 Task 2

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour and 30 minutes
- Write using black pen
- NESA approved calculators & templates may be used
- A reference sheet is provided
- In Question 6 – 12, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks: 54

Section I – 5 marks

- Attempt Questions 1–5
- Answer on the Multiple Choice answer sheet provided.
- Allow about 10 minutes for this section

Section II –49 marks

- Attempt Questions 6–12
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 1 hour 20 minutes for this section.

The answers to all questions are to be returned in separate stapled bundles clearly labelled Question 6, Question 7, etc. Each question must show your Candidate Number.

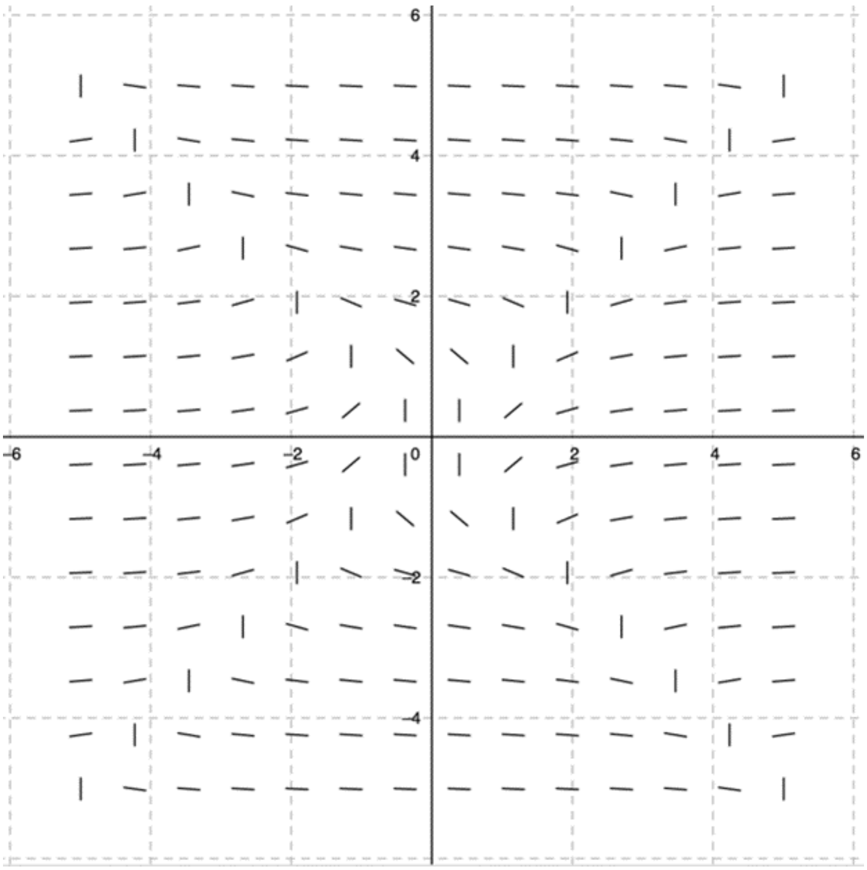
Section I

5 marks
Attempt Questions 1 to 5
Allow approximately 15 minutes for this section

Answer on the Multiple Choice sheet provided.

---

1. Which differential equation best describes the slope field shown? **1**



(A) $\dfrac{dy}{dx} = x^2 - y^2$

(B) $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$

(C) $\dfrac{dy}{dx} = \dfrac{1}{x^2 - y^2}$

(D) $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$

2. What is the value of $\tan\alpha$, when the expression $2\sin x - \cos x$ is written in the form $\sqrt{5}\sin(x-\alpha)$? **1**

(A) -2 (B) $\frac{-1}{2}$ (C) $\frac{1}{2}$ (D) 2

3. Out of a group of 10, 5 are to be selected for the final round of a competition. Three of those 5 will finish in 1st, 2nd and 3rd place. In how many ways can this be done? **1**

(A) $\frac{10!}{5!3!}$ (B) $\frac{10!}{5!}$ (C) $\frac{10!}{5!2!}$ (D) $\frac{10!}{3!3!}$

4. Consider $\int \frac{1}{2x}dx$, Which of the following is/are true?

a. $2\ln|x|+c$
b. $\frac{1}{2}\ln|x|+c$
c. $2\ln\frac{x}{2}+c$
d. $\frac{1}{2}\ln|2x|+c$ **1**

