KILLARA HIGH SCHOOL

MATHEMATICS

Extension 1

Assessment Task

December 2008

TASK WEIGHTING: **10 %**

NAME: _____

Time allowed: 70 minutes.

Total Marks 42

Instructions:

Start each section in a new booklet

Marks may be deducted for untidy or badly arranged work

Calculators may be used

All necessary working should be shown.

Put your name on booklet with your answers.

Section A (20 Marks) Start a new booklet **Marks**

1 A polynomial $P(x)$ is defined as $P(x) = (2x-1)(3x-2)(x+4)$.

(a) Expand and simplify $P(x)$ [1]

(b) What is the degree of $P(x)$? [1]

(c) What is the leading coefficient of $P(x)$? [1]

(d) What is the constant term of $P(x)$? [1]

2 For the polynomial $G(x) = x(2x+3)(x-1)(x+1)$ draw a sketch of $G(x)$. [2]

3 For the polynomial $H(x) = 2x^3 - x^2 - 8x + 4$

(a) Find the remainder when $H(x)$ is divided by $(x+3)$: [1]

(b) Show that $(x-2)$ is a factor of $H(x)$. [1]

(c) Fully factorise $H(x)$ [2]

4 The polynomial $x^3 - 2x^2 + kx + 24 = 0$ has roots α, β and γ.

(a) Find the value of $\alpha + \beta + \gamma$ [1]

(b) Find the value of $\alpha\beta\gamma$ [1]

(c) It is known that two of the roots are equal in magnitude but opposite in sign. Find the third root and hence find the value of k. [2]

5 Let $f(x) = x^3 + 5x^2 + 17x - 10$. The equation $f(x) = 0$ has only one real root.

(a) Show that the root lies between 0 and 2. [1]

(b) Use one application of the 'halving the interval' method to find a smaller interval containing the root. [2]

(c) Which end of the smaller interval found in part (b) is closer to the root? Briefly justify your answer. [1]

6 You are given that $3{\cdot}5$ is an approximate root of the equation $x^3 - 50 = 0$.
Using one application of Newton's method, find a better approximation. [2]

Section B (22 Marks) Start a new booklet **Marks**

7 In TAB betting, the 'trifecta' pays on the first three horses in correct finishing order; and the 'quinella' pays on the first two horses in either order. In a 12-horse race, what is the possible number of:

a) trifecta combinations? [1]

b) quinella combinations? [1]

8 In how many ways is it possible to select six letters, including at least one vowel, from the letters of FLABELLIFORM [2]

9 Five letters from the word SHILLING are arranged in a row.
Find the number of ways in which this can be done, when the first letter is I and the last is L,

a) if no letter may be repeated, [2]

b) if each letter may occur as many times as it does in SHILLING. [2]

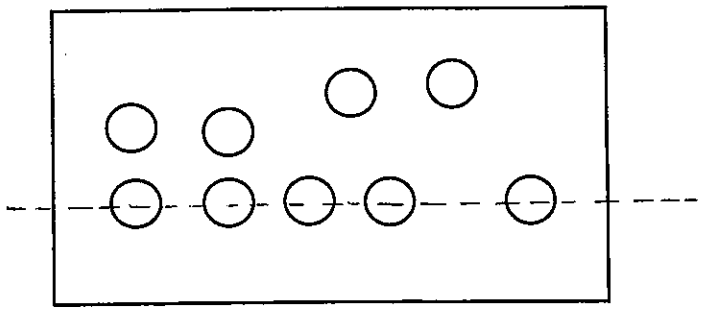
10 From four oranges, three bananas and two apples, how many selections of five pieces of fruit can be made, taking at least one of each kind? . [3]

11 Two women and five men are to be seated at a round table. How many different arrangements are possible if the two women are to sit together? [2]

12 In how many ways can a lift holding eight passengers carry a party of thirteen up a building in two journeys. [3]

Section B is continued on the next page. → →

13 The diagram shows a table with 9 balls lying on the table, 5 of which all lie in a straight line. No other group of 3 balls are collinear.



a) How many groups of 3 balls can be selected from the 5 balls lying in a straight line? [1]

b) How many different triangular patterns can be formed by using the 9 points as vertices? [2]

14 The ratio of the number of combinations of (2n+2) different objects taken n at a time to the number of combinations of (2n-2) different objects taken n at a time is 99:7. Find the value of n. [3]

End of examination

Solutions for Extension 1 Task 8/12/08

QUESTION: (1)

