Year 11 Extension 1 Mathematics

Term 4 Assessment 2006

Time allowed: 1 hour
All questions may be attempted.

Question 1

a) Find (i) $\int (5x-3)^{15}\,dx$ 2

(ii) $\int \frac{5}{\sqrt{3x+1}}\,dx$ 3

b) If $f(x) = \sqrt{\frac{2x-1}{4x+3}}$ then $f'(x) = \frac{5}{\sqrt{(2x-1)(4x+3)^3}}$. (You do not need to prove this!)

Use this fact to evaluate $\int_{3/2}^{1\frac{1}{2}} \frac{dx}{\sqrt{(2x-1)(4x+3)^3}}$ 3

Question 2

a) Find the area enclosed by the curve $y = 3 + 2x - x^2$, the x-axis and the lines $x = -1$ and $x = 2$. 3

b) Calculate the area between the curve $y = x^3 - x^2 - 2x$ and the x-axis. 5

c) The area bounded by the curve $y = x^3$, the y-axis and the lines $y = 0$ and $y = k$ is 12 units. Find the value of k. 5

d) Find the area enclosed by the curve $y = \frac{1}{4}x^3$ and the line $y = x$. 4

Question 3

Calculate the volume of the solid generated by rotating the ellipse $\frac{x^2}{4} + y^2 = 1$ about the x-axis. 4

Question 4

Calculate an approximation for the area between the curve $y = 2^x$, the x-axis and the lines $x = 1$, $x = 3$

a) by dividing the area into two strips and using the trapezoidal rule 2

b) by using Simpson's rule with three function values. 2

Question 5

For the curve $y = \dfrac{x^2}{x^2 - 4}$

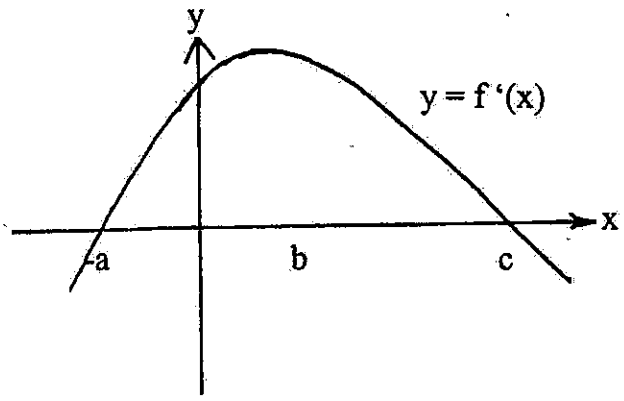
a) Show that there is a maximum turning point at (0,0). 3

b) Find, showing working, any (i) vertical asymptotes 2
(ii) horizontal asymptotes. 2

c) Is the function even or odd, give reasons? 1

d) Draw a neat sketch of the function showing its main features. 2

Question 6



The parabola drawn in the diagram above shows the graph of the gradient function $y = f'(x)$ for a given function $y = f(x)$. On separate diagrams draw possible graphs of

a) $y = f''(x)$ 2

b) $y = f(x)$ 3

Question 1

a) (i) $\int (5x-3)^{15}\,dx = \frac{1}{5} \times \frac{(5x-3)^{16}}{16}$ ✓ $+ c$

$= \frac{(5x-3)^{16}}{80} + c$

(ii) $\int \frac{5}{\sqrt{3x+1}}\,dx = \int 5(3x+1)^{-\frac{1}{2}}\,dx$

$= 5 \times \frac{(3x+1)^{\frac{1}{2}}}{\frac{1}{2}}$ ✓ $\times \frac{1}{3}$ ✓ $+ c$

$= \frac{10\sqrt{3x+1}}{3}$ ✓ $+ c$

b) $\int_{1\frac{1}{2}}^{5\frac{1}{2}} \frac{dx}{\sqrt{(2x-1)(4x+3)^3}} = \frac{1}{5}\left[\sqrt{\frac{2x-1}{4x+3}}\right]_{1\frac{1}{2}}^{5\frac{1}{2}}$ ✓

