Killara HIGH SCHOOL

STUDENT NUMBER

MATHEMATICS EXTENSION 1

ASSESSMENT TASK 1

Friday 3rd December, 2010

TIME ALLOWED: *Reading time: 5 minutes*
Working time: 60 minutes

INSTRUCTIONS: This paper consists of 3 questions
ANSWER all three questions
Start each question in a new booklet
Marks may not be given for untidy and poorly arranged work.
All relevant working should be shown.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

- ✓ PE1 appreciates the role of mathematics in the solution of practical problems
- ✓ PE3 solves problems involving permutations and combinations, inequalities, polynomials
- ✓ PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations.
- ✓ H2 constructs arguments to prove and justify results.
- ✓ HE1 appreciates interrelationships between ideas drawn from different areas of mathematics
- ✓ H6 uses the derivative to determine the features of the graph of a function
- ✓ H7 uses the features of a graph to deduce information about the derivative
- ✓ HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

	MARKS
QUESTION 1 Polynomials	/14
QUESTION 2 Parametric Equations	/12
QUESTION 3 Geometrical Application of Differentiation	/16
	/42

This assessment task constitutes 10% of the Higher School Certificate Course Assessment.

<u>QUESTION 1</u> (14 Marks) ***Start a new booklet*** **Marks**

(a) **(i)** Show that $x-3$ is a factor of $f(x)=x^3+3x^2-9x-27.$ **1**

(ii) Find the intercepts that the graph of $y=f(x)$ makes with the x axis. **2**

(iii) Without using calculus, sketch the graph of $f(x)=x^3+3x^2-9x-27.$ **1**

(b) If α, β, and γ are the roots of the equation $x^3-3x+1=0,$ find:

(i) $\alpha\beta+\beta\gamma+\gamma\alpha$ **1**

(ii) $\alpha^2+\beta^2+\gamma^2$ **1**

(iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$ **1**

.

(c) The polynomial $P(x)=2x^4-ax^2+bx-1$ is exactly divisible by $\left(x+1\right)$, **3**

and has a remainder of -3 on division by $\left(x-2\right)$.

Find the values of a and b.

(d) The polynomial $P(x)=x^3+px^2+qx+r$ has real roots $\sqrt{m}$, $-\sqrt{m}$ and α.

(i) Show that $\alpha+p=0$ **1**

(ii) Show that $m\alpha=r$ **1**

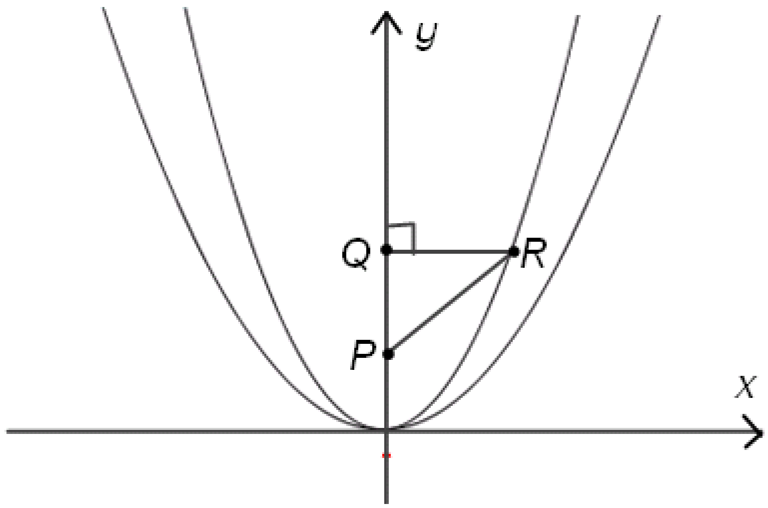
(iii) Show that $pq=r$ **2**

End of Question 1

<u>QUESTION 2</u> (12 Marks) ***Start a new booklet***

(a) P and Q are the foci of the two parabolae $x^2 = 4y$ and $x^2 = 4ay$, $a > 1$.

Copy or trace the diagram into your answer booklet.



(i) State the coordinates of P and Q. **1**

(ii) Draw in the two directrices **1**

(iii) What is the length of PR? **1**

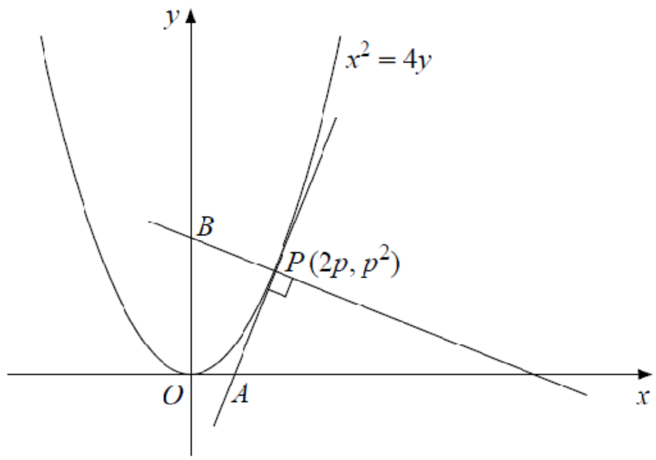
(b) The points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$.

