Name: ____________________________

Class: ____________________________

James Ruse Agricultural High School

2023 Year 12 Task 3

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 80 minutes
- Write using black pen
- Calculators approved by NESA may be used NESA Reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Write your student number at the top of every page of your answer sheets.

Total marks:35

Section I – 5 marks

- Attempt Questions 1–5 on the Multiple Choice Answer Sheet.
- Allow about 7 minutes for this section

Section II – 30 marks

- Attempt Questions 6–10
- **Start EACH Question on a new sheet of paper.**
- Allow about 1 hour and 13 minutes for this section

This paper MUST NOT be removed from the examination room.

Section I

5 marks
Attempt Questions 1 – 5
Allow about 7 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 5.

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1. Two forces $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ N and $\begin{pmatrix} -7 \\ -5 \end{pmatrix}$ N act on a particle. What is the resultant force?

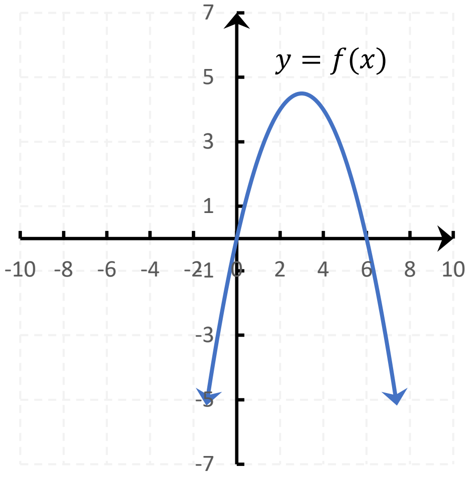
   A. $\begin{pmatrix} -21 \\ 10 \end{pmatrix}$ N

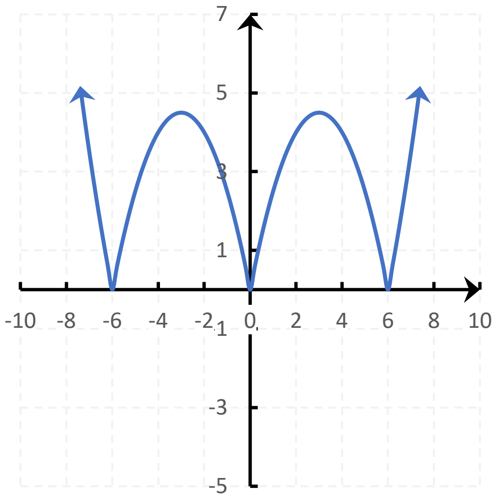
   B. $\begin{pmatrix} -4 \\ -7 \end{pmatrix}$ N

   C. $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ N

   D. $\begin{pmatrix} 10 \\ 3 \end{pmatrix}$ N

2. The graph $y = f(x)$ is sketched below, under which transformation will the diagram transform to the one on the right?





   A. $y = |f(x)|$

   B. $y = f(|x|)$

   C. $|y| = f(x)$

   D. $y = |f(|x|)|$

3. Consider a random variable that is normally distributed with a mean of 0 and standard deviation of 1. Define $\boldsymbol{\Phi(x) = P(X \leq x)}$ for $\boldsymbol{x > 0}$. Which of the following is NOT true?

   A. $\boldsymbol{\Phi(-x) = 1 - \Phi(x)}$

   B. $\boldsymbol{P(|X - x| \leq 1) = \Phi(x+1) - \Phi(x-1)}$

   C. $\boldsymbol{P(|X| \leq x) = 2\Phi(x) - 1}$

   D. $\boldsymbol{P(|X| \geq x) = 2 - \Phi(x)}$

4. Let $\underset{\sim}{a}$ and $\underset{\sim}{b}$ be two-unit vectors and θ is the angle between them. Then $\underset{\sim}{a} + \underset{\sim}{b}$ is a unit vector if

   A. $\theta = \frac{\pi}{4}$

   B. $\theta = \frac{\pi}{3}$

   C. $\theta = \frac{\pi}{2}$

   D. $\theta = \frac{2\pi}{3}$

5. Two forces, $F_A = 4\underset{\sim}{i} - 2\underset{\sim}{j}$ and $F_B = 2\underset{\sim}{i} + 4\underset{\sim}{j}$, act on a particle. The particle is initially at rest at position $\underset{\sim}{i} + \underset{\sim}{j}$. All components are measured in newtons and displacements are measured in meters. The Cartesian equation of the path of the particle is

   A. $y = -3x - 1$

   B. $y = \frac{x}{3} - \frac{2}{3}$

   C. $y = 3x + 1$

   D. $y = \frac{x}{3} + \frac{2}{3}$

Section II

42marks

Attempt Questions 6 – 12

Allow about 1 hour and 13 minutes for this section.