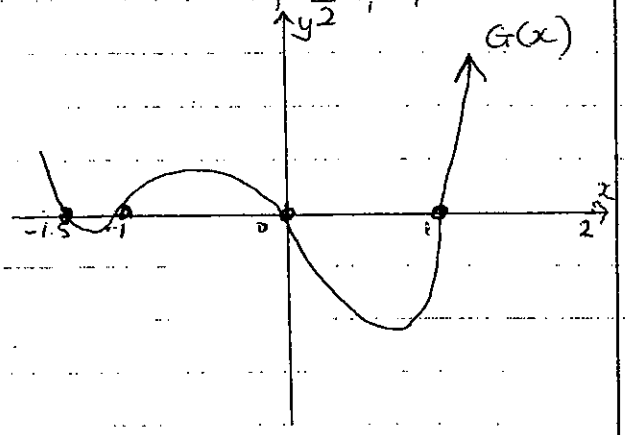
(a) $P(x) = (2x-1)(3x-2)(x+4)$
$= 6x^3 + 17x^2 - 26x + 8$ — 1

(b) degree is 3 — 1

(c) leading co-efficient is 6 — 1

(d) constant term is 8 — 1

2) $G(x) = x(2x+3)(x-1)(x+1)$
roots are $0, -\frac{3}{2}, 1, -1$ — 1 for roots



1 for graph.

1 out of 2 for right roots but wrong dirn of graph

3) $H(x) = 2x^3 - x^2 - 8x + 4$

(a) $H(-3) = 2(-3)^3 - 3(-3)^2 - 8(-3) + 4$
$= -54 - 9 + 24 + 4$
$= -35$ — 1

(b) $H(2) = 2(2)^3 - (2)^2 - 8(2) + 4$
$= 16 - 4 - 16 + 4$
$= 0$

As $H(2) = 0$, $x-2$ is a factor of $H(x)$ — 1

(c)
$$x-2 \overline{)\,2x^3 - x^2 - 8x + 4}$$
$2x^3 - 4x^2$
$3x^2 - 8x$ — 1
$3x^2 - 6x$
$-2x + 4$
$-2x + 4$

$\therefore 2x^3 - x^2 - 8x + 4$
$= (x-2)(2x^2 + 3x - 2)$
$= (x-2)(2x-1)(x+2)$ — 1

4) $x^3 - 2x^2 + kx + 24 = 0$

(a) $\alpha + \beta + \gamma = 2$ — 1

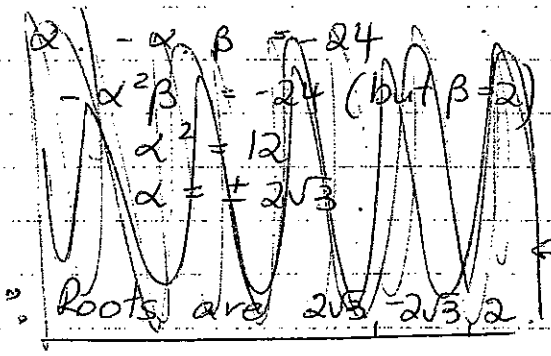
(b) $\alpha\beta\gamma = -24$ — 1

(c) Let roots be $\alpha, -\alpha, \beta$.
$\therefore$ from (i) & (ii)

$\alpha - \alpha + \beta = 2$
$\beta = 2$. — 1

(ii) $-\alpha\beta = -24$
$-\alpha^2\beta = -24$ (but $\beta = 2$)
$\alpha^2 = 12$
$\alpha = \pm 2\sqrt{3}$.

$\therefore$ Roots are $2\sqrt{3}, -2\sqrt{3}, 2$.



← extra bit for fun!!

As 2 is a root, $P(2) = 0$
$2^3 - 2 \cdot 2^2 + k \cdot 2 + 24 = 0$
$\therefore k = 12$.

$\therefore$ the value of k is 12 — 1

5) $f(x) = x^3 + 5x^2 + 17x - 10$

(a) $f(0) = -10$

$f(2) = 2^3 + 5.2^2 + 17.2 - 10$

$= 52.$

As $f(0)$ & $f(2)$ are opposite in sign, there is a root between 0 & 2 — must have reason — 1

(b) $f\left(\frac{0+2}{2}\right)$

ie $f(1) = 1 + 5 + 17 - 10$ — 1

$= 13.$

As $f(1)$ & $f(0)$ are opposite in sign, there is a root between $x = 0$ and $x = 1$ — 1

(c) $f(0) = -10$ & $f(1) = 13$

as $f(0)$ is smaller in magnitude than $f(1)$, the root is closer to $x = 0$. — 1

6) Let $f(x) = x^3 - 50$

$f'(x) = 3x^2$

$f(3.5) = -7.125$ — 1

$f'(3.5) = 36.75$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 3.5 + \frac{7.125}{36.75}$

$= 3.693877551\ldots$ — 1

$= 3.694$ (to 3 d.p)

SECTION B.

