Killara HIGH SCHOOL

STUDENT NUMBER

MATHEMATICS
EXTENSION 1

HSC HALF – YEARLY ASSESSMENT

20th March, 2013

TIME ALLOWED: *Reading time: 5 minutes*
Working time: 2 hours

INSTRUCTIONS: This paper consists of 10 multiple choices and 4 fifteen mark questions
ANSWER all questions
Multiple choice and Standard Integral formula sheets are attached.
Start each question in a new booklet
Marks may not be given for untidy and poorly arranged work.
All relevant working should be shown.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

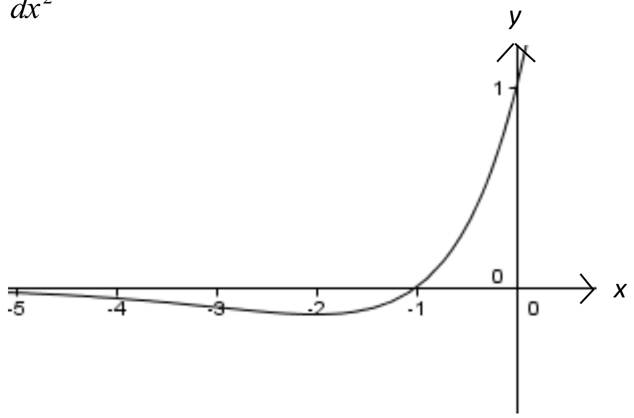
H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts
H2 - constructs arguments to prove and justify results
H4 - expresses practical problems in mathematical terms based on simple given models
H5 - applies appropriate techniques from the study of calculus and trigonometry to solve problems
H6 - uses the derivative to determine the features of the graph of a function
H7 - uses the features of a graph to deduce information about the derivative
H9 - communicates using mathematical language, notation, diagrams and graphs
Plus Preliminary from P1 to P8
HE1 - Appreciates interrelationships between ideas drawn from different areas of mathematics
HE6 – Determines integrals by reduction to a standard form through a given substitution

This assessment task constitutes 25% of the Higher School Certificate Course Assessment.

Section 1
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

1 For the sketch below where there is a stationary point at $x = -2$ and a point of inflexion at $x = -3$, find the points that satisfy both of the following inequalities

$\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$.



(A) $x < -3$

(B) $-3 < x < -2$

(C) $x > -2$

(D) $-3 < x < -1$

2 How many solutions does $\cos 3x = \frac{1}{2}$ have in the domain $0 \le x \le 2\pi$?

(A) 1
(B) 2
(C) 4
(D) 6

3 If $\int_1^4 f(x)dx = 5$, then $\int_1^4 (2f(x) - 1)dx$ is equal to

(A) 9
(B) 10
(C) 7
(D) 8

4 For the function $f(x) = xe^{-x}$. Find (e) .

(A) e^{-2e}

(B) e^{1-e}

(C) $\frac{1}{e}$

(D) $-e^{2e}$

5 What are the amplitude and period of the function $y = 1 + 2\sin\frac{1}{2}\pi x$?

(A) Amplitude 1 and period $\frac{1}{2}\pi$

(B) Amplitude 2 and period 4π

(C) Amplitude 3 and period 4

(D) Amplitude 2 and period 4

6 The domain of $y = \ln(3 - x)$ is

(A) $x < 3$

(B) $x > 3$

(C) $x > 0$

(D) $x \geq 1$

7 $\int 2cos^2 2x \ dx$ is

(A) $2\frac{sin^3 2x}{3} + C$

(B) $-2sin^2 2x \,. cos2x + C$

(C) $\frac{sin4x}{4} + x + C$

(D) $-\frac{sin4x}{4} + C$

8 What transformations need to be applied to the curve $y = \ln(-x)$ in order to obtain the curve $y = 1 - \ln(x)$

(A) Reflect in the *x-axis*, reflect in the *y-axis* then shift 1 unit up

(B) Reflect in the *y-axis*, translate 1 unit right

(C) Reflect in the *x-axis*, translate 1 unit right

(D) Reflect in the *x-axis,* reflect in the *y-axis* then shift 1 unit down

9 6 cards numbered as 1,2,3,4,5,6. One card is drawn at random and not replaced. The probability that second number is 5

(A) $\frac{1}{5}$

(B) $\frac{1}{6}$

(C) $\frac{1}{18}$

(D) $\frac{1}{30}$

10 The value of $\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x \, dx$ is

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{2-\sqrt{3}}{4}$

(D) 2

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 (15 marks) **Begin a new booklet** **Marks**

(a) Differentiate $\cos(ln x)$ with respect to x.
2

(b) The acute angle between the lines $y = (m+2)x$ and $y = mx$ is 45° .