(A) b, c only

(B) d, c only

(C) b, d only

(D) a, c only

5. Which of the following is NOT a first order linear differential equation? **1**

(A) $x\frac{dy}{dx}+y\sqrt{x^2+1}=\sin x$

(B) $\frac{dy}{dx}=-e^x y$

(C) $\frac{1}{y}\frac{dy}{dx}-4=\frac{\cot x}{y}$

(D) $\frac{dy}{dx}-\frac{x^2}{y}=\ln x$

End of Section I

Section II

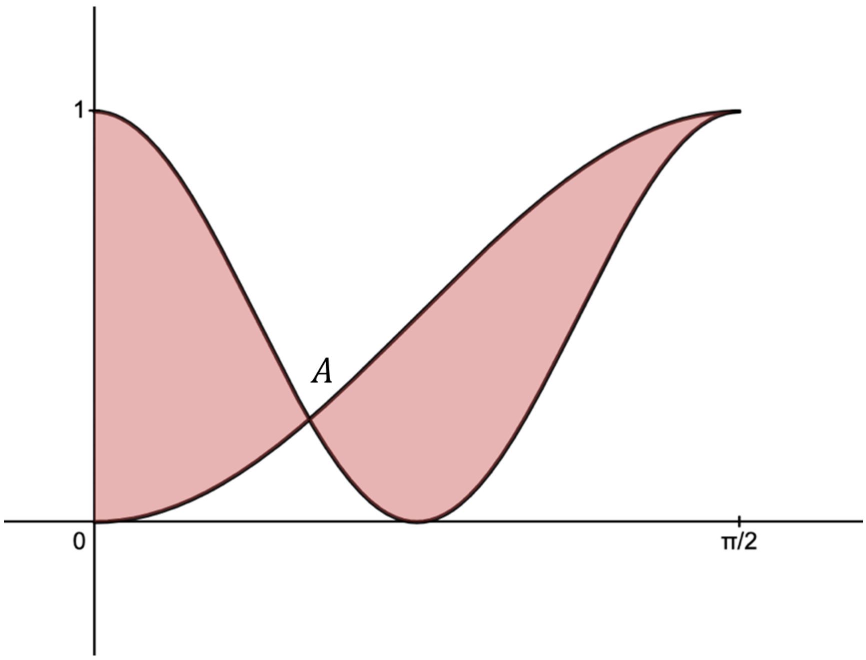
49 marks
Attempt Questions 7 to 10
Allow approximately 1 hour for this section.

Write your answers on the paper supplied.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (7 Marks)

(a) Using the substitution $u = \sqrt{x}$, Evaluate $\displaystyle\int_1^9 \frac{dx}{x+\sqrt{x}}$ **2**

(b) The curves $y = \cos^2 2x$ and $y = \sin^2 x$ are shown below for $0 \le x \le \frac{\pi}{2}$.



i. Find the x-coordinate of the point A. **2**

ii. Hence, find the area bounded by the two curves, the y-axis and the line $x = \frac{\pi}{2}$ **3**

End of Question 6

Question 7 (7 Marks)

(a) i. Given that $x = -2$ is a root of $x^3 + 12x^2 + 44x + 48 = 0$, find the other two roots. **2**

ii. Hence, solve the following equation, giving your answers to the nearest degree

$$\sec\theta(\sec^2\theta + 44) + 12(\tan^2\theta + 5) = 0, \qquad 0^\circ \leq \theta \leq 180^\circ$$

3

(b) The displacement function x travelled by a particle on a straight line in time t is given by

$$x = 100e^{-0.1t}$$

i. Derive the velocity $\dot{x}$ function in terms of t. **1**

ii. Describe the motion of the particle. **1**

End of Question 7

Question 8 (7 Marks)

(a) i. Show that $6\cos\theta - 8\sin\theta$ can be written in the form $R\cos(\theta + \alpha)$, where $R > 0$ and α is an acute angle measured in radians. **2**

ii. Hence solve the equation $3\cos\theta - 4\sin\theta - 2 = 0$ for $0 \leq \theta \leq 2\pi$, giving your answers to three significant figures. **2**

(b) Prove by mathematical induction that $2^{n+2} + 3^{3n}$ is divisible by 5 for $n \in \mathbb{Z}, \quad n \geq 1$ **3**

End of Question 8

Question 9 (7 Marks)

(a) Find the number of ways three letters from the word QUARRY can be arranged if all three letters are to be distinct. **2**

(b) Given $f(x) = 2\cos^{-1}\dfrac{x}{\sqrt{2}} - \sin^{-1}(1 - x^2), \quad -\sqrt{2} \leq x \leq \sqrt{2}$