Answer each question on a new page. Extra writing pages are available.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 Start on a new page. 7 marks

a) The parametric equations of a curve are given by

$$\begin{matrix} x = \cos\theta + \sin\theta - 2 \\ y = \sin 2\theta \end{matrix}, \qquad \mathbf{0 \le \theta \le 2\pi}$$

i) Show that the cartesian equation of the curve is **2**

$$\boldsymbol{y = x^2 + 4x + 3}$$

ii) Sketch the curve which corresponds to the above parametric equations, showing the turning points, *x*- and *y*- intercepts, and the exact coordinates of the endpoints, if applicable. **2**

b) Consider a random variable that is normally distributed with a mean of 3 and a standard deviation of σ, and $\frac{\boldsymbol{P(X<1)}}{\boldsymbol{P(X<5)}} = \frac{\mathbf{1}}{\mathbf{9}}$. **3**
Find the value of $\boldsymbol{P(|X-4|<1)}$

Question 7 Start on a new page. 7 marks

a) Consider the points $A\ (1,1)$ and $B\left(\frac{\sqrt{3}}{2}-\frac{1}{2}, \frac{\sqrt{3}}{2}+\frac{1}{2}\right)$.

i) Find the value of $\angle AOB$ where O is the origin. **2**

ii) Hence, calculate the area of triangle AOB. **2**

b) Let $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 4\underset{\sim}{j}$, where the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is θ. **3**

Find the value of $\cos 2\theta$.

Question 8 **Start on a new page.** **7 marks**

a) The probability density function of the random variable $\boldsymbol{X}$ is given by **3**

$$f(x) = \begin{cases} \dfrac{kx}{\sqrt{4-x^2}}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of constant $\boldsymbol{k}$

b) The quadrilateral $OABC$ has $\overrightarrow{OA} = 4\underset{\sim}{i} + 2\underset{\sim}{j}$, $\overrightarrow{OB} = 6\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\overrightarrow{OC} = 8\underset{\sim}{i} - 20\underset{\sim}{j}$.

Find $\overrightarrow{AB}$ and hence show that $OABC$ is a trapezium. **2**

c) The vectors $\underset{\sim}{a} + 3\underset{\sim}{b}$ and $4\underset{\sim}{a} - \underset{\sim}{b}$ are perpendicular, and $|\underset{\sim}{a}| = 2|\underset{\sim}{b}|$. Determine the angle between the nonzero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

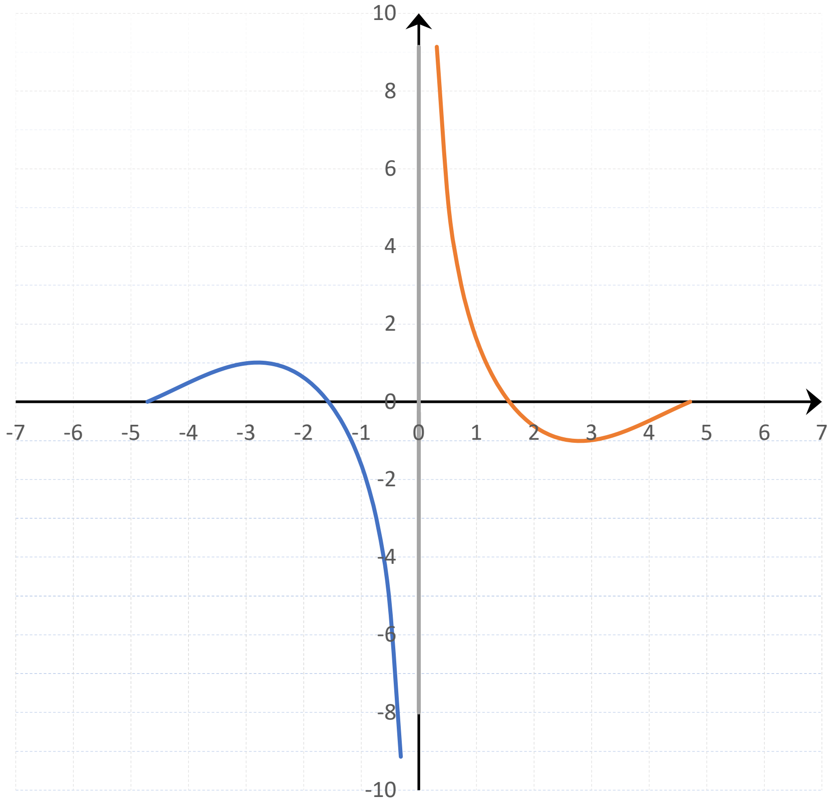
Question 9 **Start on a new page.** **7 marks**

a) Given the function $\boldsymbol{y = (x-1)(|x-3|)}$, $\boldsymbol{x \in R}$

i) Sketch the function. **2**

ii) Hence solve $\boldsymbol{(x-1)(|x-3|) = 1}$, leave your answer in exact values. **1**

b) The diagram shows the graph of a function $y = f(x)$ over its domain.



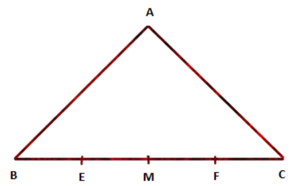
Draw the following function on the sheet provided, clearly showing any transformation of the turning points, *x*- and *y*- intercepts, and asymptotes, as applicable for:

i) $\boldsymbol{y = f(|x|)}$ **2**

ii) $\boldsymbol{y = \frac{1}{f(x)}}$ **2**

Question 10 **Start on a new page.** **7 marks**

a) In triangle ABC below, given that $|\overrightarrow{BE}| = |\overrightarrow{EM}| = |\overrightarrow{MF}| = |\overrightarrow{FC}|$



Prove that $\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{AM} + \overrightarrow{AF} + \overrightarrow{AC} = \frac{5}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$ **2**

b) A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of $N22°E$ as shown in the diagram. **2**
How far east and north is the airplane from the airport when sighted?