(i) Show that the equation of the chord PQ is $y = \frac{(p+q)}{2}x - apq$. **2**

(ii) If PQ is a focal chord, find a relationship between P and Q. **1**

Question 2 is continued on the next page

(c)



The diagram shows the graph of the parabola $x^2 = 4y$. The tangent to the parabola at $P(2p, p^2)$, $p > 0$, cuts the x axis at A. The normal to the parabola at P cuts the y axis at B.

(i) Derive the equation of the tangent AP. **2**

(ii) Show that B has coordinates $(0, p^2 + 2)$ **2**

(iii) Let C be the midpoint of AB. **2**
Find the Cartesian equation of the locus of C

End of Question 2

<u>QUESTION 3</u> (16 Marks) ***Start a new booklet***

(a) For all x in the domain $-2 \le x \le 2$, a function, $f(x)$ is such that $f(x) < 0$, $f'(x) > 0$ and $f''(x) < 0$. Sketch a possible $y = f(x)$ in this domain. **2**

(b) A curve has equation $y = 7 + 4x^3 - 3x^4$

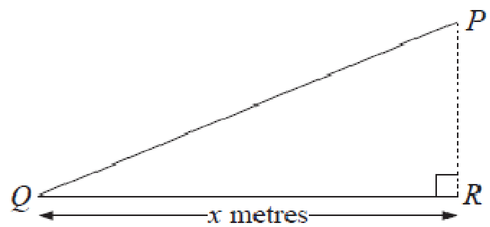
(i) Find the coordinates of the two stationary points **2**

(ii) Find all values of x for which $\dfrac{d^2y}{dx^2} = 0$ **2**

(iii) Determine the nature of the stationary points **2**

(iv) Sketch the curve for the domain $-1 \le x \le 2$, showing all main features. **2**

(c) A wire of length 5 metres is bent to form the hypotenuse and base of a right angled triangle PQR, as shown in the diagram.



Let the length of the base QR be x metres

(i) What is the length of the hypotenuse PQ in terms of x? **1**

(ii) Show that the area of the triangle PQR is $\frac{1}{2}x\sqrt{25-10x}$ square metres. **2**

(iii) What is the maximum possible area of the triangle? **3**

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

YR12 EXT 1 TASK 1. Dec. 2010.

Question 1.

a) (i) $f(3) = 3^3 + 3.3^2 - 9.3 - 27$ | 1 mark WITH SUBSTITUTION

$= 0$

$\therefore x-3$ is a factor of $f(x)$

(ii) Intercepts occur when $f(x) = 0$.

$$\begin{array}{r} x^2+6x+9 \\ x-3\,\overline{)\,x^3+3x^2-9x-27} \\ x^3-3x^2 \\ \hline 6x^2-9x \\ 6x^2-18x \\ \hline 9x-27 \\ 9x-27 \\ \hline 0 \end{array}$$

MUST HAVE WORKING!

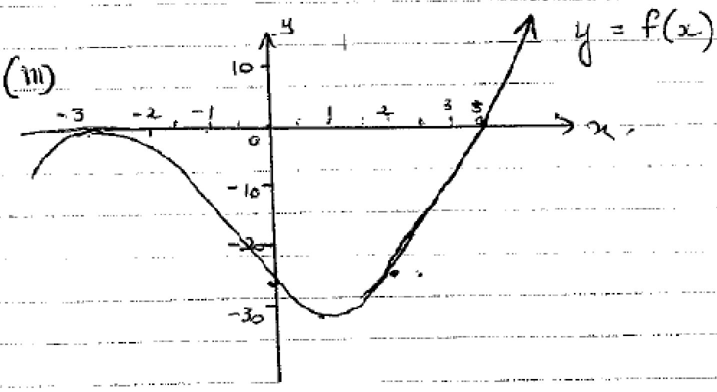
$\therefore f(x) = (x-3)(x^2+6x+9)$ | 1 MARK

$= (x-3)(x+3)^2$

$f(x) = 0 \Rightarrow (x-3)(x+3)^2 = 0$

$\therefore x = 3$ or $x = -3$. | 1 MARK.

(iii)



1 MARK FOR CORRECT X + Y INTERCEPTS & GRAPH.

0 FOR NO SCALE

b) $x^3 + 0x^2 - 3x + 1 = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\alpha\beta\gamma = -1$

(i) $\alpha\beta + \alpha\gamma + \beta\gamma = -3$. | 1 MARK.

(ii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ | MUST HAVE THIS.

$= 0 - 2.-3$

$= 6$ | 1 MARK.

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$ ← MUST HAVE THIS

$= \frac{-3}{-1}$ | 1 mark.