(i) Show that $\left|\dfrac{2}{m^2+2m+1}\right| = 1$ 1

(ii) Hence find any values of m. 2

(c) Solve the inequality $\dfrac{x}{x+1} \le 2$. 3

(d) Find $\lim\limits_{x\to 0} \dfrac{\sin\frac{x}{4}}{3x}$. 2

(e) Show that $\dfrac{d}{dx}3^x = 3^x \ln 3$ 2

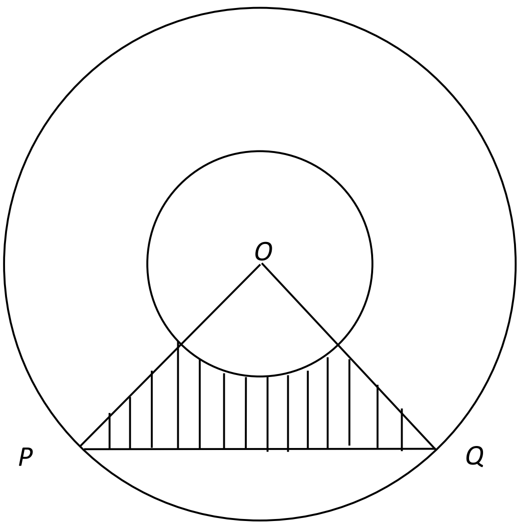
(f) A monic polynomial $P\,(x)$ of degree 4 has zeros 2 and -2.

(i) What down the general form of $P\,(x)$. 1

(ii) Find that particular $P\,(x)$ such that $P(0) = 4$ and the remainder is 3 when $P\,(x)$ is divided by $x - 1$. Write equation of the monic polynomial of degree 4. 2

Question 12 (15 marks) **Begin a new booklet** **Marks**

(a) Two concentric circles with centre O have radii 2cm and 4cm. The points P and Q lie on the larger circle and $\angle POQ = x$, where $0 \leq x \leq \dfrac{\pi}{2}$.



(i) If the area A cm^2 of the shaded region is $\frac{1}{16}$ the area of the larger circle, show that x satisfies the equation $8sinx - 2x - \pi = 0$. 3

(ii) Show that this equation has solution $x = \alpha$, where $0.5 < \alpha < 0.6$ 2

(iii) Taking 0.6 as the first approximation for α, use one application of Newton's method to find the second approximation, giving the answer correct to two decimal places. 2

(b) Integrate with respect to x $\dfrac{e^{\sqrt{x}}}{\sqrt{x}}$ 2

(c) Integrate with respect to x $\int \frac{2x-1}{x+1}\,dx$ 2

(d) (i) Express $\sqrt{3}sinx + cosx$ in the form $Asin(x+\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$. 2

(ii) Hence, or otherwise, solve $\sqrt{3}sinx + cosx = 1$ for $0 \le x \le 2\pi$. 2

Question 13 (15 marks) **Begin a new booklet** **Mark**

(a) Consider the function $f(x) = e^{-x} - xe^{-x}$.

(i) Find $f'(x)$. 2

(ii) The graph $y = f(x)$ has one minimum turning point. 2
Find the coordinates and nature of the minimum turning point.

(iii) Describe the behaviour of $f(x)$ as $x \to \infty$. 1

(iv) Find the x and y – intercepts of the graph $= f(x)$. 2

(v) Sketch the graph of $y = f(x)$ showing the features from parts (ii) - (iv) 2
You don't need to find the point of inflexion.

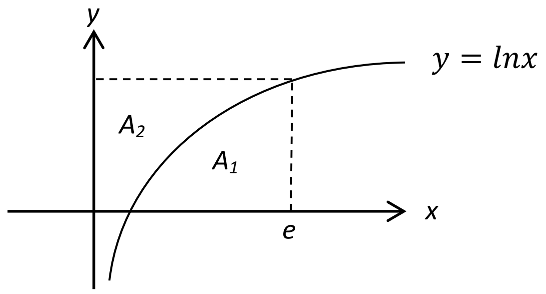
(b) Use the substitution $u = x + 1$ to evaluate $\int_0^3 \frac{x-2}{\sqrt{x+1}}\,dx$. 3

(c) Use mathematical induction to prove that, for ≥ 1 , 3

$$4 - 8 + 16 \; . \; . \; . \; . + 4\,(-2)^{n-1} = -\,\frac{4[(-2)^n - 1]}{3} \quad \text{for all } n \geq 1.$$

Question 14 (15 marks) **Begin a new booklet** **Mark**

(a) (i) Find the area enclosed between the curve $y = lnx$, x – axis and $x = e$. 2