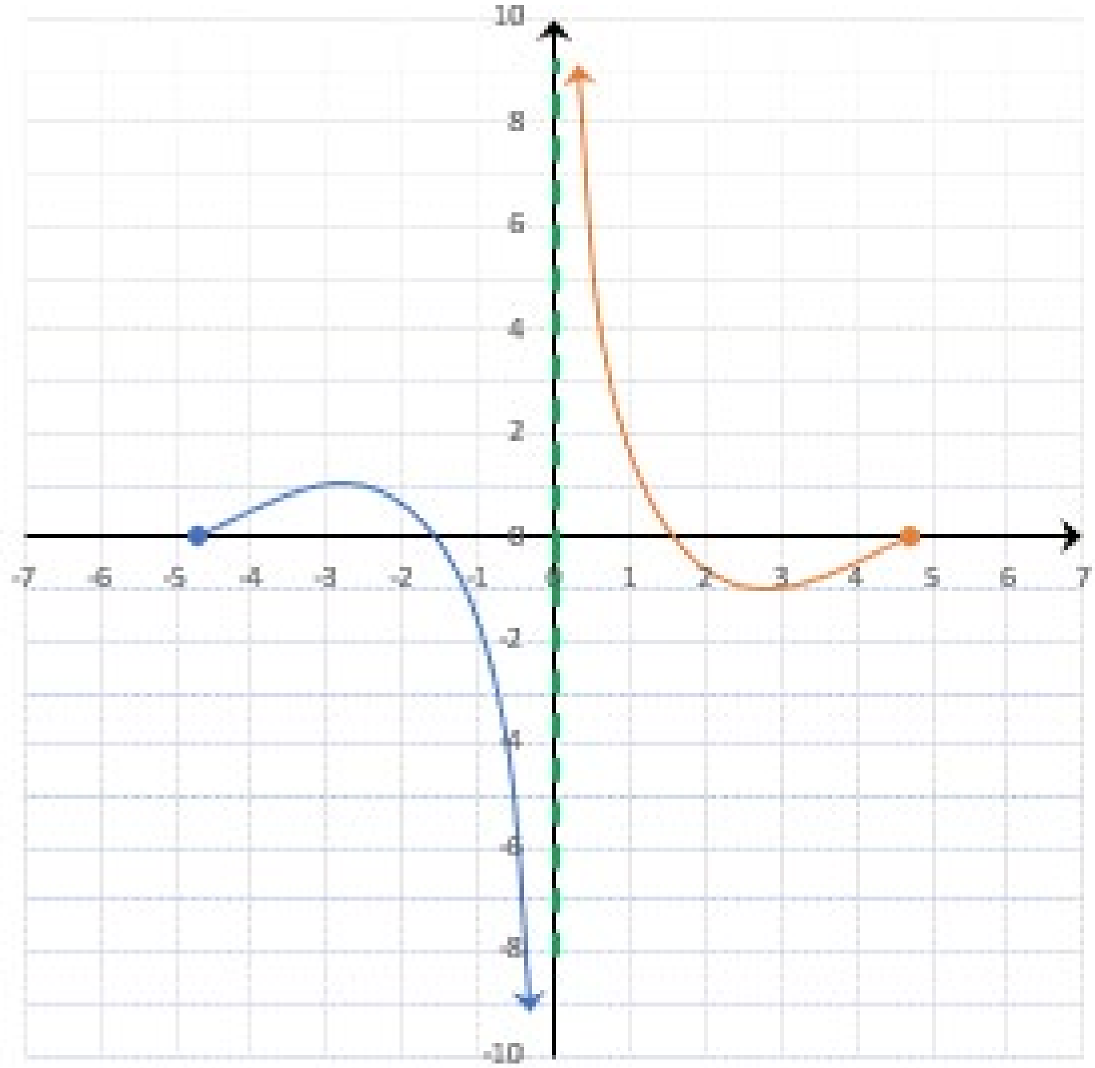
c) The point A and B have respective position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, relative to a fixed origin 0. The point C lies on $\overrightarrow{AB}$ produced such that $|\overrightarrow{AB}|:|\overrightarrow{AC}| = 1:4$. The point D lies on $\overrightarrow{OB}$produced such that $|\overrightarrow{OB}|:|\overrightarrow{OD}| = 1:k$, where k is a scalar constant. Given that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$, show that: **3**

$$k = \frac{3|\underset{\sim}{a}|^2 - 7\underset{\sim}{a}.\underset{\sim}{b} + 4|\underset{\sim}{b}|^2}{|\underset{\sim}{b}|^2 - \underset{\sim}{a}.\underset{\sim}{b}}$$