$= 3.$

c) $P(x) = 2x^4 - ax^2 + bx - 1$

$P(-1) = 2 - a - b - 1$

$= 0$

$\therefore \boxed{a+b=1}$ —— ① | 1 mark

$P(2) = 2.2^4 - a.2^2 + b.2 - 1$

$= -3.$

$\therefore 32 - 4a + 2b - 1 = -3.$

$2b - 4a = -34.$

$\boxed{\therefore b - 2a = -17}$ —— ② | 1 mark.

① - ② $3a = 18$

$a = 6$ —— (*)

(*) → ① $6 + b = 1$

$b = -5$

$\therefore a = 6, b = -5$ | 1 mark.

d) $P(x) = x^3 + px^2 + qx + r$ | MUST HAVE ALL WORKING.

i) Sum of roots $\alpha + \sqrt{m} - \sqrt{m} = -p$ | 1 mark

$\therefore \alpha = -p$

$\alpha + p = 0$

(ii) Product of roots $\alpha . \sqrt{m} . -\sqrt{m} = -r$ | 1 mark.

$\therefore -m\alpha = -r$

$m\alpha = r$

(iii) $\alpha.\sqrt{m} - \alpha.\sqrt{m} - \sqrt{m}.\sqrt{m} = q.$ | 1 mark for $r - m = q$.

$-m = q$

from (ii) $m = \frac{r}{\alpha}$. $\therefore -\frac{r}{\alpha} = q.$

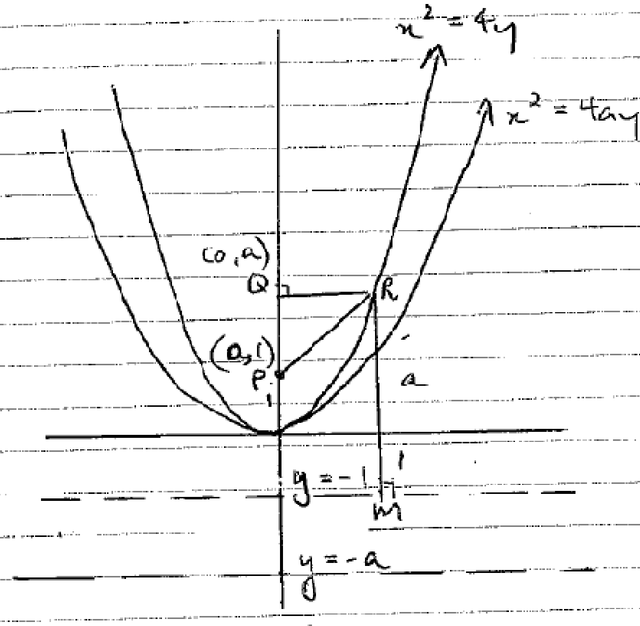
but $\alpha = -p$.

$\therefore \frac{-r}{-p} = q.$ | 1 mark for soln.

$\therefore pq = r$

Question 2

a)



(ii) | 1 mark.

(i) $P(0,1)$ $Q(0,a)$ | 1 mark.

(iii) $PR = RM$ (def'n of parabola). | 1 mark.
$\therefore PR = a+1$

b) $(2ap, ap^2)P$ & $Q(2aq, aq^2)$ | MUST HAVE ALL WORKING

(i) $m_{PQ} = \dfrac{aq^2 - ap^2}{2aq - 2ap}$

$= \dfrac{a(\cancel{q-p})(q+p)}{2a(\cancel{q-p})}$

$m_{PQ} = \dfrac{q+p}{2}$ | 1 mark

$y - aq^2 = \dfrac{q+p}{2}(x - 2aq)$

$y - \cancel{aq^2} = \left(\dfrac{q+p}{2}\right)x - \dfrac{\cancel{2aq^2}}{\cancel{2}} - \dfrac{2apq}{2}$

$\therefore y = \left(\dfrac{q+p}{2}\right)x - apq.$ | 1 mark.

(ii) If focal chord, passes through $(0,a)$

$\therefore a = \left(\dfrac{p+q}{2}\right)0 - apq$ | 1 mark

$a = -apq$ $\boxed{pq = -1}$

Question 2(c)

(i) AP: $y = \frac{x^2}{4}.$

$\frac{dy}{dx} = \frac{2x}{4}$

$= \frac{x}{2}.$

at $P(2p, p^2)$ $\frac{dy}{dx} = \frac{2p}{2}$

$= p.$

$\therefore M_{AP} = p.$ — 1 mark for tangent

$y - p^2 = p(x - 2p)$

$y - p^2 = px - 2p^2.$

$\boxed{\therefore y = px - ap^2 \quad \ldots \text{ eq'n of AP}}$ — 1 mark for tangent eq'n.

ii) Eq'n of normal PB.