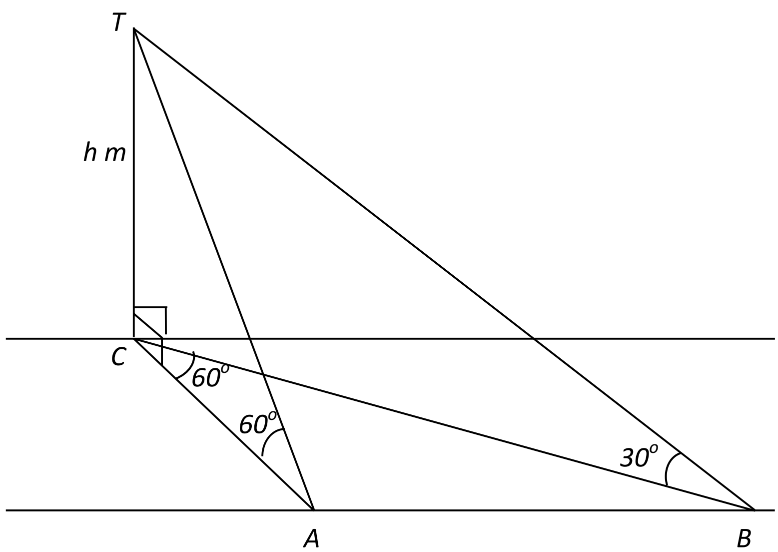
(ii) Area A_2 in part (i) is rotated about y – axis. Find the volume of the solid of revolution formed. 2

(b) Prove that $\dfrac{1-\sin^2\theta \ \cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$ 2

(c) A tower is constructed on the bank of a long straight river. From a point A on the opposite bank the angle of elevation of the top of the tower is 60°. From another point 50 m further down the river the angle of elevation of the tower is 30°.

(i) Find the exact height of the tower 3

(ii) Find the exact width of the river. 2

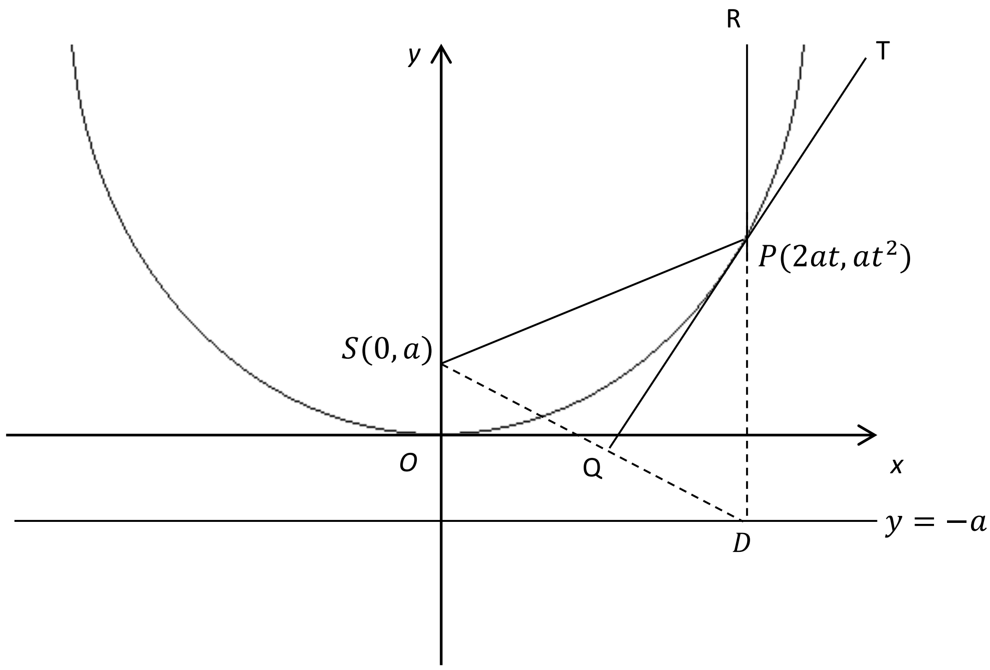


50 m

(d) The diagram shows the parabola $x^2 = 4ay$ with focus S $(0, a)$ and direxstrix $y = -a$. The point $P(2at, at^2)$ is a point on the parabola and the line RP is drawn parallel to the y axis, neeting the directrix at D. The tangent QPT to the parabola at P intersects SD at Q.