$= \frac{1}{5}\left[\sqrt{\frac{10}{25}} - \sqrt{\frac{2}{9}}\right]$ ✓

$= \frac{1}{5}\left[\frac{\sqrt{10}}{5} - \frac{\sqrt{2}}{3}\right]$ ✓

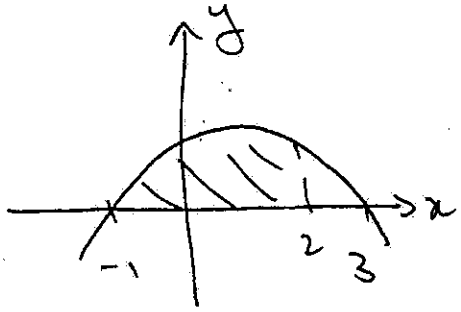
Question 2

a) $3+2x-x^2=0$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$x=-1,3$



$$A=\int_{-1}^{2}(3+2x-x^2)\,dx$$

$$=\left[3x+x^2-\frac{x^3}{3}\right]_{-1}^{2} \quad \checkmark$$

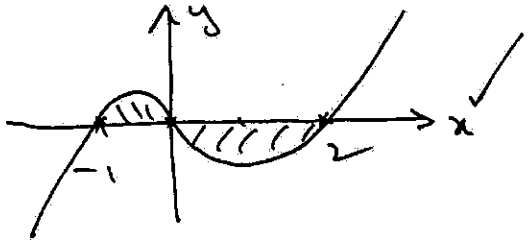
$$=\left(6+4-\frac{8}{3}\right)-\left(-3+1+\frac{1}{3}\right) \quad \checkmark$$

$$=9 \quad \checkmark$$

b) $x(x^2-x-2)=0$

$x(x+1)(x-2)=0$

$x=0,-1,2$



$$A=\int_{-1}^{0}(x^3-x^2-2x)\,dx+\left|\int_{0}^{2}(x^3-x^2-2x)\,dx\right| \quad \checkmark$$

$$=\left[\frac{x^4}{4}-\frac{x^3}{3}-x^2\right]_{-1}^{0}+\left|\left[\frac{x^4}{4}-\frac{x^3}{3}-x^2\right]_{0}^{2}\right| \quad \checkmark$$

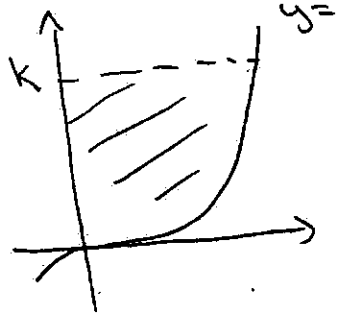
$$=\left[0-\left(\frac{1}{4}+\frac{1}{3}-1\right)\right]+\left|\left[\left(4-\frac{8}{3}-4\right)-0\right]\right|$$

$$=\frac{5}{12}+\left|\left(-\frac{8}{3}\right)\right|$$

$$=\frac{37}{12} \quad \checkmark$$

Q.2

c)



$\Rightarrow x = y^{\frac{1}{3}}$ ✓

$$\int_0^K y^{\frac{1}{3}}\,dy = 12 \quad \checkmark$$

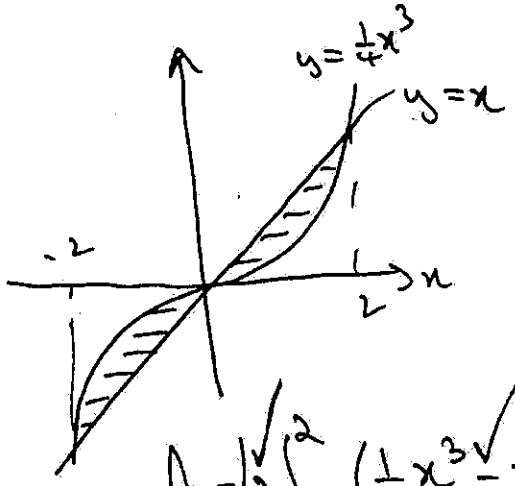
$$\left[\frac{3}{4}y^{4/3}\right]_0^K = 12 \quad \checkmark$$

$$\frac{3}{4}K^{4/3} = 12$$

$$K^{4/3} = 16 \quad \checkmark$$

$$K = 8 \quad \checkmark$$

d)



$\frac{1}{4}x^3 = x$ ✓

$\frac{1}{4}x^3 - x = 0$

$x\left(\frac{1}{4}x^2 - 1\right) = 0$

$x = 0,\ x = \pm 2$ ✓

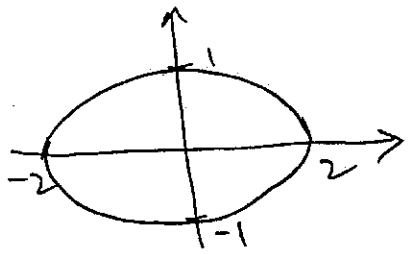
$$A = \left|2\int_0^2 \left(\frac{1}{4}x^3 - x\right).dx\right| \quad \checkmark$$

$$= \left|2\left[\frac{x^4}{16} - \frac{x^2}{2}\right]_0^2\right|$$

$$= |2[1-2]|$$

$$= 2 \quad \checkmark$$

Question 3



limits

$$V = \pi \int_{-2}^{2} \left(1 - \frac{x^2}{4}\right) dx \quad \checkmark$$

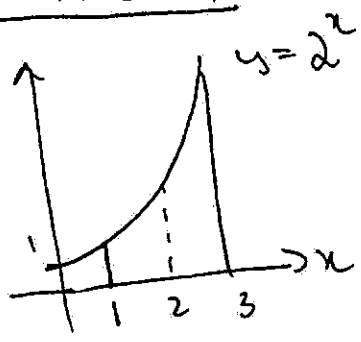
$$= 2\pi \int_{0}^{2} \left(1 - \frac{x^2}{4}\right) dx \quad \checkmark$$