$M_{AP} = p \quad \therefore m_{PB} = -\frac{1}{p}.$

$y - p^2 = -\frac{1}{p}(x - 2p)$

$py - p^3 = -x + 2p.$

$\boxed{x + py = 2p + p^3 \ldots \text{ eq'n of PB}}$ — 1 mark

at B, $x = 0$ $\therefore 0 + py = 2p + p^3$

$y = 2 + p^2.$

$\therefore \boxed{B\ (0, p^2 + 2)}$ — 1 mark.

iii) A : $y = 0$ $\therefore 0 = px - ap^2.$

$\therefore x = ap$ $\therefore \boxed{A\ (ap, 0)}$

Midpoint of AB: $\left(\frac{ap + 0}{2}, \frac{p^2 + 2}{2}\right) = C$ — 1 mark.

$\therefore \boxed{C : \left(\frac{ap}{2}, \frac{p^2 + 2}{2}\right)}$

$x = \frac{ap}{2}$ $\therefore$ $p = \frac{2x}{a}$ —— ①

$\therefore$ $y = \frac{1}{2}p^2 + 1$ —— ②

① → ② $y = \frac{1}{2}\left(\frac{2x}{a}\right)^2 + 1$

$y = \frac{2^4 x^2}{2a} + 1$

$ay = 2x^2 + a.$

$\therefore$ $2x^2 = ay - a$

$\therefore$ $2x^2 = a(y-1)$

1 mark for $y = \frac{2x^2}{a} + 1$

L.S.E.

QUESTION 3:

a) $-2 \le x \le 2$. ①

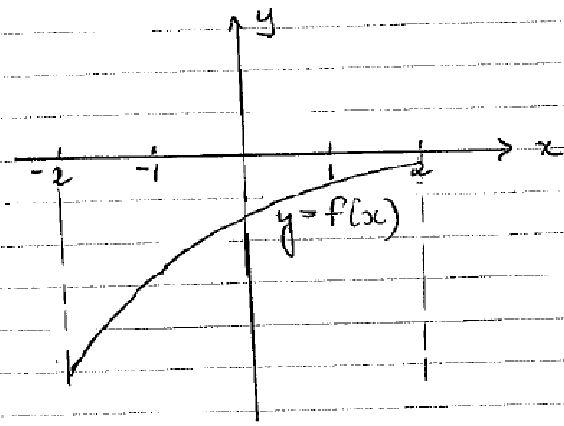
$f(x) < 0$ (negative y values) ②

$f'(x) > 0$ ($\therefore$ increasing) ③

$f''(x) < 0$ ($\therefore$ concave down) ④

Must have all 4 correct for 2 marks

Award 1 for 2 correct.



) $y = 7 + 4x^3 - 3x^4$

(i) Stationary points occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 12x^2 - 12x^3$.

$\frac{dy}{dx} = 0 \Rightarrow 12x^2(1-x) = 0$

$\therefore x = 0$ or $x = 1$

when $x = 0$, $y = 7$ $\therefore (0,7)$

when $x = 1$, $y =$ $\therefore (1,8)$

(ii) $\frac{d^2y}{dx^2} = 24x - 36x^2$. | 1 mark for $\frac{d^2y}{dx^2}$.

$\frac{d^2y}{dx^2} = 0 \Rightarrow 12x(2-3x) = 0$

$x = 0$ or $x = \frac{2}{3}$. | 1 mark for $\therefore$ 2 values

(iii) $\frac{d^2y}{dx^2} = 24x - 36x^2$.

at $(0,0)$ $\frac{d^2y}{dx^2} = 0$, $\therefore$ could be hor. pt of inflexion.

x	0^- -0.1	0	0.1 0^+
$\frac{d^2y}{dx^2}$	<0	0	>0

$24 \times -0.1 - 36(-0.1)^2$ <0 | $24 \times 0.1 - 36 \times 0.1$ >0

1 mark ... MUST SHOW SUBSTITUTIONS.

Change in concavity

$\therefore$ $\boxed{\text{horizontal pt of inflexion at } (0,7)}$

at $(1,8)$ $\frac{d^2y}{dx^2} = 24 \times 1 - 36.1^2$.

$= -12$

<0 | 1 mark.

$\therefore$ $\boxed{\text{Maximum at } (1,8)}$

(iv)

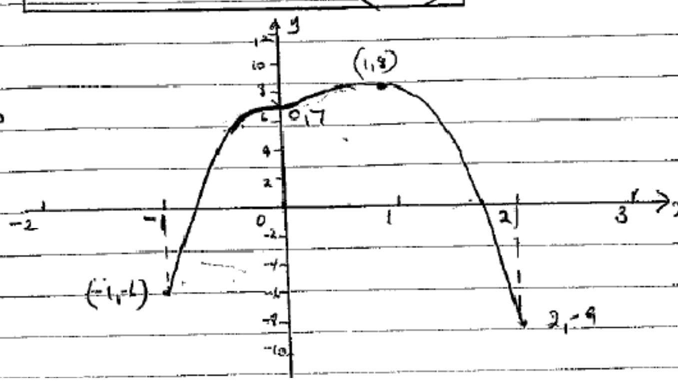
$x = -1, y = -6$

$\therefore (-1,-6)$

$x = 2$,

$y = -9$

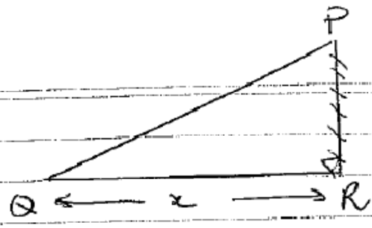
$\therefore (2,-9)$



2 mark 4 correct correct curve + SCALE + labels.