- End of examination -

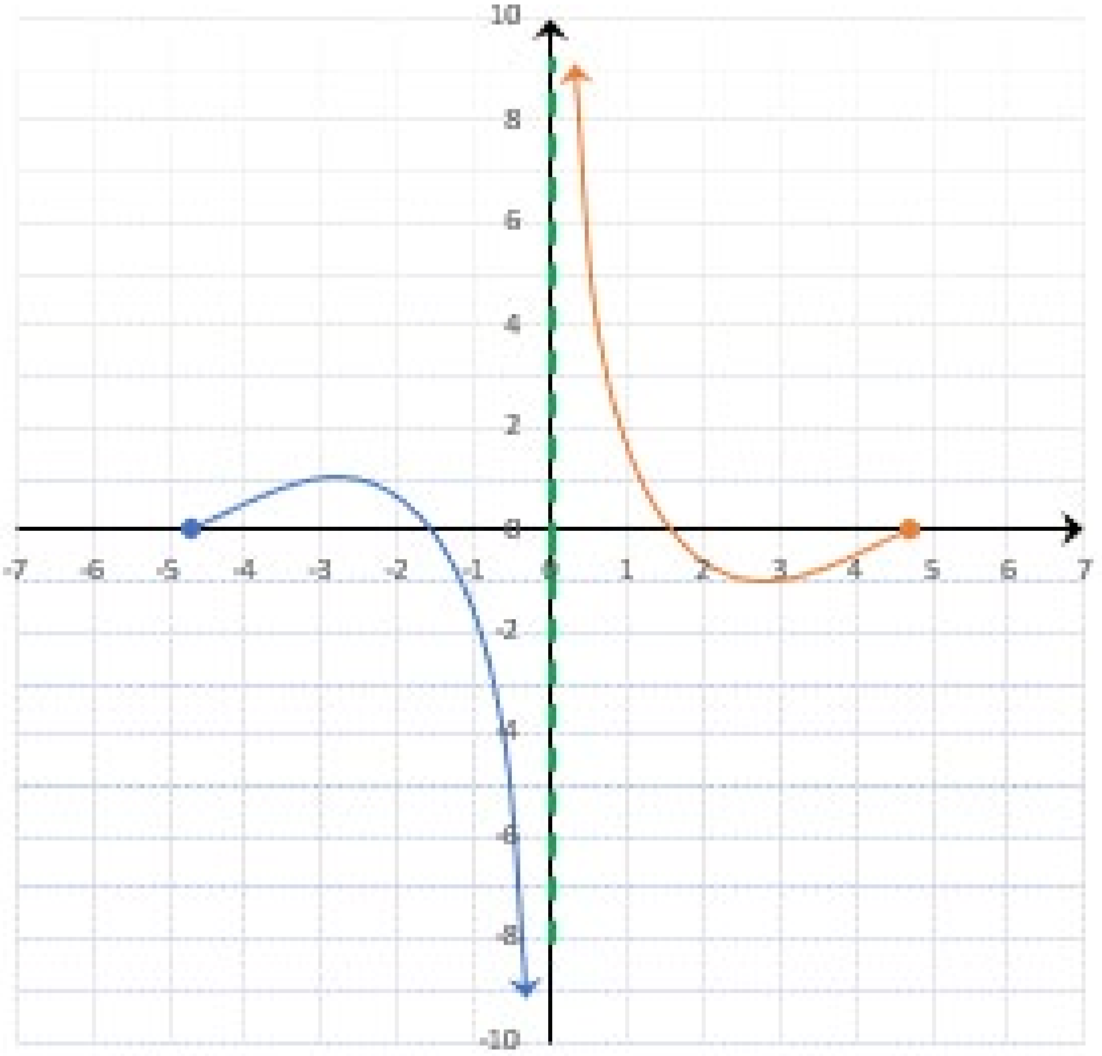
Question 9b)

$y = f(|x|)$



Question 9b)

$$y = \frac{1}{f(x)}$$



Section I **Student ID number:**

5 Marks
Attempt Question 1 – 5.
Allow approximately 7 minutes for this section.

Use the multiple choice answer sheet below to record your answers to Question 1 – 5.

Select the alternative: A, B, C or D that best answers the question.

Colour in the response oval completely.

Sample:

$2+4=?$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, draw a cross through the incorrect answer and colour in the new answer

ie A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word "*correct*" and draw an arrow as follows:

correct

A ● B ~~●~~ C ○ D ○

Year 12 Examination 2023
Mathematics Extension 1

Multiple Choice Answer Sheet

Completely colour in the response oval representing the most correct answer.

1	A	○	B	●	C	○	D	○
2	A	○	B	○	C	○	D	●
3	A	○	B	○	C	○	D	●
4	A	○	B	○	C	○	D	●
5	A	○	B	○	C	○	D	●

Mark: /5

Yr 12 X1 2023 Task 3

(i) $x + 2 = \cos\theta + \sin\theta$

$y = \sin 2\theta = 2\sin\theta\cos\theta$

$(x+2)^2 = (\cos\theta + \sin\theta)^2$

1 m $(x+2)^2 = \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta$

$(x+2)^2 = 1 + y$ $\quad (\cos^2\theta + \sin^2\theta = 1)$

$x^2 + 4x + 4 = 1 + y$

1 m $\therefore y = x^2 + 4x + 3$ #

Show Q :
must show
$\sin^2\theta + \cos^2\theta = 1$
$y = 2\sin\theta\cos\theta$
$(x+2)^2 = x^2 + 4x + 4$
otherwise max 1 m

Since $y = \sin 2\theta$

$\therefore -1 \le y \le 1$

For $y = 1$ $\quad 1 = x^2 + 4x + 3$

$0 = x^2 + 4x + 2$

$x = \dfrac{-4 \pm \sqrt{16 - 4(2)}}{2}$

$x = \dfrac{-4 \pm \sqrt{8}}{2} = \dfrac{-4 \pm 2\sqrt{2}}{2}$

$x = -2 \pm \sqrt{2}$

1 m
end pts $(-2 \pm \sqrt{2}, 1)$
or
parabola with some essential points
TP $(-2, -1)$
x-intercepts $x = -3, -1$
end pts $(-2 \pm \sqrt{2}, 1)$

