KILLARA HIGH SCHOOL

YEAR 11X1-MATHEMATICS
HSC Assessment 1- Term 4, 2007

TOTAL TIME: 70 minutes

TOTAL MARKS: 40

Read each question thoroughly.

Attempt all questions.

Start each question on a new page.

Non-Programmable Calculator may be used.

All necessary working out must be shown to obtain full marks.

Question 1 **marks**

(a) Evaluate $\sum_{n=3}^{20} 4n-2$ **2**

(b) The number 2, a, b are in arithmetic progression and a, b, 9 are in geometric progression. Find the values of a and b. **3**

(c) Pat borrows \$45000 at 7.2% per annum reducible interest, calculated monthly. The loan is to be repaid in 70 equal monthly instalments. Let M be the value of each instalment.

i. Show that the amount (A) owing after the 70^{th} installment is **3**

$$A_{70} = 45000(1.006)^{70} - \frac{M(1.006^{70}-1)}{.006}.$$

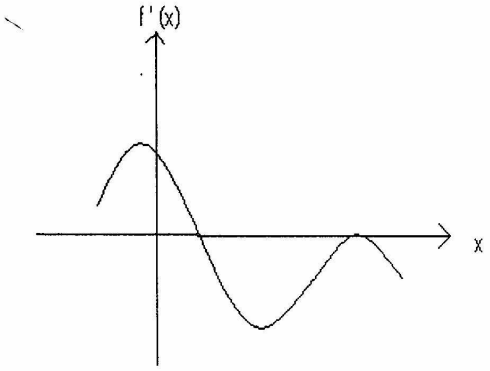
ii. Show that the monthly installment is \$789.18. **1**

iii. Find the amount owing after the 11^{th} payment. **1**

iv. With the twelfth payment, Pat pays an additional \$8000, so this payment is \$8789.18. After this, repayments continue at \$789.18 per month. How many more repayments will be needed? **3**

Question 2 **Marks**

(a). The following diagram shows the graph of $f'(x)$. Copy the diagram into your writing booklet and sketch a possible graph of f(x). Label clearly any stationary points and points of inflexion. **4**



(b) Given that $\frac{d^2y}{dx^2} = x - 2$. The point (0, 3) is on the curve and the tangent at this point is parallel to the line $y - 2x + 5 = 0$. Find the equation of the curve. **2**

(c) A closed cylindrical can is to be made out of 294π square centimeters of sheet metal. If h is the height and r is the radius show that

i. $h = \frac{147}{r} - r$. **1**

ii. Hence find an expression for the volume in term of r **1**

iii. Find the greatest possible volume of this can (answer to the nearest whole number). **3**

(d) If $y = (2 - x)\sqrt{x}$

i. State the domain of the function and find the axes intercepts. **2**

ii. Show that $\frac{dy}{dx}$ is undefined at x = 0 and that the stationary point occurs at $x = \frac{2}{3}$. Determine its nature. **3**

iii. Use these results to sketch the curve $y = (2 - x)\sqrt{x}$. **2**

HSC Assessment 1

Question 1 (13 marks)

(a) $10, 14, 18$

$a = 10,\ d = 4,\ l = 78$ (1)

$S_{18} = \frac{18}{2}(10 + 78)$

$= 792$ (1)

(b) $b - a = a - 2$

$2a = b + 2$

$\frac{b}{a} = \frac{9}{b}$

$b^2 = 9a$ Sub. $2a$

$\Rightarrow b^2 = \frac{9}{2}(b + 2)$ (1)

$2b^2 - 9b - 18 = 0$

$(2b + 3)(b - 6) = 0$

$\therefore \begin{cases} b = -\frac{3}{2} \text{ or } b = 6 \\ a = \frac{1}{4} \text{ or } a = 4 \end{cases}$ (1) (1)

i) $A_1 = 45000(1.006) - M$

$A_2 = 45000(1.006)^2 - M(1.006) - M$ (1)

$\vdots$

$A_{70} = 45000(1.006)^{70} - M(1.006^{69} + 1.006^{68} + \cdots 1)$ (1)

$= 45000(1.006)^{70} - \frac{M(1.006^{70} - 1)}{.006}$ (1)

ii) After 70^{th} payment, $A_{70} = 0$

$\therefore 45000(1.006)^{70} - \frac{M(1.006^{70} - 1)}{.006} = 0$

$M = \frac{45000(1.006)^{70} \times .006}{1.006^{70} - 1} = \$789.18$ (1)

iii) $A_{11} = 45000 \times 1.006^{11} - \frac{789.18(1.006^{11} - 1)}{.006} = \$39114.57$ (1)

iv) After 12^{th} payment, the amount owing is:

$39114.57 \times 1.006 - 8789.18 = \$30,560.58$

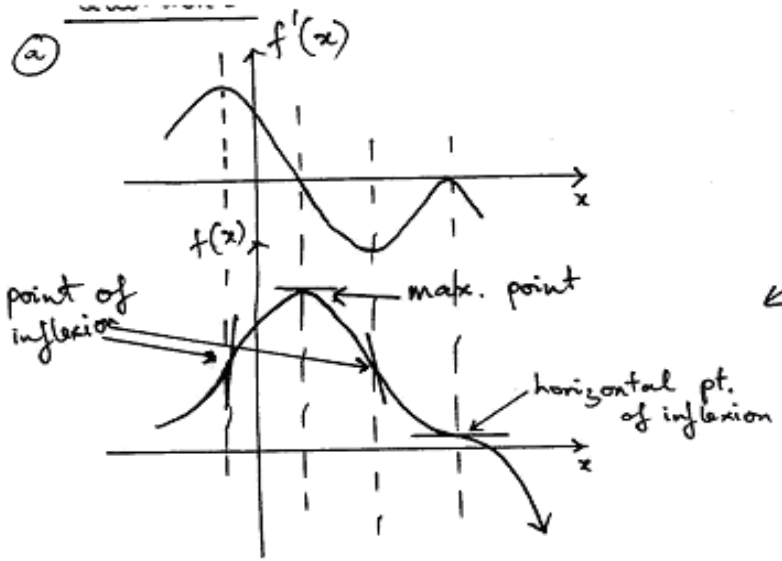
$A_n = 30560.58 \times 1.006^n - \frac{789.18(1.006^n - 1)}{.006} = 0$ (1)

$\therefore 1.006^n\left(30560.58 - \frac{789.18}{.006}\right) = -\frac{789.18}{.006}$

$-100969.42 \times 1.006^n = -131530$ (1)

$1.006^n = 1.30267 \Rightarrow n = 44.2$ months. (1)

(a)



Marking for (a)

(1) - correct intervals where the curve is increasing & decr.
(1) - horizontal pt. of inflexion
(1) - points of inflexion
(1) - maximum point and correct shape

(-1) for each error

(b)

$$\frac{dy}{dx} = \frac{x^2}{2} - 2x + C$$

$\frac{dy}{dx} = 2$ at $x = 0$

$\therefore \quad x^2 - 2x + C_1 = 2$ at $x = 0$

$\Rightarrow C_1 = 2$

$$\frac{dy}{dx} = \frac{x^2}{2} - 2x + 2 \qquad (1)$$

$$y = \frac{1}{6}x^3 - x^2 + 2x + C_2 \quad \text{Sub } (0,3)$$

$\Rightarrow 3 = C_2$

hence the equation of the curve

$$y = \frac{1}{6}x^3 - x^2 - 2x + 3. \qquad (1)$$

(c)(i) $294\pi = 2\pi r^2 + 2\pi rh$

$$h = \frac{294\pi - 2\pi r^2}{2\pi r}$$

$$= \frac{147}{r} - r \qquad (1)$$

(ii) $V = \pi r^2 h$

$$= \pi r^2 \left(\frac{147}{r} - r\right)$$

$$= 147\pi r - \pi r^3 \qquad (1)$$

(iii) $\frac{dV}{dr} = 147\pi - 3\pi r^2$

$147\pi - 3\pi r^2 = 0 \qquad (1)$

$r^2 = 49$

$\therefore r = 7 \qquad (1)$

hence $h = \frac{147}{7} - 7 = 14$

$\therefore V = \pi \times 7^2 \times 14 = 2155\,cm^3 \qquad (1)$

$f''(r) = -6\pi r$

$f''(7) < 0$

$\therefore r = 7$ is a max.

(-1) if not showing $f''(7) < 0$

(a) i) $\frac{4}{3}x^3 + \frac{6}{\sqrt{x}} + 2x + C$ (2)

ii) $-\frac{2}{9}(5-3x)^{3/2} + C$ (1)

iii) $\frac{-3}{8(4x^2-1)^2} + C$

(b) $\int_2^5 \frac{3x^4 - 5x^3}{x^2}\,dx = \int_2^5 3x^2 - 5x\,dx$

$= \left[x^3 - \frac{5}{2}x^2\right]_2^5$ (1)

$= \left(125 - \frac{125}{2}\right) - \left(8 - \frac{10}{2}\right) = 59.5$ (1)

(c)

x	0	0.5	1	1.5	2
f(x)	$\frac{1}{4}$	.2353	$\frac{1}{5}$	.16	$\frac{1}{8}$

(1)

$n = 4$

$h = \frac{2-0}{4} = 0.5$

$A \doteq \frac{0.5}{3}\left(\frac{1}{4} + \frac{1}{5} + 4\times.2353\right) + \frac{0.5}{3}\left(\frac{1}{5} + \frac{1}{8} + 4\times.16\right)$ (1)

$\doteq$ 0.39 Sq. units. (1)

d) $y = (2-x)\sqrt{x}$

i) domain: $x \geq 0$ (1)

when $x=0$, $y=0$

when $y=0$, $x=2$ (1)

ii) $\frac{dy}{dx} = -1\times\sqrt{x} + \frac{2-x}{2\sqrt{x}}$ (1)

$\therefore \frac{dy}{dx}$ is undefined at $x=0$

for stationary point $\frac{dy}{dx} = 0$

$\Rightarrow -\sqrt{x} + \frac{2-x}{2\sqrt{x}} = 0$

$-2x + 2 - x = 0$

$-3x = -2$

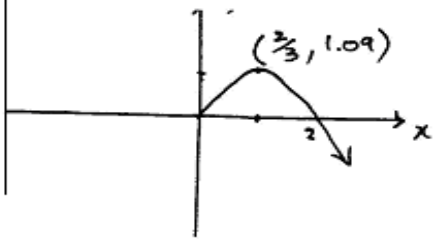
$x = \frac{2}{3}$ (1)

$y = 1.09$

x	$\frac{1}{3}$		$\frac{2}{3}$		1
$\frac{dy}{dx}$	↗		0		↘

(1)

$(\frac{2}{3}, 1.09)$ is a maximum



(1) - labelling

(1) - Correct shape