1 mark 4 correct curve if labels or scale missing.

c)



MUST HAVE ALL WORKING.

(i) $PQ + QR = 5m.$

$\therefore PQ + x = 5$

$PQ = 5 - x.$ — 1 mark

(ii) Area $= \frac{1}{2} \times x \times PR$ (area of Δ PQR)

$$PR^2 = (5-x)^2 - x^2$$
$$= 25 - 10x + x^2 - x^2$$
$$= 25 - 10x.$$
$$\therefore PR = \sqrt{25-10x}$$ — 1 mark for PR.

$\therefore$ Area $= \frac{1}{2} \times x \times \sqrt{25-10x}$ — 1 mark

(iii) Maximum area when $\frac{dA}{dx} = 0$

$$\frac{dA}{dx} = \frac{1}{2}x \cdot \frac{1}{2}(25-10x)^{-\frac{1}{2}} \cdot -10 + (25-10x)^{\frac{1}{2}} \cdot \frac{1}{2}$$ — 1 mark for correct use of product rule to find derivative

$$= \frac{-5x}{2\sqrt{25-10x}} + \frac{1}{2}\sqrt{25-10x}.$$

$\frac{dA}{dx} = 0 \Rightarrow \frac{5x}{2\sqrt{25-10x}} = \frac{1}{2}\sqrt{25-10x}$

$$5x = 25 - 10x.$$
$$15x = 25$$
$$x = \frac{5}{3}.$$ — 1 mark

When $x = \frac{5}{3}$, $A = \frac{1}{2} \cdot \frac{5}{3} \sqrt{25 - 10 \cdot \frac{5}{3}} = \frac{25}{6\sqrt{3}}$ units^2

To show maximum: — 1 mark

x	$\frac{5}{3}^-$ (1)	$5/3$	$\frac{5}{3}^+$ (2)
$\frac{dy}{dx}$	$\frac{-10}{2\sqrt{15}} + \frac{1}{2} \cdot \sqrt{15} = 0.65$ $= > 0$	0	$\frac{-5}{\sqrt{5}} + \frac{\sqrt{5}}{2} = -1.11$ < 0

$\therefore$ Maximum area when $x = \frac{5}{3}$

Area is $\frac{25}{6\sqrt{3}} m^2$

YR12 EXT 1 TASK 1. Dec. 2010.

Question 1.

a) (i) $f(3) = 3^3 + 3.3^2 - 9.3 - 27$ | 1 mark WITH SUBSTITUTION

$= 0$

$\therefore x - 3$ is a factor of $f(x)$

(ii) Interceps occur when $f(x) = 0$.

$$\begin{array}{r} x^2 + 6x + 9 \\ x - 3 \overline{) x^3 + 3x^2 - 9x - 27} \\ x^3 - 3x^2 \\ \hline 6x^2 - 9x \\ 6x^2 - 18x \\ \hline 9x - 27 \\ 9x - 27 \\ \hline 0 \end{array}$$

MUST HAVE WORKING!

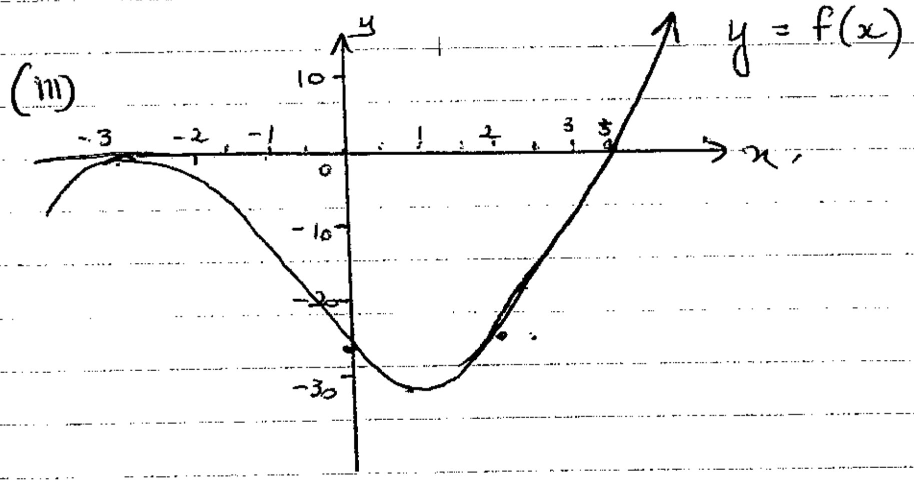
$\therefore f(x) = (x-3)(x^2+6x+9)$ | 1 MARK

$= (x-3)(x+3)^2$

$f(x) = 0 \Rightarrow (x-3)(x+3)^2 = 0$

$\therefore x = 3$ or $x = -3$. | 1 MARK.