For $y = -1$ $\quad -1 = x^2 + 4x + 3$

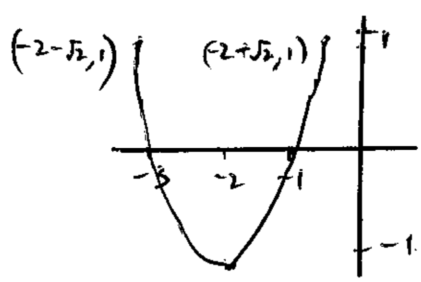
$0 = x^2 + 4x + 4$

$0 = (x+2)^2$

$\therefore x = -2$ $\therefore$ Vertex at $(-2, -1)$

2m perfect graph with all essential pts

(ii)



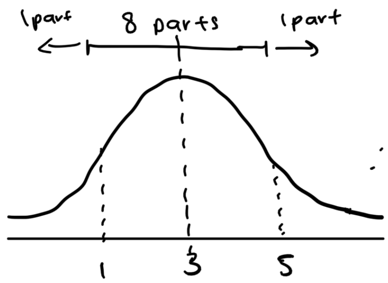
$y = x^2 + 4x + 3$

$y = (x+3)(x+1)$

$0 = (x+3)(x+1)$

$x = -3, -1$ (x-intercepts)

MATHEMATICS Extension 1 : Question ...6...

Suggested Solutions	Marks	Marker's Comments
b) $P(\lvert x-4\rvert<1) = P(-1<x-4<1)$		
$= P(3<x<5)$	1	
$= P(x<5) - P(x<3)$		
		
$\therefore P(x<5) = \frac{9}{10}$		1 for getting $P(x<5) = \frac{9}{10}$
OR by symmetry		
$P(x<1) = 1 - P(x<5)$		
as $9P(x<1) = P(x<5)$		
$\therefore 9(1-P(x<5)) = P(x<5)$		
$9 - 9P(x<5) = P(x<5)$		
$9 = 10P(x<5)$		
$\therefore P(x<5) = \frac{9}{10}$		
$\therefore P(\lvert x-4\rvert<1) = \frac{9}{10} - \frac{1}{2}$		
$= \frac{2}{5}$	1	

Question 7 Ext 1 Task 3, 2023

MATHEMATICS EXTENSION 1 : Question 12		
Suggested Solutions	**Marks**	**Marker's Comments**
a) $A(1,1)$ $B\left(\frac{\sqrt{3}}{2}-\frac{1}{2}, \frac{\sqrt{3}}{2}+\frac{1}{2}\right)$ i) $\angle AOB$? $\overrightarrow{OA} = \begin{pmatrix}1\\1\end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix}\frac{\sqrt{3}}{2}-\frac{1}{2}\\ \frac{\sqrt{3}}{2}+\frac{1}{2}\end{pmatrix} = \frac{\sqrt{3}}{2}-\frac{1}{2}+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)$ $= \sqrt{3}$ $\overrightarrow{OA}\cdot\overrightarrow{OB} = \lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert \cos\angle AOB$ $\lvert\overrightarrow{OA}\rvert = \sqrt{1^2+1^2} = \sqrt{2}$ $\lvert\overrightarrow{OB}\rvert = \sqrt{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)^2} = \sqrt{2}$ $\therefore \angle AOB = \cos^{-1}\frac{\sqrt{3}}{2}$ $= 30^\circ$		1 — angle 1 — $\lvert\overrightarrow{OA}\rvert$ & $\lvert\overrightarrow{OB}\rvert$ & $\overrightarrow{OA}\times\overrightarrow{OB}$
ii) Hence calc area of ΔAOB. $A = \frac{1}{2}ab\sin C$ $a = \lvert\overrightarrow{OA}\rvert$, $b = \lvert\overrightarrow{OB}\rvert$ $= \frac{1}{2}(\sqrt{2})\times(\sqrt{2})\times\frac{1}{2}$ $= \frac{1}{2}\,u^2$		1 — answer 1 — correct a & b & formula some students forgot the $\frac{1}{2}$ &/used cos C

Question 7

b)

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$

1 mark for correct expansion of $\cos 2\theta$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = (1)(2) + (2)(-4)$$
$$= 2 - 8$$
$$= -6$$

$$\left|\underset{\sim}{a}\right| = \sqrt{1^2 + 2^2}$$
$$= \sqrt{5}$$

$$\left|\underset{\sim}{b}\right| = \sqrt{2^2 + (-4)^2}$$
$$= \sqrt{4 + 16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

1 mark for finding $\underset{\sim}{a} \cdot \underset{\sim}{b}$, $\left|\underset{\sim}{a}\right|$, and $\left|\underset{\sim}{b}\right|$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \left|\underset{\sim}{a}\right| \left|\underset{\sim}{b}\right| \cos\theta$$
$$-6 = \sqrt{5} \times 2\sqrt{5} \times \cos\theta$$
$$= 10\cos\theta$$
$$\cos\theta = -\frac{3}{5}$$
$$\therefore \cos 2\theta = 2\left(-\frac{3}{5}\right)^2 - 1$$
$$= \frac{18}{25} - 1$$
$$= -\frac{7}{25}$$

1 mark for final answer

MATHEMATICS EXT 1: Question 8

Suggested Solutions	Marks	Marker's Comments
a, $f(x) = \begin{cases} \dfrac{kx}{\sqrt{4-x^2}} & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$		
Since $f(x)$ is a PDF		
$\displaystyle\int_0^1 \frac{kx}{\sqrt{4-x^2}}\,dx = 1$	1 Knowing property of PDF	
$\displaystyle -\frac{1}{2}\int_0^1 -2kx\,(4-x^2)^{-1/2}\,dx = 1$		
$\displaystyle -\frac{1}{2}k\left[\frac{(4-x^2)^{1/2}}{1/2}\right]_0^1 = 1$	1 Correct integration	
$-k(\sqrt{3}-2) = 1$		
$k = \dfrac{1}{2-\sqrt{3}}$	1 Final	
$= 2+\sqrt{3}$		

COMMENTS:

- dropping the x in the numerator was a common error → led to arcsine integral
- Please stop overcomplicating things!! There is no need to choose your own sub or use IBP in an Ext 1 exam
  - ↳ if you did this and made an error, no CF marks awarded.