i. Find $f'(x)$. **3**

ii. Sketch $f(x)$. **2**

End of Question 9

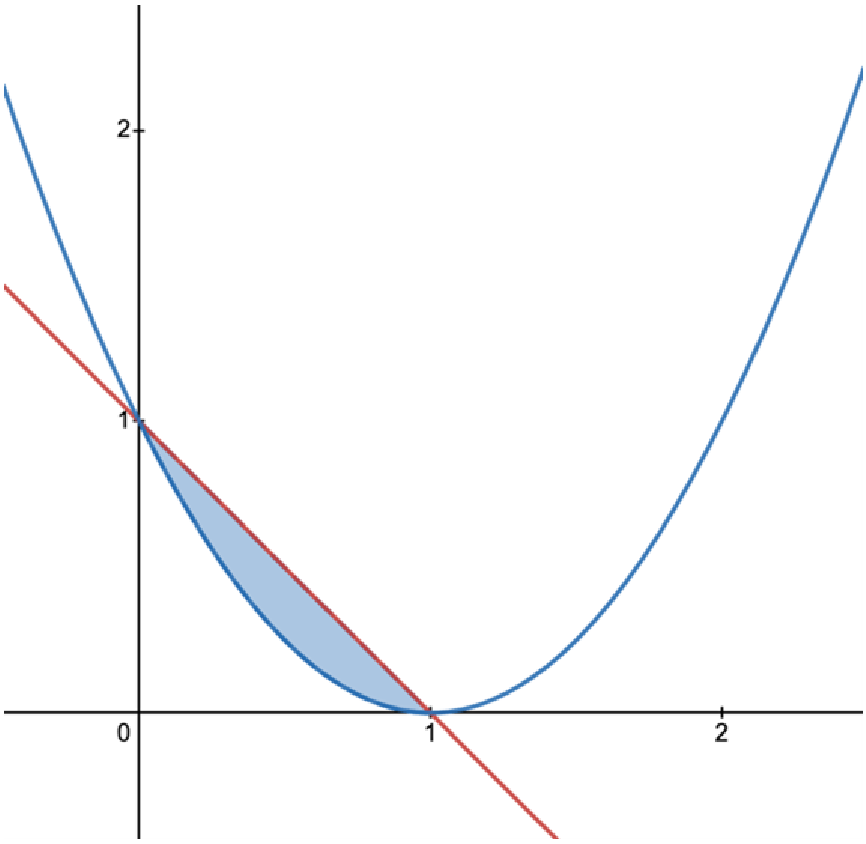
Question 10 (7 Marks)

(a) Nine shirts for a baseball team are numbered from 1 to 9. Five players are allowed to take one shirt each. Find the number of ways this can be done is

i. There are no restrictions. **1**

ii. The first three players all decide to choose an even numbered shirt. **2**

(b) The diagram below shows the region bound by $y = 1 - x$ and $y = (x-1)^n$ for even $n \leq 2$.



i. The region is rotated about the x-axis to form a solid. Show that the volume of the solid formed is $V_n = \pi[\frac{1}{3} - \frac{1}{(2n+1)}]$. **2**

ii. Find the limit of V_n as $n \to \infty$, and describe the physical significance of this result. **2**

End of Question 10

Question 11 (7 Marks)

(a) Find $\int \frac{\sin 2x}{\sqrt{1-\sin^2 x}} dx$. **3**

(b) Prove by mathematical induction that for n positive integer,

$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3++n\left(\frac{1}{2}\right)^{n-1}=4-\frac{(n+2)}{2^{n-1}}$$

4

End of Question 11

Question 12 (7 Marks)

(a) Given that $\sin^2 2\theta(\cot^2\theta-\tan^2\theta)=4\cos 2\theta$.

Solve the equation: $\sin^2 2\theta(\cot^2\theta-\tan^2\theta)=2$ for $0^\circ \leq \theta \leq 360^\circ$

2

(b) The curve $y=-\ln(x+1)$, the y-axis and the line $y=1$ enclose a region.

i. Sketch this region, clearly showing intercepts with the axes and asymptotes. **2**

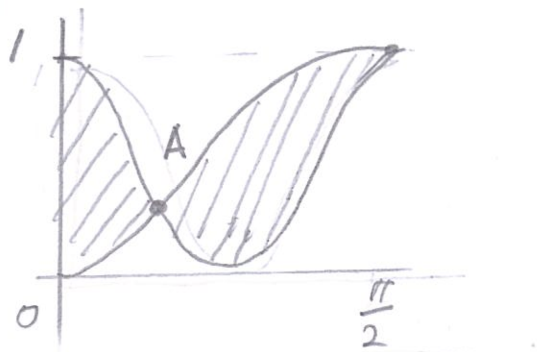
ii. This region is rotated around the y-axis to form a solid of revolution. Find the volume of this solid. **3**

End of Question 12

End of paper.

MC Answers
1C 2C 3C 4C 5D

MATHEMATICS Extension 1: Question …6 Task 2 2023

Suggested Solutions	Marks Awarded	Marker's Comments
a) $u=\sqrt{x} \Rightarrow x=u^2$, $\frac{dx}{du}=2u$ $\int_1^9 \frac{dx}{x+\sqrt{x}}$ when $x=1$, $u=1$; $x=9$, $u=3$ $\int_1^3 \frac{1}{u^2+u} \cdot 2u\,du$ $\int_1^3 \frac{1}{\cancel{u}(u+1)} \cdot 2\cancel{u}\,du$ $=2\left[\ln(1+u)\right]_1^3$ $=2(\ln 4-\ln 2)$ $=2\ln 2$ or 1.39	$u=1$ $u=3$ 1 mark.	1 mark for correct boundaries and expression in terms of u NOT a mix of u and x
b)  i) $\cos^2 2x=\sin^2 x$ $(\cos 2x)^2=\sin^2 x$	METHOD 1	