(i) Prove that PQ is perpendicular to SD. 2

(ii) Prove that $\angle RPT = \angle SPQ$ 2



End of test

Section 1

11

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Select the alternative A, B, C and D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

	A	**B**	**C**	**D**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

STANDARD INTEGRALS

12

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

ANSWERS

BDCBDACABB

SECTION I - MULTIPLE CHOICE

1 B, 2 Domain $0 \leq 3x \leq 6\pi$, $\therefore$ 6 solutions D

3 $\int_1^4 (2f(x)-1)\,dx = \int_1^4 2f(x)\,dx - \int_1^4 dx = 2\times 5 - [x]_1^4 = 10-(4-1) = 7$ C

4 $f(e) = e \times e^{-e} = e^{1-e}$ B, 5 Period $= \frac{2\pi}{\frac{1}{2}\pi} = 4$, $A = 2$ D

6 $3-x>0$ $-x>-3$, $x<3$, A,

7 $\int 2\cos^2 2x\,dx = \int(\cos 4x + 1)\,dx = \frac{\sin 4x}{4} + x + C$ C, 8 A

9 $P(\text{2nd no } 5) = \frac{1}{6}\times\frac{1}{5}\times 6 = \frac{1}{5}$ A, 10 B

Section 2

11 a) $\frac{d\cos(\ln x)}{dx} = -\sin \ln x \cdot \frac{1}{x}$

$= -\frac{\sin \ln x}{x}$

a) • Finds derivative of $\cos x$ (1 mark)

• Finds derivative of $\ln x$ (1 mark)

Many students did this as product rule. "$u = \cos$" cannot be differentiated

b) $y = (m+2)x$, $y = mx$

i) $m_1 = m+2$, $m_2 = m$

$\tan 45^\circ = \left|\frac{(m+2)-m}{1+m(m+2)}\right|$

$1 = \left|\frac{2}{m^2+2m+1}\right| \Rightarrow \left|\frac{2}{(m+1)^2}\right| = 1$

b) i) • uses the formula to obtain the required equation (1 mark)

$\left|\frac{2}{m^2+2m+1}\right| = 1.$

You must explain that $m^2+2m+1 > 0$ for all m. $\therefore 2 = |m+1|^2$ ie $(m+1)^2 = 2$ and solve.

ii) $(m+1)^2 = 2$ from part (i)

$m+1 = \mp\sqrt{2}$

$m = -1 \mp \sqrt{2}$

Else, You must Consider $m^2+2m+1 = 2$ or $m^2+2m+1 = -2$.

ii) • writes the equation in part (i) as a perfect square (1 mark)

• Evaluates "m" (1 mark)

Start here

c) $\frac{x}{x+1} \leq 2 \quad x(x+1)^2$

$x \neq -1$

$x(x+1) \leq 2(x+1)^2$

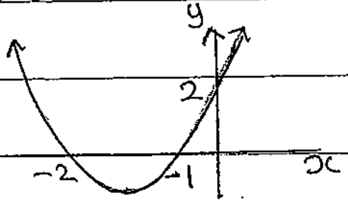
$x^2+x \leq 2x^2+4x+2$

$x^2+3x+2 \geq 0$

$(x+2)(x+1) \geq 0$

~~$x > -1$~~

$x \leq -2$



- Eliminates the denomin and $x \neq -1$ (1 mark)
- Progress towards solution (1 mark)
- Correct response (1 m

Most students did this question well.

e) $\frac{d}{dx} 3^x = 3^x \ln 3$

$3^x = e^{\ln 3^x}$

$= e^{x \ln 3}$

$\therefore \frac{d}{dx} e^{\ln 3 . x} = e^{\ln 3 . x} . \ln 3$

$= 3^x . \ln 3$

$\left[\text{Since } e^{\ln 3 . x} = 3^x\right]$

e)
- Expresses $3^x = e^{\ln 3 . x}$ (1 mark)
- Differentiates correctly (1 mark)

Very poorly done. Many students skipped this question.

f) i) $P(x) = (x^2-4)(x^2+bx+c)$

ii) $P(0) = 4$, $P(2) = P(-2) = 0$

$\therefore P(0) = (-4)(0+c) = 4$

$c = -1$

$P(1) = 3$, $(1-4)(1+b-1) = 3$

$b = -1$

Hence $P(x) = (x^2-4)(x^2-x-1)$

$P(x) = x^4 - x^3 - x^2 - 4x^2 + 4x + 4$

$P(x) = x^4 - x^3 - 5x^2 + 4x + 4$

i • Correct $P(x)$ (1 mark)

ii • Evaluates "b" and "c" (1 mark)

- Writes equation of monic polynomial to fourth degree (1 mark)

"Too many students wasted time on this question, by starting $P(x) = ax^4 + bx^3 + cx + d$. Division transformation is the intent of this question."