(iii)



1 MARK FOR CORRECT X + Y INTERCEPTS & GRAPH.

0 FOR NO SCALE

b) $x^3 + 0x^2 - 3x + 1 = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\alpha\beta\gamma = -1$

(i) $\alpha\beta + \alpha\gamma + \beta\gamma = -3$. | 1 MARK.

(ii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ | MUST HAVE THIS

$= 0 - 2.-3$

$= 6$ | 1 MARK.

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$ ← MUST HAVE THIS

$= \frac{-3}{-1}$

1 mark.

$= 3.$

c) $P(x) = 2x^4 - ax^2 + bx - 1$

$P(-1) = 2 - a - b - 1$

$= 0$

$\therefore \boxed{a + b = 1}$ —— ① 1 mark

$P(2) = 2.2^4 - a.2^2 + b.2 - 1$

$= -3.$

$\therefore 32 - 4a + 2b - 1 = -3.$

$2b - 4a = -34.$

$\boxed{\therefore b - 2a = -17}$ —— ② 1 mark.

① - ② $3a = 18$

$a = 6$ —— (*)

(*) → ① $6 + b = 1$

$b = -5$

$\therefore a = 6, b = -5$ 1 mark.

d) $P(x) = x^3 + px^2 + qx + r$ MUST HAVE ALL WORKING.

i) Sum of roots $\alpha + \sqrt{m} - \sqrt{m} = \frac{-p}{1}$ 1 mark

$\therefore \alpha = -p$

$\alpha + p = 0$

(ii) Product of roots $\alpha . \sqrt{m} . -\sqrt{m} = -r$ 1 mark.

$\therefore -m\alpha = -r$

$m\alpha = r$

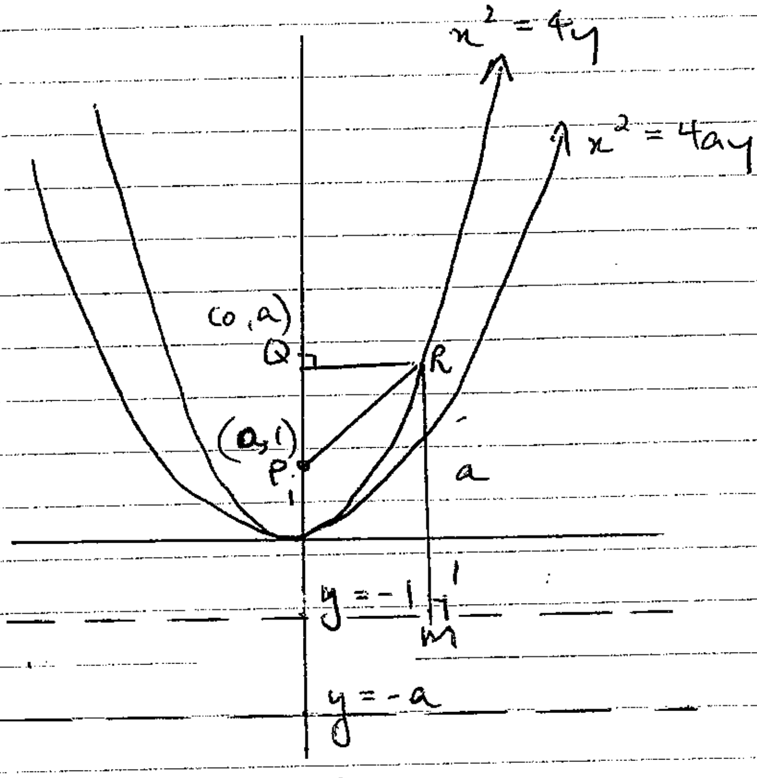
(iii) $\alpha . \sqrt{m} - \alpha . \sqrt{m} - \sqrt{m}.\sqrt{m} = q.$ but $\alpha = -p.$ 1 mark for $r - m = q$.

$-m = q.$ $\therefore \frac{-r}{-p} = q.$ 1 mark for soln.

from (ii) $m = \frac{r}{\alpha}.$ $\therefore -\frac{r}{\alpha} = q.$ $\therefore pq = r$

Question 2

a)



(ii) | 1 mark.

(i) $P(0,1)$ $\quad Q(0,a)$ | 1 mark.

(iii) $PR = RM$ (def'n of parabola) | 1 mark.

$\therefore PR = a+1$

b) $(2ap, ap^2)P$ & $Q(2aq, aq^2)$ | MUST HAVE ALL WORKING

(i) $m_{PQ} = \dfrac{aq^2 - ap^2}{2aq - 2ap}$

$= \dfrac{a\cancel{(q-p)}(q+p)}{2a\cancel{(q-p)}}$

$m_{PQ} = \dfrac{q+p}{2}$ | 1 mark

$y - aq^2 = \dfrac{q+p}{2}(x - 2aq)$

$y - \cancel{aq^2} = \left(\dfrac{q+p}{2}\right)x - \cancel{\dfrac{2aq^2}{2}} - \dfrac{2apq}{2}$

$\therefore y = \left(\dfrac{q+p}{2}\right)x - apq.$ | 1 mark.