$$= 2\pi \left[x - \frac{x^3}{12}\right]_0^2$$

$$= 2\pi \left[\left(2 - \frac{8}{12}\right) - 0\right]$$

$$= \frac{8\pi}{3} \quad \checkmark$$

Question 4



Area trapez. ✓

a) $A \doteq \frac{1}{2}(2^1+2^2) + \frac{1}{2}(2^2+2^3)$

$= 9 \ u^2$ ✓

b) $A = \frac{3-1}{6}\{2^1 + 4 \times 2^2 + 2^3\}$ ✓

$= \frac{1}{3}\{26\}$

$= 8\frac{2}{3} \ u^2$ ✓

Question 5

a) $y' = \dfrac{(x^2-4)\times 2x - x^2 \times 2x}{(x^2-4)^2}$

$= \dfrac{2x^3 - 8x - 2x^3}{(x^2-4)^2}$

$= \dfrac{-8x}{(x^2-4)^2}$ ✓

$y' = 0 \Rightarrow x = 0$ ✓

x	-1	0	1
y'	$+\frac{8}{9}$	0	$-\frac{8}{9}$

✓

∴ Max t.p. at $(0,0)$

b) (i) Vert. asymptotes where $x^2 - 4 = 0$ ✓

ie. $x = \pm 2$ ✓

(ii) $\lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2-4}$ ✓ $= \lim\limits_{x\to\pm\infty} \dfrac{1}{1-\frac{4}{x^2}}$

$= 1$ ✓

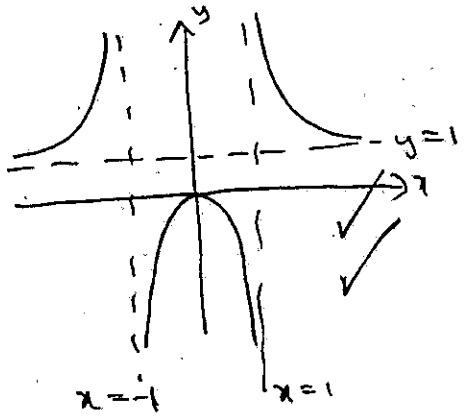
ie. $y = 1$ is horizontal asymptote

c) $f(-x) = \dfrac{(-x)^2}{(-x)^2 - 4}$

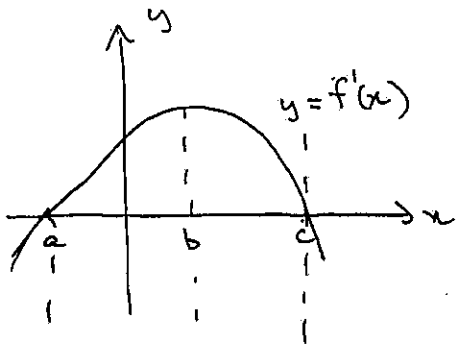
$= \dfrac{x^2}{x^2-4} = f(x)$ ✓

∴ $y = f(x)$ is an even fn.

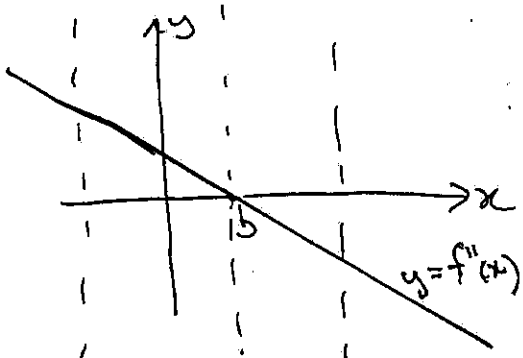
(d)



Q.6

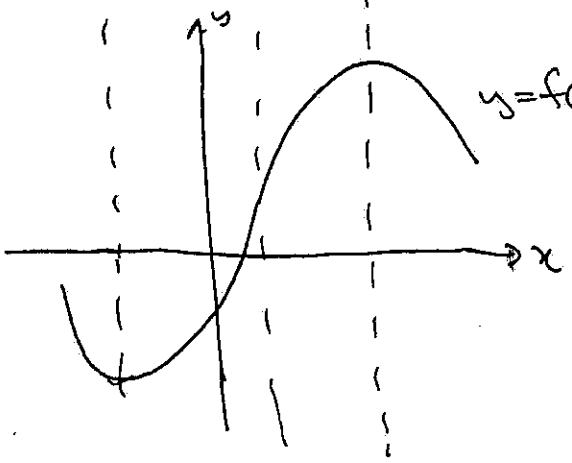


a)



St. Line correct slope ✓
Through (b,0) ✓

b)



Turning pts at x = a (Min) ✓
x = b (Max) ✓

Inflection at x = b ✓