Question 12

a) Start here

i) A of large circle $= \pi(4)^2$

Area of $\Delta OPQ = \frac{1}{2} \cdot 4 \cdot 4 \sin x$

$= 8\sin x$

Area of sector $OAB = \frac{x}{2\pi} \cdot \pi \cdot 2^2$

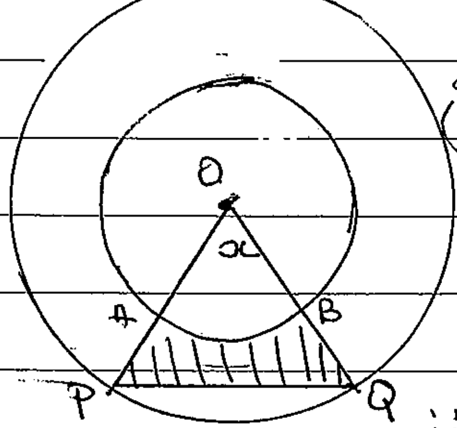
$= 2x$

$A_{shaded} = 8\sin x - 2x$

$A_{shaded} = \frac{1}{16}\pi \cdot 4^2$ (Given)

$\therefore\ 8\sin x - 2x = \pi$

$8\sin x - 2x - \pi = 0$



i) • Finds area of ΔOPQ or Area of large circle (1 mark)

• Finds area of sector OAB (1 mark)

• Shows x satisfies given equation (1 mark)

Generally, well done

ii) $f(x) = 8\sin x - 2x - \pi$,

$f(0.5) = 8\sin 0.5 - 2(0.5) - \pi$, $f(0.6) = 8\sin 0.6 - 2(0.6) - \pi$

$= -0.31 < 0$ $\qquad = 0.18 > 0$

$\therefore$ since $f(x)$ is continuous & $f(0.5) < 0$, $f(0.6) > 0$

then $f(\alpha) = 0$ for $0.5 < \alpha < 0.6$

Substituting different values of x in the domain $[0.5, 0.6]$ is not the way to approach this problem.

ii) • Shows $f(0.5)$ and $f(0.6)$ have different signs (1 mark)

• Deduces existance of a root (1 mark)

iii) $\alpha = 0.6 - \frac{f(0.6)}{f'(0.6)}$ $\qquad f'(x) = 8\cos x - 2$

$= 0.6 - \frac{8\sin 0.6 - 2(0.6) - \pi}{8\cos 0.6 - 2}$

$= 0.56$ (2 decimal place)

iii) • Applies Newton's rule (1 mark)

• calculates the root (1 mark).

Again many students refuse to learn formulae.

Write the • formula

• Show the substitution

• Write calculator display up to 5 d.p.

• round off

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b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$, let $\sqrt{x}=u$, $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$

$\Downarrow$

$du = \frac{dx}{2\sqrt{x}}$

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx$

$= 2\int e^{u}\,du$

$= 2e^{u} + C$

$\therefore \int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2e^{\sqrt{x}} + C$

b) • Uses correct substitution converts dx into du (1mark)

• Finds the primitive function (1mark)

Poorly done. Incorrect substitution

c) $\int \frac{2x-1}{x+1}dx = \int \frac{2x-1+2-2}{x+1}dx$

$= \int \frac{2(x+1)-3}{x+1}dx$

$= \int \left(2-\frac{3}{x+1}\right)dx$

$= 2x - 3\ln(x+1) + C$

c) • Writes $\frac{2x-1}{x+1} = 2 - \frac{3}{x+1}$ (1mark)

• Finds the primitive function (1mark)

If deg (Numerator) $\geq$ deg (denominator) first do polynomial division, then you integrate

d) i) $\sqrt{3}\sin x + \cos x$ where

$A = \sqrt{(\sqrt{3})^2+1} = 2$, $\tan\alpha = \frac{1}{\sqrt{3}}$, $\alpha = \frac{\pi}{6}$

$\therefore \sqrt{3}\sin x + \cos x = 2\sin\left(x+\frac{\pi}{6}\right)$

i) • Correctly finds A and (1mark)

• Correct answer 1mark.