MATHEMATICS EXT 1: Question

Suggested Solutions	Marks	Marker's Comments
b, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$		
$= (6\underset{\sim}{i} - 3\underset{\sim}{j}) - (4\underset{\sim}{i} + 2\underset{\sim}{j})$		
$= 2\underset{\sim}{i} - 5\underset{\sim}{j}$	1	
$\overrightarrow{OC} = 8\underset{\sim}{i} - 20\underset{\sim}{j}$		
$= 4(2\underset{\sim}{i} - 5\underset{\sim}{j})$		
$= 4\overrightarrow{AB}$		
$\therefore \overrightarrow{OC} \parallel \overrightarrow{AB}$		
One pair of sides are parallel, meaning the quadrilateral is a trapezium.	1	
COMMENTS		
• The majority of students are abusing vector notation.		
• $(2,-5)$ is a point, not a vector. Must have $2\underset{\sim}{i} - 5\underset{\sim}{j}$ or $\binom{2}{-5}$.		
• This was marked <u>very</u> leniently → defn of trapezium		

Question 8

c)

$$\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)=\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$$

$$=2\left|\underset{\sim}{b}\right|^2\cos\theta \quad \left(\left|\underset{\sim}{a}\right|=2\left|\underset{\sim}{b}\right|\right)$$

$$\left(\underset{\sim}{a}+3\underset{\sim}{b}\right)\cdot\left(4\underset{\sim}{a}-\underset{\sim}{b}\right)=0\Rightarrow\left(\underset{\sim}{a}\cdot 4\underset{\sim}{a}\right)-\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)+\left(3\underset{\sim}{b}\cdot 4\underset{\sim}{a}\right)-\left(3\underset{\sim}{b}\cdot\underset{\sim}{b}\right)=0$$

$$4\left|\underset{\sim}{a}\right|^2-\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)+12\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)-3\left|\underset{\sim}{b}\right|^2=0$$

$$4\left|\underset{\sim}{a}\right|^2+11\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)-3\left|\underset{\sim}{b}\right|^2=0$$

$$\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)=\frac{-4\left|\underset{\sim}{a}\right|^2+3\left|\underset{\sim}{b}\right|^2}{11}$$

$$\therefore 2\left|\underset{\sim}{b}\right|^2\cos\theta=\frac{-4\left|\underset{\sim}{a}\right|^2+3\left|\underset{\sim}{b}\right|^2}{11}$$

$$\cos\theta=\frac{-4\left(2\left|\underset{\sim}{b}\right|\right)^2+3\left|\underset{\sim}{b}\right|^2}{22\left|\underset{\sim}{b}\right|^2} \quad \left(\left|\underset{\sim}{a}\right|=2\left|\underset{\sim}{b}\right|\right)$$

$$=-\frac{13}{22}$$

$$\theta=126°13'$$

1 mark for completing **both:**

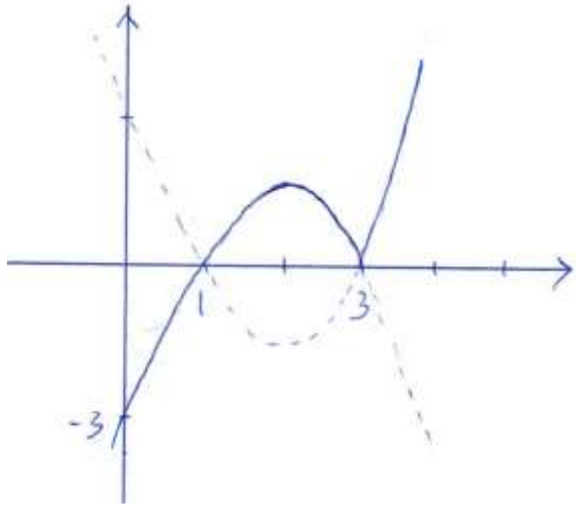
- correctly expanding to $4\left|\underset{\sim}{a}\right|^2-\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)+12\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)-3\left|\underset{\sim}{b}\right|^2=0$
- $\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)=\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$ or $\cos\theta=\frac{\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)}{\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|}$

1 mark for getting the correct answer. (Obtuse and not acute!)

Students who left their answer as $\theta=\cos^{-1}\left(-\frac{13}{22}\right)$ unfortunately don't receive the 2nd mark.

