Killara HIGH SCHOOL

Student Number

2015

HALF YEARLY
EXAMINATION

Extension 1 Mathematics

20th March 2015

General Instructions
- Reading time – 5 minutes
- Working time 2 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 70

Section I - Pages 2 - 6

10 marks
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 7 - 10

60 marks
- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
Total	/70

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 25% of the Higher School Certificate Course Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

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1 The value of k, if $x+1$ is a factor of x^3-2x^2-kx+5 is:

(A) 4

(B) 2

(C) –2

(D) –6

2 The solution to $x(x-2)^2(3+x)^3<0$ is:

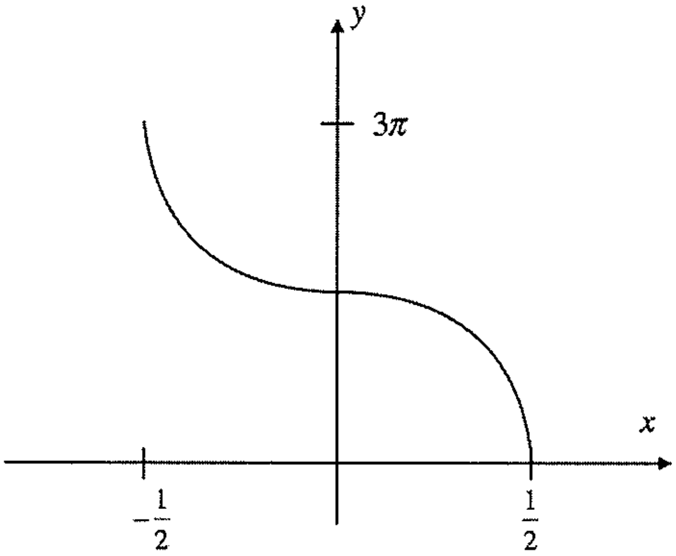
(A) $-3<x<0$

(B) $x<-3$ or $x>0$

(C) $x<-3$ or $x>2$

(D) $-3<x<2$

3



The graph sketched above represents:

(A) $y = 3\cos^{-1} 2x$

(B) $y = 3\sin^{-1}\frac{x}{2}$

(C) $y = \cos^{-1}\frac{x}{3}$

(D) $y = 3\cos^{-1}\frac{x}{2}$

4 What is the exact value of $\sin\left(2\cos^{-1}\frac{3}{4}\right)$

(A) $\frac{3\sqrt{7}}{8}$

(B) $\frac{15}{8}$

(C) $\frac{\sqrt{7}}{2}$

(D) $\frac{21}{8}$

5 What is the gradient of the tangent to the curve $y=\sin^{-1}2x$ at the point where $x=-\frac{1}{4}$

(A) $\frac{\sqrt{56}}{7}$

(B) $\frac{4\sqrt{3}}{3}$

(C) $\frac{2\sqrt{3}}{3}$

(D) $\frac{8\sqrt{3}}{3}$

6 Two roots of the equation $2x^3 + x^2 - px + 6 = 0$ are reciprocals of one another. The value of p is:

(A) 13

(B) 23

(C) -13

(D) -23

7 What is the exact value of $\sin 36° \cos 24° + \cos 36° \sin 24°$?

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $\sqrt{3}$

8 What can the expression $\frac{1 + \cos x + \cos 2x}{\sin x + \sin 2x}$ be simplified to:

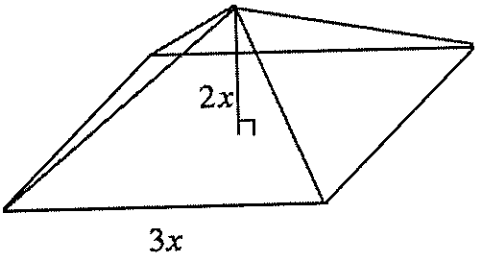
(A) $1 + \cot x + \cot 2x$

(B) $\cot x - \operatorname{cosec} x$

(C) $\cot x$

(D) $\operatorname{cosec} 3x + \cot 3x$

9 The diagram shows a square based pyramid. The ratio of the perpendicular height to the side of the base is 2:3. The angle between a side face and the base is closest to:



(A) 38°

(B) 43°

(C) 53°

(D) 59°

10 What is the exact value of $\int_0^{\frac{\pi}{8}} \sin^2 2x dx$

(A) $\frac{\pi}{16}$

(B) $\frac{1}{16}(\pi-2)$

(C) $\frac{1}{16}(\pi-8)$

(D) $\frac{1}{16}(\pi+8)$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a) Solve $\frac{2x+1}{x-1} \geq 3, \quad x \neq 1$ **2**

(b) Differentiate

(i) e^{6x+3} **1**

(ii) $e^{2x^2} + \log 3x$ **2**

(c) Simplify $\int \frac{6x+12}{x^2+4x} dx$ **2**

(d) Simplify $\int \sin x \cos^2 x \, dx$ **2**

(e) Simplify $\int 5^x \, dx$ **2**

(f) Evaluate $\int_0^3 \frac{1}{9+x^2} dx$ **2**

(g) If α, β, γ and δ are the roots of the equation $x^4 - 3x^3 + x - 7 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ **2**

End of Question 11

Question 12(15 marks) Use a SEPARATE writing booklet

(a) Using the substitution $u = 1 + t$ find: **3**

$$\int \frac{t}{\sqrt{1+t}} dt$$

(b) Using the substitution $u = 2 - x$ evaluate: **3**

$$\int_1^2 x\sqrt{2-x}\, dx$$

(c) Show that $\frac{x+7}{x-1}$ can be written as $1 + \frac{8}{x-1}$ and hence show that **3**

$$\int_2^{e+1} \frac{x+7}{x-1} dx = e + 7$$

(d) Using the result $\lim_{x \to 0} \frac{\sin x}{x} = 1$, find the value of $\lim_{x \to 0} \frac{1-\cos 2x}{x^2}$ **3**

(e) The function $h(x)$ is given by

$$h(x) = \sin^{-1} x + \cos^{-1} x \ , \ 0 \le x \le 1$$

i.) Find $h'(x)$ **1**

ii.) Sketch the graph of $y = h(x)$ **2**

End of Question 12

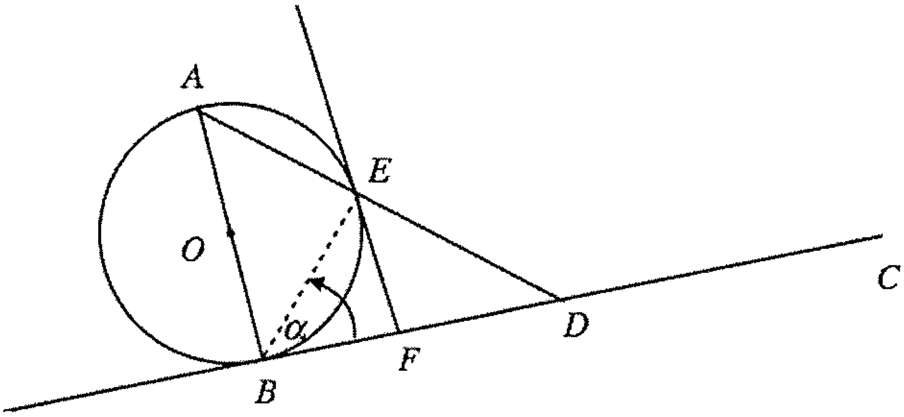
Question 13(15 marks) Use a SEPARATE writing booklet

(a) Find the volume of the solid generated when the area bounded by the curve $y = 1 + \sin x$, the x axis and the ordinates $x = 0, x = 2\pi$, is rotated about the x axis. **4**

(b) Evaluate $\int_0^{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{4-2x^2}} dx$ **3**

(c) Express $\sqrt{3}\sin x - \cos x$ in the form $r\sin(x-\alpha)$ and hence or otherwise solve $\sqrt{3}\sin x - \cos x = 1$ for $0 \le x \le 2\pi$ **4**

(d)



In the diagram, AB is a diameter of the circle, centre O, and BC is a tangent to the circle at B. The line AED intersects the circle at E and BC at D. The tangent to the circle at E intersects BC at F. Let $\angle EBF = \alpha$ radians.