Quite well done

ii) $\sqrt{3}\sin x + \cos x = 1$

Domain: $0 \leq x \leq 2\pi$, $\frac{\pi}{6} \leq x+\frac{\pi}{6} \leq 2\pi + \frac{\pi}{6}$

$2\sin\left(x+\frac{\pi}{6}\right) = 1$ From part (i)

$\sin\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$ $\quad$ sin>0 it is 1st & 2nd quadrant

$x + \frac{\pi}{6} = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$

$x = 0, \pi, 2\pi$

• Any acceptable attempt towards solution (1mark)

• correct solution (1mark)

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Question 13

a) $f(x) = e^{-x} - x e^{-x}$

Many students did not use product rule.

i) $f'(x) = -e^{-x} - [e^{-x} - x e^{-x}]$

$= -2e^{-x} + x e^{-x}$

$= e^{-x}(x-2)$

i) • Differentiates e^{-x} (1 mark)

2 • Uses chain rule and differentiates xe^{-x} (1 mark)

ii) Stationary point $f'(x) = 0$

$e^{-x}(x-2) = 0$

$e^{-x} > 0$, $x - 2 = 0$, $x = 2$

$x = 2$, $f(2) = e^{-2} - 2e^{-2} = -\frac{1}{e^2}$

$\therefore$ Stationary point $\left(2, -\frac{1}{e^2}\right)$

ii) • Finds coordinates of stationary pt (1 mark)

2 • Finds the nature of the stationary pt. (1 mark)

Well done by most students.

The nature of stationary point

$f''(x) = -e^{-x}(x-2) + e^{-x}$

$= e^{-x}(1 - x + 2)$

$f''(x) = e^{-x}(3-x)$

$f''(2) = e^{-2} = 0.135 > 0$ $\cup$ min turning pt

iii) $f(x) = \frac{1}{e^x} - \frac{x}{e^x} = \frac{1}{e^x}(1-x)$

$f(x) \to 0$ (from below) as $x \to \infty$

iii) Describes the behaviour (1 mark)

Very few wrote from below and so were given zero.

iv) x-intercept: $y = 0$, $e^{-x}(1-x) = 0$, $x = 1$

Well done!

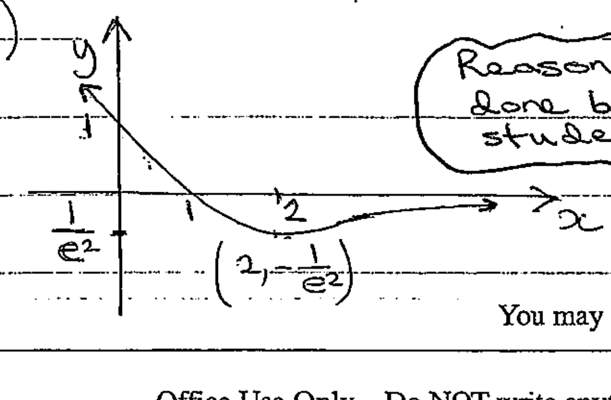
y-intercept: $x = 0$, $y = e^0 - 0 = 1$

$\therefore$ x-intercept $(1, 0)$, y-intercept $(0, 1)$

iv) • Finds x-intercept (1 mark)

2 • Finds y-intercept (1 mark)

v)



Reasonably well done by most students

v) • All important features shown (1 mark)

2 • Correct shape (1 mark) uniform scaling

You may ask for an extra Writing Booklet if you need more space.

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Question 13 continue

b) $\int_0^3 \frac{x-2}{\sqrt{x+1}}\,dx$,

Let $u=x+1$, $x=u-1$, $\frac{dx}{du}=1$

when $x=3$ $u=4$, $x=0$ $u=1$

$\therefore \int_1^4 \frac{(u-1)-2}{\sqrt{u}}\,du = \int_1^4 \frac{u-3}{\sqrt{u}}\,du$

$= \int_1^4 \left(\sqrt{u}-\frac{3}{\sqrt{u}}\right)du$

$= \left[\frac{2}{3}\sqrt{u^3}-3\sqrt{u}.2\right]_1^4$

$= \left[\frac{2}{3}.8-12\right]-\left[\frac{2}{3}-6\right]$

$= -\frac{4}{3}$

- Rewrites the integral in terms of u. (1 mark)
- Finds the new limits (1 mark)
- Evaluates integral correctly (1 mark)

3

Some students forgot to change limits and a few still don't know their index laws. Most did this question well.

c) $4-8+16 \quad +4(-2)^{n-1} = \frac{-4[(-2)^n-1]}{3}$

Step 1 Prove formula is true for $n=1$

$LHS = 4(-2)^0 = 4$, $RHS = \frac{-4[(-2)^1-1]}{3}$

$= \frac{-4[(-2)-1]}{3}$

$= 4$

Some students did not show working for step 1. They should

$\therefore$ formula is true for $n=1$.