Year 12 Ext. 1 Task 3 Q9

(a) (i)



1 mark for general shape

1 mark for details – including;

- all three intercepts
- scale on x-axis
- sharp at x=3
- correct concavity on both ends

For the responses with bouncing behaviour at x=3, 1 mark for the general shape was allowed ONLY IF the symmetry was evident (i.e. clearly shown max at x=2)

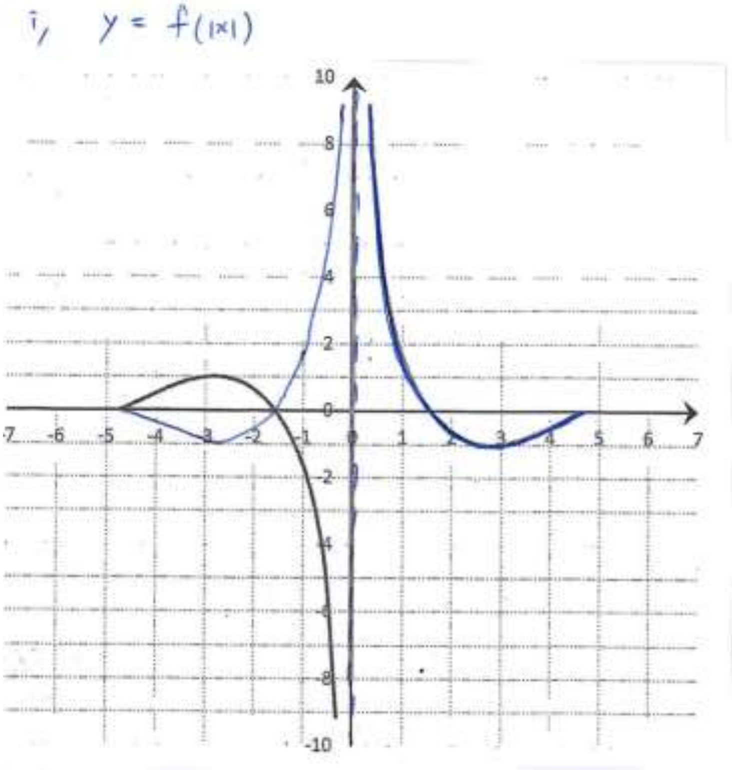
responses with any other wrong graphs awarded zero mark

(a) (ii)

$x = 2,\ 2+\sqrt{2}$

1 mark for BOTH correct solutions in EXACT values

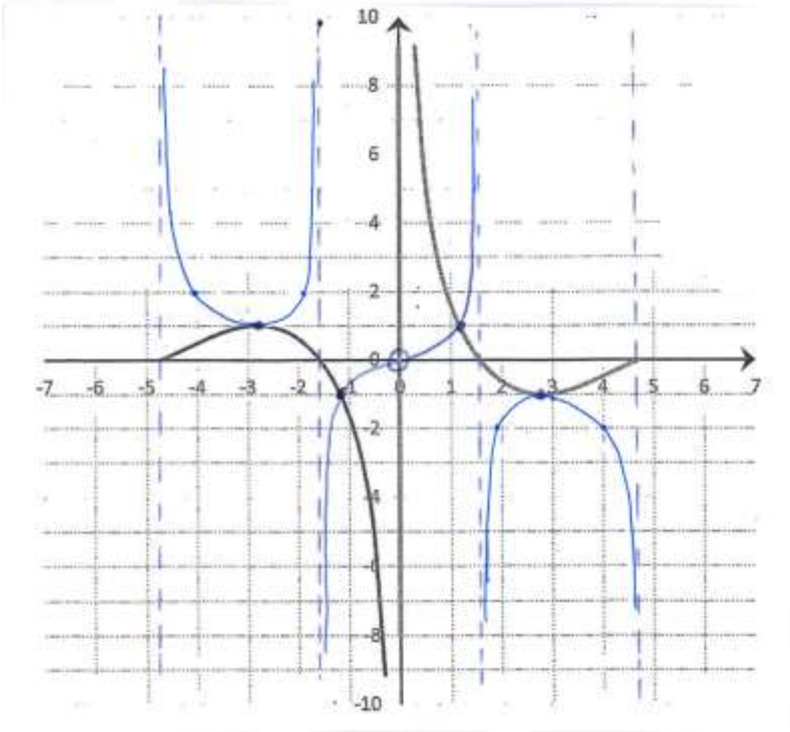
(b)



1 mark for general shape – must show BOTH branches

1 mark for details – including intercepts and scales
For the scale, it was particularly considered at;
when y=2, the graph must be on the right-hand-side of the grid line (i.e. x>-1)
when y=1, the graph must be on the left-hand-side of the grid line (i.e. x<-1)

ii, $y = \frac{1}{f(x)}$



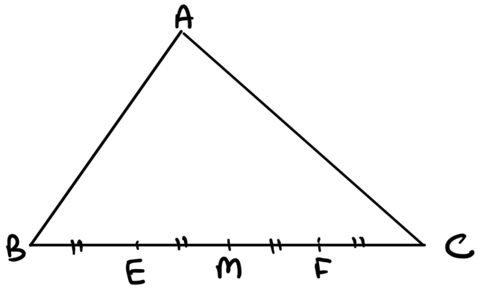
1 mark for the general shape

1 mark for details – including asymptotes, open-circle at x=0 and intersection at $x = \pm 1$

MATHEMATICS Extension 1 : Question 10

Suggested Solutions	Marks	Marker's Comments

a)