MATHEMATICS Extension 1 : Question 6		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
$(2\cos^2 x - 1)^2 = 1 - \cos^2 x$ $4\cos^4 x - 4\cos^2 x + 1 = 1 - \cos^2 x$ $4\cos^4 x - 3\cos^2 x = 0$	1	
$\cos^2 x(4\cos^2 x - 3) = 0$ $\cos^2 x = 0$ or $2\cos^2 x = 3$ $\cos x = 0$ $\quad \cos x = \pm\frac{\sqrt{3}}{2}$ $0 \le x \le \frac{\pi}{2}$ $\quad \therefore$ x coordinate of $A = \frac{\pi}{6}$	1	have to show all solution and reject those not in domain.
METHOD 2 $(1 - 2\sin^2 x)^2 = \sin^2 x$ $1 - 4\sin^2 x + 4\sin^4 x = \sin^2 x$ $1 - 5\sin^2 x + 4\sin^4 x = 0$	1	
$4\sin^2 x \quad -1$ $\sin^2 x \quad -1$ $4\sin^2 x = 1$ $\quad \sin^2 x = 1$ $\sin x = \pm\frac{1}{2}$ $\quad \sin x = \pm 1$ $\therefore A = \left(\frac{\pi}{6}, \frac{1}{4}\right)$	1	including all possible solutions

3/3

MATHEMATICS Extension 1: Question 6

Suggested Solutions	Marks Awarded	Marker's Comments
ii Area bounded by the 2 curves		correct bounds and correct curves subtraction.
$\int_0^{\frac{\pi}{6}} \cos^2 2x - \sin^2 x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 x - \cos^2 2x \, dx$		1
* $\cos 2x = \cos^2 x - \sin^2 x$ $= 1 - \sin^2 x - \sin^2 x$ $= 1 - 2\sin^2 x$.	$\sin^2 x + \cos^2 x = 1$, $\cos^2 x = 1 - \sin^2 x$	
$2\sin^2 x = 1 - \cos 2x$		
$\sin^2 x = \frac{1 - \cos 2x}{2}$ — i		
* $\cos^2 2x - \sin^2 2x = \cos 4x$		
$\cos^2 2x - (1 - \cos^2 2x) = \cos 4x$		
$2\cos^2 2x - 1 = \cos 4x$		
$\cos^2 2x = \frac{\cos 4x + 1}{2}$ — ii		
$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (1 + \cos 4x) - (1 - \cos 2x) dx + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - \cos 2x) - (1 + \cos 4x) dx$		
$= \frac{1}{2}\left[\cancel{x} + \frac{\sin 4x}{4} - \cancel{x} + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{6}} + \frac{1}{2}\left[\cancel{x} - \frac{\sin 2x}{2} - \left(\cancel{x} + \frac{\sin 4x}{4}\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$		
$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{6}} + \frac{1}{2}\left[-\frac{\sin 2x}{2} - \frac{\sin 4x}{4}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$		1
$= \frac{1}{2}\left(\frac{\sin 240}{4} + \frac{\sin 120}{2} - 0\right) + \frac{1}{2}\left[\left(-\frac{\sin 120}{2} - \frac{\sin 240}{4}\right) - \left(-\frac{\sin 180}{2} - \frac{\sin 360}{4}\right)\right]$		

$= \frac{3\sqrt{3}}{8}$ or 0.6495 — 1

MATHEMATICS EXT 1: Question 7

Suggested Solutions	Marks	Marker's Comments
ai, Since $x=-2$ is a zero, then		
$x^3+12x^2+44x+48=(x+2)(Ax^2+Bx+C)$		
Comparing coefficients:		
$x^3: A=1$		
$x^0: 2C=48 \Rightarrow C=24$	1	Progress
$x^2: B+2A=12 \Rightarrow B=10$		
$\therefore P(x)=(x+2)(x^2+10x+24)$		Could have used
$=(x+2)(x+4)(x+6)$		long division too.
$\therefore$ roots are $-2,-4,-6$	1	Final
ii, $\sec\theta(\sec^2\theta+44)+12(\tan^2\theta+5)=0$		
$\sec^3\theta+44\sec\theta+12(\sec^2\theta+4)=0$, using $1+\tan^2\theta=\sec^2\theta$	1	Using identity
$\sec^3\theta+12\sec^2\theta+44\sec\theta+48=0$		
$\therefore \sec\theta=-2,-4,-6$, from (i)	1	using (i)
$\cos\theta=-\frac{1}{2},-\frac{1}{4},-\frac{1}{6}$		
$\therefore \theta=100°, 104°, 120°$ for $0\le\theta\le180°$	1	final
		Not marked down if you didn't give answer to nearest degree, but READ THE QUESTION!