- Proves it is true for $n=1$ and shows for $n=k$. (1 mark)
- Writes the formula for $n=k+1$ and some progress towards solution (1 mark)
- Correct solution (1 mark)

3

Step 2 Assume that the formula is true for $n=k$

$4-8+16 \quad +4(-2)^{k-1} = \frac{-4[(-2)^k-1]}{3}$

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Question 13 continue

Step 3 Prove that the formula is true for $n=k+1$

$$4-8+16 \ldots +4(-2)^{k-1}+4(-2)^k = \frac{4[(-2)^{k+1}-1]}{3}$$

$$\text{LHS} = \frac{4[(-2)^k-1]}{3} + 4(-2)^k \quad [\text{from step 2}]$$

Quite a large group of students did not set this step out correctly. They did not write the R.H.S. in the first line but then went ahead and changed the LHS. to equal what the RHS should have been. ie. they left this out

A few wrote the LHS in the initial line as $4(-2)^{k-1}+4(-2)^k$ and left out all the other terms. THIS IS WRONG!

$$= \frac{-4[(-2)^k-1]+12(-2)^k}{3}$$

MOST that set it out correctly managed to get this far at least

$$= \frac{-4(-2)^k+4+12(-2)^k}{3}$$

$$= \frac{8(-2)^k+4}{3}$$

$$= \frac{(-4)(-2)(-2)^k+4}{3}$$

$$= \frac{-4(-2)^{k+1}+4}{3}$$

$$= \frac{-4[(-2)^{k+1}-1]}{3}$$

$= \text{RHS}$

$\therefore$ If the formula is true for $n=k$, then it is true for $n=k+1$

We know that the formula is true for $n=1$, so it must be true for $n=2$. If it is true for $n=2$, then it must be true for $n=3$, and so on.

$\therefore$ It is true for any positive integer $n \geq 1$.

You may ask for an extra Writing Booklet if you need more space.

Question 14 continue

Start here ii) ~~Find area of river~~

~~$A = \frac{1}{2} b.h$ where h=width, $A = \frac{1}{2} AC.BC.\sin 60°$~~

~~$A = \frac{1}{2}.50.w$~~ ~~$A = \frac{1}{2}\frac{h}{\sqrt{3}}.\sqrt{3}h \times \frac{\sqrt{3}}{2}$~~

$\therefore\ 25w = \frac{\sqrt{3}}{4}h^2$, $h^2 = \frac{3\times 50^2}{7}$ [from (i)]

$25w = \frac{\sqrt{3}}{4}\cdot\frac{3\times 50^2}{7} \Rightarrow \cancel{100}w = \frac{3\sqrt{3}}{7}50^{\cancel{2}}$

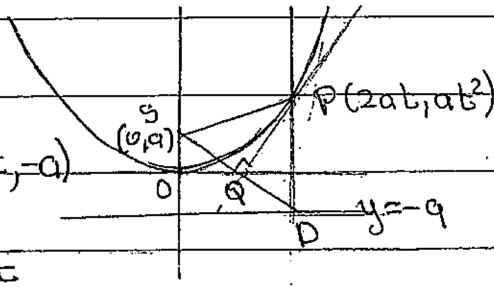
$w = \frac{75\sqrt{3}}{7}$ m

ii) • ~~Shows the two~~ ~~ways of writing the~~ area (1 mark)

• Calculates width ~~of the river~~ (1 mark)

This was poorly done. Few students found the two areas & solved the resulting eq'ns.

d) ~~$x^2 = 4ay$~~



i) ~~Coordinates D(2at, −a)~~

~~$m_{DS} = \frac{a - -a}{0 - 2at} = -\frac{1}{t}$~~

m_{PQ}: $y' = \frac{2x}{4a} = \frac{x}{2a}$, at P(2at, at²)

The gradient at P $m_{PQ} = \frac{2at}{2a} = t$

~~$m_{PQ} \times m_{DS} = -\frac{1}{t} \times t = -1 \therefore PQ \perp SD$~~

This was well done but SHOW means show all working

i)

• Uses coordinates of D to find m_{SD} and

• Uses derivative to find m_{PQ} (1 mark)

• Shows ~~the~~ condition for ~~perpendicularity~~ (1 mark)

ii) ~~Prove $\angle RPT = \angle SPQ$.~~

~~From definition of parabola: A moving point~~
~~equidistant from a point (0, a) and a line y=−a.~~

~~$\therefore$ PS = PD. Hence $\triangle PSQ$ is isosceles.~~

~~PQ ⊥ to base SD bisects ∠SPD.~~

~~$\therefore$ ∠SPQ = ∠DPQ~~, also:

~~∠DPQ = ∠RPT (Vertically opposite ∠'s)~~

~~Hence ∠SPQ = ∠RPT~~

ii)