Q7. a) ${}^{12}P_3 = 12 \times 11 \times 10$

$= 1320$ ✓ [1]

b) ${}^{12}C_2 = \frac{12 \times 11}{1 \times 2}$

$= 66$ ✓ [1]

Q8. FLB, LLF, RM — AE10 — 4 — 8

If NO vowel ${}^8C_6 = {}^8C_2$

$= \frac{8 \times 7}{1 \times 2} = 28$

any other combination least least 1 vowel

${}^{12}C_6 = 924$ All combinations

∴ Combinations $= 924 - 28$

$= 896$ ✓ [2]

Q9 SHILLING

a) I _ _ _ L ✓ correct method

$1 \times 4 \times 3 \times 2 \times 1$ or $1 \times {}^4P_3 \times 1$

no repeats ∴ No. of Ways = 24 ✓ [2]

b) I _ _ _ L

$1 \times 6 \times 5 \times 4 \times 1$ or $1 \times {}^6P_3 \times 1$ ✓ correct expression

$= 120$ ✓ [2]

Q10.

4o 3B 2A.

2	2	1	$^4C_2 \times {}^3C_2 \times {}^2C_1 = 36$
2	1	2	$^4C_2 \times {}^3C_2 \times {}^2C_2 = 18$
1	2	2	$^4C_1 \times {}^3C_2 \times {}^2C_2 = 12$
3	1	1	$^4C_3 \times {}^3C_1 \times {}^2C_1 = 24$
1	3	1	$^4C_1 \times {}^3C_3 \times {}^2C_1 = 8$
			98

✓ for correct attempt To itemise combinations.
Then −1 for each error
✓ for bare answer [3]

Q11 round table.

2 5
♀ ♂

consider the 2 women as 1.

1 5 6
♀ ♂ Total

∴ arrangements in a circle
$= (6-1)! \times 2$
↑ ways to arrange the women

$= 5! \times 2$
$= 120 \times 2$
$= 240$ ways [2]

✓ for correct understanding of $(n-1)!$

Q12

left corn carry 8 out of 13
2 Trips

∴ 8+5	$^{13}C_8 \times 1$	= 1287
7+6	$^{13}C_7 \times 1$	= 1716
6+7	$^{13}C_6 \times 1$	= 1716
5+8	$^{13}C_5 \times 1$	= 1287
		6006

[3]

✓ for understanding
✓ for working
✓ for solution (or bare answer)

Q13

a) groups of 3 from 5 in a line

Ways $= {}^5C_3 = 10$ ✓ [1]

b) Total combinations
$= {}^9C_3 = 84$

but 10 of these are from the straight line (see a)

∴ No. of Triangles $= 84 - 10$
$= 74$ [2]

✓ for correct understanding to give $^9C_3 - {}^5C_3$
✓ for bare answer

Q 14

$^{2n+2}C_n : {}^{2n-2}C_n = 99 : 7$

[✓ bonus answer.]

✓ for simplification of $^{2n+2}C_n$

✓ " " " " $^{2n-2}C_n$

$$\frac{\left[\frac{(2n+2)!}{(2n+2-n)!\,n!}\right]}{\left[\frac{(2n-2)!}{(2n-2-n)!\,n!}\right]} = \frac{99}{7}$$

$$\frac{(2n+2)!}{(n+2)!\,n!} \times \frac{(n-2)!\,n!}{(2n-2)!} = \frac{99}{7}$$

$$\frac{(2n+2)(2n+1)(2n)(2n-1)}{(n+2)(n+1)(n)(n-1)} = \frac{99}{7}$$

$$\frac{4(4n^2-1)}{(n+2)(n-1)} = \frac{99}{7}$$

$$28(4n^2-1) = 99(n^2+n-2)$$

$$112n^2 - 28 = 99n^2 + 99n - 198$$

$$13n^2 - 99n + 170 = 0$$

$$13 \times 1 \quad \begin{matrix} 17 & 34 & 5 & 10 \\ 10 & 5 & 34 & 17 \end{matrix}$$

$$(13n - 34)(n - 5) = 0$$

$n = \frac{34}{13}, 5$ but n is integral

$\therefore n = 5$ [3]