MATHEMATICS EXT 1: Question 7

Suggested Solutions	Marks	Marker's Comments
b i, $x = 100e^{-0.1t}$		
$\dot{x} = -0.1 \times 100e^{-0.1t}$		
$\dot{x} = -10e^{-0.1t}$	1	Final
ii, $t=0$, $x = 100$		
$t \to \infty$, $x \to 0^+$		
$t \to \infty$, $\dot{x} \to 0^-$		
∴ The particle starts at a displacement of 100 and gradually approaches, but never reaches, the origin.	1	Must have underlined phrases.
The particle doesn't reach the origin at $t = \infty$.		

MATHEMATICS EXT2: Question 8

Suggested Solutions	Marks	Marker's Comments
c i) $4\cos x - 8\sin x = (2B - A)\cos x + (-2A - B)\sin x$		
$\Rightarrow \begin{cases} 4 = 2B - A \\ -8 = -2A - B \end{cases}$		
Solving simultaneously:		
$A = \frac{12}{5} \quad B = \frac{16}{5}$	2	One for each.
ii) $\int \frac{4\cos x - 8\sin x}{2\cos x - \sin x}\,dx$		
$= \int \frac{\frac{12}{5}(-2\sin x - \cos x) + \frac{16}{5}(2\cos x - \sin x)}{2\cos x - \sin x}\,dx$	1	Using part (i)
$= \frac{12}{5}\ln\lvert 2\cos x - \sin x\rvert + \frac{16}{5}x + C$	1	Final answer.

Question 8

a) i)
$$6\cos\theta - 8\sin\theta = R\cos(\theta+\alpha)$$
$$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

$$\therefore \begin{cases} R\cos\alpha = 6 & (1) \\ R\sin\alpha = 8 & (2) \end{cases}$$

$(1)^2 + (2)^2$:
$$R^2\cos^2\alpha + R^2\sin^2\alpha = 6^2 + 8^2$$
$$R^2 = 100$$
$$\therefore R = 10 \quad (R>0)$$ ✓

$R = \sqrt{6^2+8^2}$: not accepted
'Show Q'

$\frac{(2)}{(1)}$:
$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{8}{6}$$
$$\tan\alpha = \frac{4}{3}$$
$$\therefore \alpha = \tan^{-1}\frac{4}{3}$$
$$\approx 0.927$$

$\angle$ in deg : not accepted
(read Q)

$$\therefore 6\cos\theta - 8\sin\theta = 10\cos(\theta + 0.927)$$ ✓

$\tan^{-1}\frac{4}{3}$
$\cos^{-1}\frac{3}{5}$ or $\sin^{-1}\frac{4}{5}$ allowed

ii)
$$3\cos\theta - 4\sin\theta - 2 = 0 \quad , \quad 0 \le \theta \le 2\pi$$
$$3\cos\theta - 4\sin\theta = 2$$
$$5\cos(\theta+0.927) = 2 \quad , \quad 0.927 \le \theta + 0.927 \le 2\pi + 0.927$$
$$\cos(\theta+0.927) = \frac{2}{5}$$ ✓

$$\theta + 0.927 = \cos^{-1}\frac{2}{5} \quad , \quad 2\pi - \cos^{-1}\frac{2}{5}$$
$$= 1.159 \quad , \quad 5.124$$

4.197 allowed this time

$$\therefore \theta = 0.232 \quad , \quad 4.20 \quad (3 \text{ sig.fig.}) \quad , \quad 0 \le \theta \le 2\pi$$ ✓

$\cos^{-1}\frac{2}{5} - \tan^{-1}\frac{4}{3}$, $2\pi - \cos^{-1}\frac{2}{5} - \tan^{-1}\frac{4}{3}$

b) $2^{n+2} + 3^{3n}$ is divisible by 5, $n \in \mathbb{Z}$, $n \geq 1$

- Test for $n = 1$

$2^{1+2} + 3^{3(1)} = 8 + 27$

$= 35$

$5 \mid 35$ $\therefore$ true for $n=1$ ✓

- Assume the statement is true for $n=k$, $k \in \mathbb{Z}$, $k \geq 1$

i.e. $2^{k+2} + 3^{3k} = 5P$, where $P \in \mathbb{Z}$ *

- Prove that the statement is true for $n = k+1$

i.e. $2^{(k+1)+2} + 3^{3(k+1)} = 5Q$, $Q \in \mathbb{Z}$

$$\begin{aligned}
LHS &= 2^{k+3} + 3^{3k+3} \\
&= 2^1(2^{k+2}) + 3^3(3^{3k}) \\
&= 2(5P - 3^{3k}) + 27(3^{3k}) \quad \text{using assumption ✓} \\
&= 10P - 2(3^{3k}) + 27(3^{3k}) \\
&= 10P + 25(3^{3k}) \\
&= 5(2P + 5(3^{3k})) \\
&= 5Q, \quad \text{where } Q = 2P + 5(3^{3k}) \in \mathbb{Z} \text{ *} \\
&= RHS
\end{aligned}$$

not stating : max 2

✓

$\therefore$ The statement is true for $n = k+1$, if it is true for $n=k$.