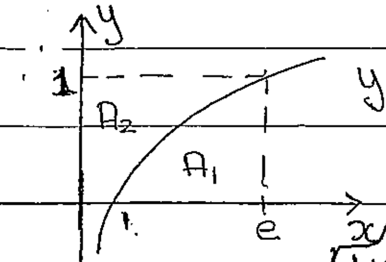
• Uses definition of parabola to show $\triangle PSQ$ is isosceles (1 mark)

• Shows the relationship between ~~the angles.~~ (1 mark)

Students need to work on geometry proofs. as setting out was a problem for many students

Question 14

a)



* Many students tried to integrate $\log_e x$ so thought $\int \log_e x \, dx = \frac{1}{x}$ & then incorrectly substitute

* Students who realised that they needed to do the area of rectangle minus the area between y axis generally did this well. Some did forget to change terminals

i) $A_1 = \int_1^e \ln x \, dx$, $y=0$ when $x=1$.

$A_1 = A_{rectangle} - A_2$

When $x=e$, $y = \ln e = 1$

$A_{rectangle} = e \times 1$

$= e$ unit^2

$y = \ln x \Rightarrow x = e^y$

$\therefore A_2 = \int_0^1 e^y \, dy$

$= [e^y]_0^1$

$= e - e^0$

$= e - 1$ unit^2

$\therefore A_1 = e - (e-1)$

$= 1$ unit^2

i) • Finds area of rectangle or A_2 (1 mark)

• Finds A_1 (1 mark)

ii) $V = \pi \int_0^1 x^2 \, dy$, $x = e^y$

$V = \pi \int_0^1 e^{2y} \, dy$

$V = \pi \left[\frac{e^{2y}}{2}\right]_0^1$

$V = \frac{\pi}{2} [e^2 - e^0]$

$V = \frac{\pi}{2}(e^2 - 1)$ cubic unit.

ii) • Finds x^2 and Limits (1 mark)

• Evaluates volume (1 mark)

Some students tried to use $y = \ln x$ ~~Some didn't know how to find the~~ the volume of ~~the~~ formula.

Students who did (i) well did (ii) well as well.

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Question 14 continue

b) $\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$

LHS $= \frac{1}{\cos^2\theta} - \sin^2\theta$

$= \sec^2\theta - \sin^2\theta \quad (\sec^2\theta = \tan^2 + 1)$

$= \tan^2\theta + 1 - \sin^2\theta \quad (1-\sin^2\theta = \cos^2\theta)$

$= \tan^2\theta + \cos^2\theta$

LHS = RHS

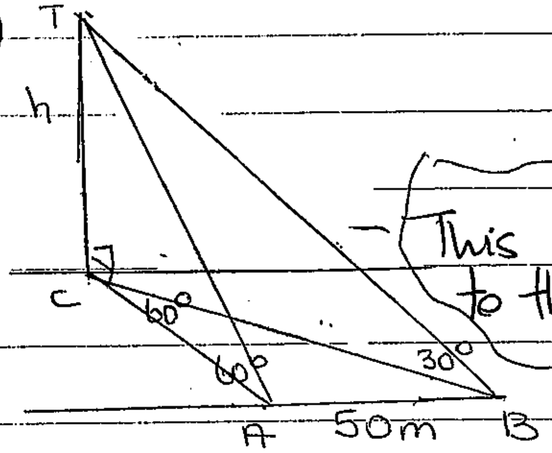
$\therefore \frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$

b) • divides by $\cos^2\theta$ to get $\sec^2\theta - \sin^2\theta$ (1 mark

• Proves the identity (1 mark)

This was well done. Students are reminded that SHOW means that you must show every line of working

c)



This was not well done. Most students need to thoroughly revise 3D trig

i)

$\triangle ACT \quad \tan 60° = \frac{h}{AC} \Rightarrow AC = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}$

$\triangle BCT \quad \tan 30° = \frac{h}{BC} \Rightarrow BC = \frac{h}{\tan 30°} = \sqrt{3}h$

$\triangle ABC \quad 50^2 = AC^2 + BC^2 - 2AC.BC\cos 60°$

$50^2 = \frac{h^2}{3} + 3h^2 - 2\frac{h}{\sqrt{3}}.\sqrt{3}h.\frac{1}{2}$

$50^2 = \frac{10h^2}{3} - h^2$

$50^2 = \frac{7h^2}{3}$

$h^2 = \frac{3\times 50^2}{7}, \quad h = \frac{50\sqrt{3}}{\sqrt{7}}m, \; h>0$

c)

i) • Finds AC & BC (1 mark)

• Substitute correctly using cos rule (1 mark)

• Finds exact value of h (1 mark)

Careless errors were expensive.

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