(ii) If focal chord, passes through $(0,a)$

$\therefore a = \left(\dfrac{p+q}{2}\right).0 - apq$ | 1 mark

$a = -apq$

$\boxed{pq = -1}$

Question 2(c)

(i) AP: $y = \frac{x^2}{4}$.

$\frac{dy}{dx} = \frac{2x}{4}$

$= \frac{x}{2}$.

at $P(2p, p^2)$ $\frac{dy}{dx} = \frac{2p}{2}$

$= p$.

$\therefore M_{AP} = p$. — 1 mark for tangent

$y - p^2 = p(x - 2p)$

$y - p^2 = px - 2p^2$.

$\boxed{\therefore y = px - ap^2 \quad \ldots \text{ eq'n of AP}}$ — 1 mark for tangent eq'n.

ii) Eq'n of normal PB.

$M_{AP} = p \quad \therefore m_{PB} = \frac{-1}{p}$.

$y - p^2 = -\frac{1}{p}(x - 2p)$

$py - p^3 = -x + 2p$.

$\boxed{x + py = 2p + p^3 \quad \ldots \text{ eq'n of PB}}$ — 1 mark

at B, $x = 0$ $\therefore 0 + py = 2p + p^3$

$y = 2 + p^2$.

$\therefore \boxed{B\ (0, p^2 + 2)}$ — 1 mark.

iii) A: $y = 0$ $\therefore 0 = px - ap^2$.

$\therefore x = ap$ $\therefore \boxed{A\ (ap, 0)}$

Midpoint of AB: $\left(\frac{ap + 0}{2}, \frac{p^2 + 2}{2}\right) = C$ — 1 mark.

$\therefore \boxed{C: \left(\frac{ap}{2}, \frac{p^2 + 2}{2}\right)}$

$x = \frac{ap}{2}$ $\therefore$ $p = \frac{2x}{a}$ — ①

$\therefore$ $y = \frac{1}{2}p^2 + 1$ — ②

① → ② $y = \frac{1}{2}\left(\frac{2x}{a}\right)^2 + 1$

$y = \frac{2\cancel{4}x^2}{2a} + 1$

$ay = 2x^2 + a.$

$\therefore$ $2x^2 = ay - a$

$\therefore$ $2x^2 = a(y-1)$

1 mark for $y = \frac{2x^2}{a} + 1$

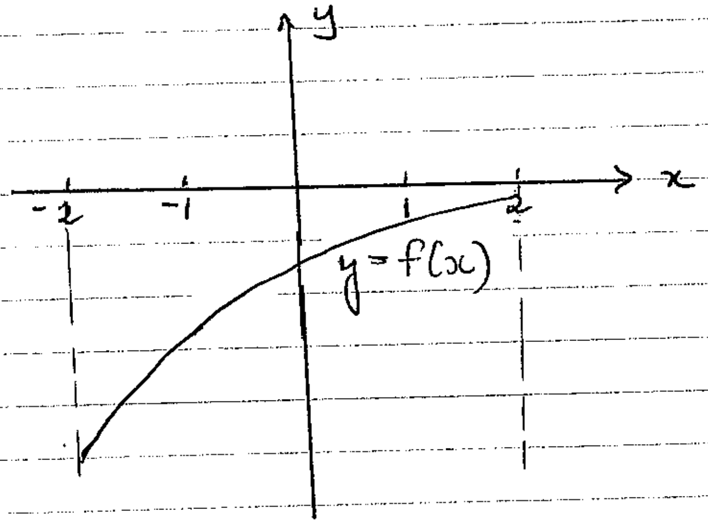
I.S.E.

QUESTION 3:

a) $-2 \le x \le 2$. ①
$f(x) < 0$ (negative y values) ②
$f'(x) > 0$ ($\therefore$ increasing) ③
$f''(x) < 0$ ($\therefore$ concave down) ④

Must have all 4 correct for 2 marks.

Award 1 for 2 correct.



) $y = 7 + 4x^3 - 3x^4$

(i) Stationary points occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 12x^2 - 12x^3$.

$\frac{dy}{dx} = 0 \Rightarrow 12x^2(1-x) = 0$

$\therefore x = 0$ or $x = 1$

when $x = 0$, $y = 7$ $\therefore (0,7)$

when $x = 1$, $y =$ $\therefore (1,8)$

(ii) $\frac{d^2y}{dx^2} = 24x - 36x^2$. | 1 mark for $\frac{d^2y}{dx^2}$.