- Hence, by mathematical induction, the statement is true for all integers $n \geq 1$.

Question 9 :

(a)

QUA $3!$ UAR $3!$

QUY $3!$ YAR $3!$

QAY $3!$ YUR $3!$

UAY $3!$ QYR $3!$

QUR $3!$ QAR $3!$

$3! \times 10 = 60$ ways

$$\frac{\binom{6}{3}}{2!} \times 3! = 60 \text{ ways}$$

$$\binom{5}{3} \times 3! = 60 \text{ ways.}$$

2 marks for working + correct answer

-1 mark for not showing enough working or the final answer is wrong.

(6) $f(x) = 2\cos^{-1}\left(\frac{x}{\sqrt{2}}\right) - \sin^{-1}(1-x^2) \qquad -\sqrt{2} \le x \le \sqrt{2}$

i) $f'(x) = 2\left(\frac{-1}{\sqrt{1-\frac{x^2}{2}}} \times \frac{1}{\sqrt{2}}\right) - \frac{-2x}{\sqrt{1-(1-x^2)^2}}$

$f'(x) = \frac{-\sqrt{2}}{\sqrt{\frac{2-x^2}{2}}} + \frac{2x}{\sqrt{1-(1-x^2)^2}}$

$= \frac{-2}{\sqrt{2-x^2}} + \frac{2x}{\sqrt{\cancel{1}-\cancel{1}(1-2x^2+x^4)}}$

$= \frac{-2}{\sqrt{2-x^2}} + \frac{2x}{\sqrt{-x^4+2x^2}}$

$f'(x) = \frac{-2}{\sqrt{2-x^2}} + \frac{2x}{|x|\sqrt{2-x^2}}$

If $x>0$, $f'(x) = \frac{-2}{\sqrt{2-x^2}} + \frac{2 \cdot x}{x\sqrt{2-x^2}}$

If $x>0$, $f'(x) = \frac{-2}{\sqrt{2-x^2}} + \frac{2}{\sqrt{2-x^2}} = 0$

If $x<0$, $f'(x) = \frac{-2}{\sqrt{2-x^2}} + \frac{2x}{-x\sqrt{2-x^2}} = \frac{-4}{\sqrt{2-x^2}} < 0$

$x=0$ $f'(x)$ is undefined.

ii) Sketch $f(x)$

$x > 0 \quad f'(x) = 0 \quad \therefore f(x)$ is a constant.

When $x = 1$, $f(1) = 2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) - \cancel{\sin^{-1}(0)}$ 0

$f(1) = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$

$\therefore f(x) = \frac{\pi}{2}$ for $x > 0$

When $x = 0$, $f(0) = 2\cos^{-1}(0) - \sin^{-1} 1$

$f(0) = 2\frac{\pi}{2} - \frac{\pi}{2} = \frac{\pi}{2}$

When $x = -1$, $f(-1) = 2\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) - \cancel{\sin^{-1}(0)}$ 0

$f(-1) = 2\left(\frac{3\pi}{4}\right) - 0 = \frac{3\pi}{2}$

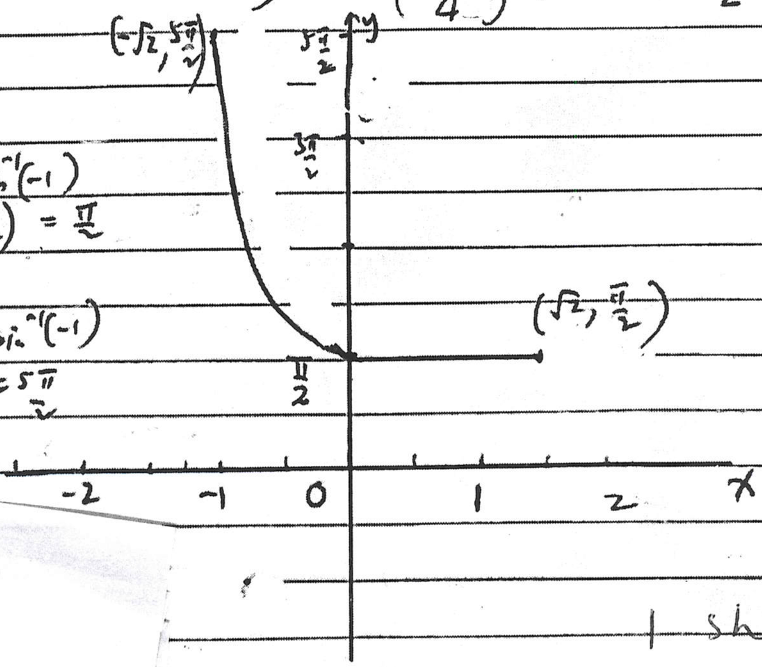
When $x = \sqrt{2}$

$f(\sqrt{2}) = 2\cos^{-1}(1) - \sin^{-1}(-1)$

$= 2(0) - \left(\frac{-\pi}{2}\right) = \frac{\pi}{2}$

$f(-\sqrt{2}) = 2\cos^{-1}(-1) - \sin^{-1}(-1)$

$= 2(\pi) + \frac{\pi}{2} = \frac{5\pi}{2}$



1 shape

1 ~~two exact value in different branch~~

Question 10

a)

i. $^{9}P_5 = 15120$ 1 mark for answer

ii. $^{4}P_3 \times {}^{6}P_2 = 720$ 1 mark for logical explanation

1 mark for final answer

b)

i.