RTP: $\vec{AB} + \vec{AE} + \vec{Am} + \vec{AF} + \vec{AC} = \frac{5}{2}(\vec{AB} + \vec{AC})$

$$\text{LHS} = \vec{AB} + (\vec{AB} + \vec{BE}) + (\vec{AB} + \vec{BM}) + (\vec{AB} + \vec{BF}) + \vec{AC}$$

$$= 4\vec{AB} + \tfrac{1}{4}\vec{BC} + \tfrac{1}{2}\vec{BC} + \tfrac{3}{4}\vec{BC} + \vec{AC}$$

as $|\vec{BE}| = |\vec{Em}| = |\vec{MF}| = |\vec{FC}|$

$$= 4\vec{AB} + \vec{AC} + \tfrac{3}{2}\vec{BC}$$

1 for getting to this step or any other form. ie. $5\vec{AB} + 10\vec{BE}$ or $4\vec{AB} + 6\vec{BE} + \vec{AC}$ etc.

$$= 4\vec{AB} + \vec{AC} + \tfrac{3}{2}(\vec{AC} - \vec{AB})$$

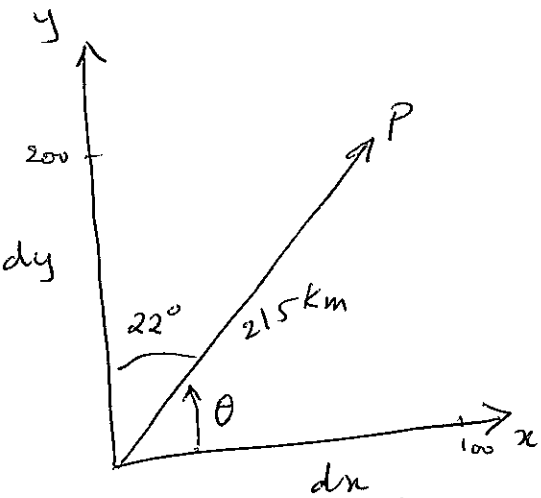
$$= \tfrac{5}{2}\vec{AB} + \tfrac{5}{2}\vec{AC}$$

$$= \tfrac{5}{2}(\vec{AB} + \vec{AC})$$

1

Question 10

(b) $\theta = 68^\circ$



$dx = d\cos\theta$
$= 215\cos 68$
$= 81$ km (1 mark)

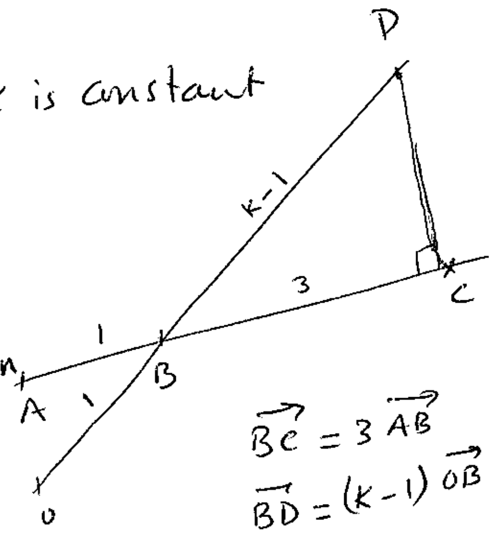
$dy = d\sin\theta$
$= 215\sin 68$
$= 199$ km (1 mark)

So the airplane was spotted 199 km North and 81 km East of the airport.

(c) $\underset{\sim}{a}$, $\underset{\sim}{b}$ position vectors of A and B

C lies on $\overrightarrow{AB}$, $|\overrightarrow{AB}| : |\overrightarrow{AC}| = 1 : 4$

D lies on OB, $|\overrightarrow{OB}| : |\overrightarrow{OD}| = 1 : k$, k is constant



$\overrightarrow{BC} = 3\overrightarrow{AB}$

$\overrightarrow{BD} = (k-1)\overrightarrow{OB}$

$\overrightarrow{OC} = 4\overrightarrow{OB} - 3\overrightarrow{OA}$
$= 4\underset{\sim}{b} - 3\underset{\sim}{a}$

$\overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{BC}$
$= (k-1)\underset{\sim}{b} - 3(\underset{\sim}{b} - \underset{\sim}{a})$
$= (k-4)\underset{\sim}{b} + 3\underset{\sim}{a}$

1 mark
Correct expression for $\overrightarrow{OC}$ or $\overrightarrow{CD}$

$\overrightarrow{AB} \perp \overrightarrow{CD}$ $\therefore$ $\overrightarrow{AB} \cdot \overrightarrow{CD} = 0$

$(\underset{\sim}{b} - \underset{\sim}{a}) \cdot ((k-4)\underset{\sim}{b} + 3\underset{\sim}{a}) = 0$

$(k-4)|\underset{\sim}{b}|^2 + 3\underset{\sim}{a}\cdot\underset{\sim}{b} - (k-4)\underset{\sim}{a}\cdot\underset{\sim}{b} - 3|\underset{\sim}{a}|^2 = 0$ (1 mark) progress

$k(|\underset{\sim}{b}|^2 - \underset{\sim}{a}\cdot\underset{\sim}{b}) = 4|\underset{\sim}{b}|^2 - 7\underset{\sim}{a}\cdot\underset{\sim}{b} + 3|\underset{\sim}{a}|^2$

$$k = \frac{4|\underset{\sim}{b}|^2 - 7\underset{\sim}{a}\cdot\underset{\sim}{b} + 3|\underset{\sim}{a}|^2}{|\underset{\sim}{b}|^2 - \underset{\sim}{a}\cdot\underset{\sim}{b}}$$

(1 mark) work leading to the correct answer