$\frac{d^2y}{dx^2} = 0 \Rightarrow 12x(2-3x) = 0$

$x = 0$ or $x = \frac{2}{3}$. | 1 mark for 2 values

(iii) $\frac{d^2y}{dx^2} = 24x - 36x^2$.

at (0,0) $\frac{d^2y}{dx^2} = 0$, $\therefore$ could be hor. pt of inflexion.

x	0^- -0.1	0	0.1 0^+
$\frac{d^2y}{dx^2}$	<0	0	>0

$24\times -0.1 - 36(0.1)^2$ <0 | $24\times 0.1 - 36\times 0.1^2$ >0

1 mark ... MUST SHOW SUBSTITUTIONS.

Change in concavity

$\therefore$ $\boxed{\text{horizontal pt of inflexion at } (0,7)}$

at (1,8) $\frac{d^2y}{dx^2} = 24\times 1 - 36.1^2$. | 1 mark.

$= -12$

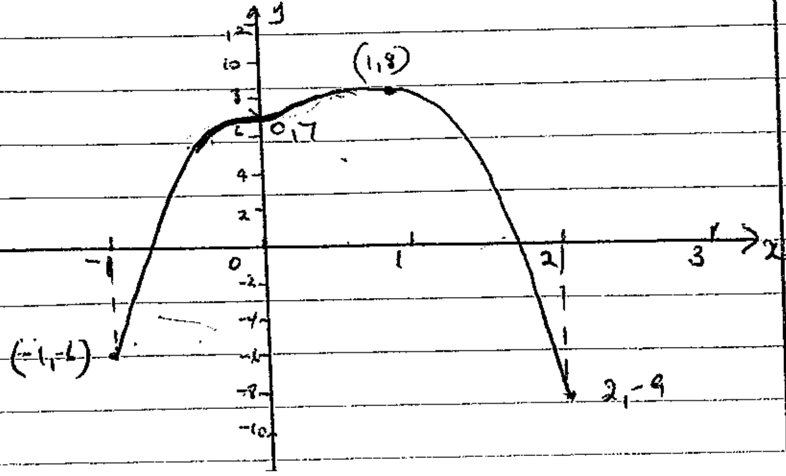
< 0

$\therefore$ $\boxed{\text{Maximum at } (1,8)}$

(iv)

$x = -1, y = -6$

$\therefore (1,-6)$



$x = 2$,

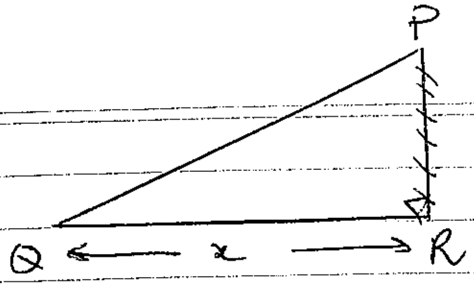
$y = -9$

$\therefore (2,-9)$

2 mark 4 correct correct curve + SCALE + labels.

1 mark 4 correct curve if labels or scale missing.

c)



MUST HAVE ALL WORKING.

(i) $PQ + QR = 5m.$

$\therefore PQ + x = 5$

$PQ = 5 - x.$

1 mark

(ii) Area $= \frac{1}{2} \times x \times PR$ (area of Δ PQR)

$$PR^2 = (5-x)^2 - x^2$$
$$= 25 - 10x + x^2 - x^2$$
$$= 25 - 10x.$$
$$\therefore PR = \sqrt{25-10x}$$

1 mark for PR.

$$\therefore \text{Area} = \frac{1}{2} \times x \times \sqrt{25-10x}$$

1 mark

(iii) Maximum area when $\frac{dA}{dx} = 0.$

$$\frac{dA}{dx} = \frac{1}{2}x \cdot \frac{1}{2}(25-10x)^{-\frac{1}{2}} \cdot -10 + (25-10x)^{\frac{1}{2}} \cdot \frac{1}{2}.$$

1 mark for correct use of product rule to find derivativ

$$= \frac{-5x}{2\sqrt{25-10x}} + \frac{1}{2}\sqrt{25-10x}.$$

$$\frac{dA}{dx} = 0 \Rightarrow \frac{5x}{2\sqrt{25-10x}} = \frac{1}{2}\sqrt{25-10x}$$

$$5x = 25 - 10x.$$
$$15x = 25$$
$$x = \frac{5}{3}.$$

1 mark

When $x = \frac{5}{3}$, $A = \frac{1}{2} \cdot \frac{5}{3} \sqrt{25 - 10 \cdot \frac{5}{3}} = \frac{25}{6\sqrt{3}}$ units^2

To show maximum:

x	$\frac{5}{3}^-$ (1)	$\frac{5}{3}$	$\frac{5}{3}^+$ (2)
$\frac{dy}{dx}$	$\frac{-10}{2\sqrt{15}} + \frac{1}{2} \cdot \sqrt{15} = 0.65$ > 0	0	$\frac{-5}{\sqrt{5}} + \frac{\sqrt{5}}{2} = -1.11$ < 0

1 mark

$\therefore$ Maximum area when $x = \frac{5}{3}$

Area is $\frac{25}{6\sqrt{3}} m^2$