$$V_n = \pi \int_0^1 (1-x)^2\,dx - \pi \int_0^1 (x-1)^{2n}\,dx$$

1 mark for correct integral

$$= \pi\left[\frac{-(1-x)^3}{3}\right]_0^1 - \pi\left[\frac{(x-1)^{2n+1}}{2n+1}\right]_0^1$$

$$= \pi\left[0-\left(-\frac{1}{3}\right)\right] - \pi\left[0-\left(\frac{-1}{2n+1}\right)\right]$$

$(\text{since } (-1)^{2n+1} = -1 \text{ as } 2n+1 \text{ is odd})$

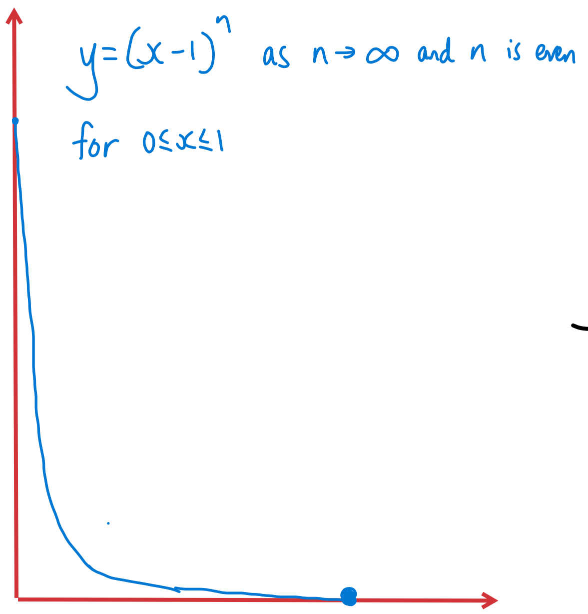
$$= \pi\left(\frac{1}{3} - \frac{1}{2n+1}\right)$$

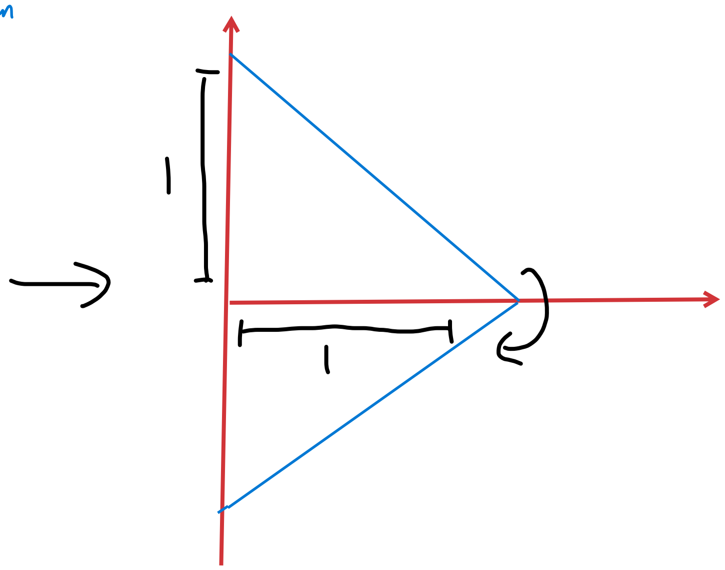
1 mark for arriving to answer with all steps

ii. $\lim_{n\to\infty}\left[\pi\left(\frac{1}{3}-\frac{1}{2n+1}\right)\right]$

$= \frac{\pi}{3}$

1 mark





As $n \to \infty$, the graph $y = (x-1)^n$ for $0 \le x \le 1$ approaches a horizontal line, therefore the area bound by the two curves approaches a right-angled triangle. This means when the area is rotated about the x-axis, the solid form approaches that of a cone with radius 1 and height 1. The volume of that is given by $V = \frac{1}{3}\pi(1)^2(1) = \frac{\pi}{3}$

1 mark for clear explanation of how the cone is derived, as well as its radius and height.

MATHEMATICS Extension 1 : Question 11

Suggested Solutions	Marks	Marker's Comments
a) $\int \frac{\sin 2x}{\sqrt{1-\sin^2 x}}\,dx$	1	Cases not penalised.
$= \int \frac{2\sin x\cos x}{\lvert\cos x\rvert}\,dx$	1	No sub given means no sub necessary.
Now for $\cos x > 0$ (i.e. Quadrants 1 & 4)		$-2\sqrt{1-\sin^2 x}+C$ also paid full marks.
$\int 2\sin x\,dx = -2\cos x + C$	1	
For $\cos x < 0$ (i.e. Quadrants 2 & 3)		
$\int -2\sin x\,dx = 2\cos x + C$		
b) Show true for $n=1$		
LHS $=1$		
RHS $= 4 - \frac{(1+2)}{2^0}$		
$= 4-3$		
$=1$	1	
$\therefore$ True for $n=1$		
Assume true for $n=k$		
i.e. $1+2(\frac{1}{2})+3(\frac{1}{2})^2+\ldots+k(\frac{1}{2})^{k-1} = 4-\frac{(k+2)}{2^{k-1}}$		
Prove true for $n=k+1$		
$1+2(\frac{1}{2})+3(\frac{1}{2})^2+\ldots+k(\frac{1}{2})^{k-1}+(k+1)(\frac{1}{2})^k$		
$= 4-\frac{(k+2)}{2^{k-1}}+\frac{(k+1)}{2^k}$ by assumption	1	Must state by assumption.
$= 4-\frac{2(k+2)}{2^k}+\frac{(k+1)}{2^k}$	1	
$= 4-\frac{2(k+2)-(k+1)}{2^k}$		
$= 4-\frac{k+3}{2^k}$ = LHS	1	

MATHEMATICS Extension 1 : Question 11

Suggested Solutions	Marks	Marker's Comments
OR		
Prove true for $n=k+1$		
$1+2(\frac{1}{2})+3(\frac{1}{2})^2+\dots+k(\frac{1}{2})^{k-1}+(k+1)(\frac{1}{2})^k$		
$=4-\frac{(k+2)}{2^{k-1}}+\frac{(k+1)}{2^k}$ by assumption	1	Must state by assumption.
$=4-\frac{1}{2^{k-1}}\left[k+2-\frac{(k+1)}{2}\right]$	1	
$=4-\frac{1}{2^{k-1}}\left(\frac{k}{2}+\frac{3}{2}\right)$		CARE needed in algebra and negatives.
$=4-\frac{k+3}{2^k}=\text{LHS}$	1	DON'T fudge your way through. Go back and check!
$\therefore$ true for $n=k+1$ if true for $n=k$		
$\therefore$ Since $n=1$ is true then from above $n=1+1=2$ is true, $n=2+1=3$ is true and so on for all positive integers n.		

1 Question 12

(a) Since $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 4\cos 2\theta$ (given)

To solve $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 2$, for $0° \le \theta \le 360°$

i.e. $4\cos 2\theta = 2$, for $0° \le 2\theta \le 720°$, (and $\theta \ne n \times 90°$, $n \in Z$)

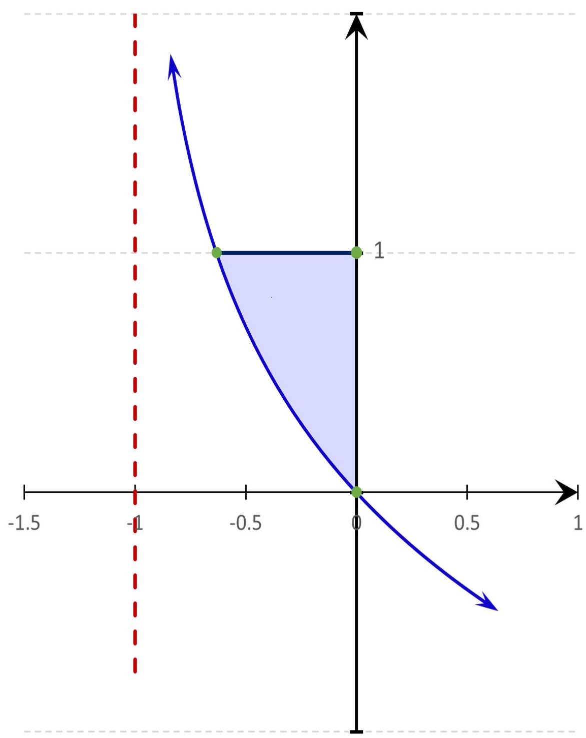
$$\cos 2\theta = \frac{1}{2}$$

$$2\theta = 60°, 300°, 420°, 660° \qquad \theta = 30°, 150°, 210°, 330°$$

2 marks for all the correct answers

(b)

(i)



1 mark for correct region

1 mark for correct shape, intercepts and asymptotes

(ii)

$$y = -ln(x+1)$$
$$x = e^{-y} - 1$$

1 mark

$$\therefore \qquad dv = \pi(e^{-y} - 1)^2 dy$$

$$V = \int_0^1 \pi(e^{-y} - 1)^2 \, dy$$

1 mark

$$= \pi \left[-\frac{1}{2}e^{-2y} + 2e^{-y} + y \right]_0^1$$

$$= \pi \left[\frac{2}{e} - \frac{1}{2e^2} - \frac{1}{2} \right] \ \text{unit}^3$$

1 mark

$$\text{or} = 0.528 \ \text{unit}^3$$