Copy the diagram into your writing booklet.

(i) Prove $\angle FED = \frac{\pi}{2} - \alpha$ **2**

(ii) Prove that $BF = FD$ **2**

End of Question 13

Question 14(15 marks) Use a SEPARATE writing booklet

(a) (i) Sketch the graphs of $y = 2\sin x$ and $y = x$ on the same set of axes. Show that $x - 2\sin x = 0$ has a root near $x = 2$. **2**

(ii) Using one application of Newton's method, find a better approximation, correct to 2 decimal places. **2**

(b) P $(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.

i.) Show that the equation of the normal to the parabola at the point *P* is: **2**

$$x + py = 2ap + ap^3$$

ii.) Find the coordinates of the point *Q* where the normal at *P* meets the *y*-axis. **1**

iii.) Show that the point *R* which divides *PQ* **externally** in the ratio 2: 1 has coordinates $\left(-2ap,\, 4a + ap^2\right)$ **2**

iv.) Find the Cartesian equation of the locus of *R*. Show that the locus is a parabola and write down the coordinates of its vertex and focus. **3**

(c) The straight lines $3x - 5y - 1 = 0$, $bx - 2y + 2 = 0$ are inclined to each other at an angle of $45°$. Find the possible values of b. **3**

End of Question 14

End of Assessment

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Killara
HIGH SCHOOL

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Student Number

Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct ↓

A ☒ B ☒ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

YR. 12. MATHEMATICS EXTENSION 1.
MID COURSE ASSESSMENT 2015.
SOLUTIONS / MARKING SCHEME

SECTION 1.
QUESTIONS 1 – 10
SUMMARY.

1. ~~A~~C 2. A 3. A. 4. A. 5. B — 1 mark each
6. A. 7. B. 8. C 9. C 10. B. — = 10 marks.

SOLUTIONS.

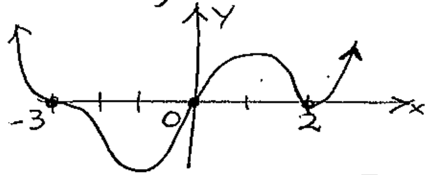
1. $(x+1)$ is a factor then $P(-1)=0$

$P(-1) = (-1)^3 - 2(-1)^2 - k \times -1 + 5$

$\therefore 0 = -1 - 2 + k + 5$

$-2 = k$ Ⓒ

2. $x(x-2)^2(3+x)^3 < 0$



$\therefore -3 < x < 0$ Ⓐ

3. $-1 < 2x < 1$; $0 \leq y \leq \pi$

$-\frac{1}{2} < x < \frac{1}{2}$; $0 \leq 3y \leq 3\pi$.

$\cos^{-1} x$

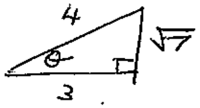
$\therefore y = 3\cos^{-1} 2x$ Ⓐ

4. $\sin\left(2\cos^{-1}\frac{3}{4}\right)$

$\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2 \times \frac{\sqrt{7}}{4} \times \frac{3}{4} = \frac{3\sqrt{7}}{8}$ Ⓐ



5. $y = \sin^{-1} 2x$.

$y' = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$

$= \frac{2}{\sqrt{1-4x^2}}$

$f'(-\tfrac{1}{4}) = \frac{2}{\sqrt{1 - 4 \times \frac{1}{16}}}$

$= \frac{2}{\sqrt{3/4}}$

$= 2 \times \frac{2}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{4\sqrt{3}}{3}$ Ⓑ

6. $\alpha, \beta, \frac{1}{\beta}$; $2x^3 + x^2 - px + 6 = 0$

$\therefore$ product $= \alpha$

$\therefore \frac{-d}{a} = \frac{-6}{2} = -3$

$\therefore \alpha = -3$

subst. $\alpha = -3$

$2(-3)^3 + (-3)^2 - p \times -3 + 6 = 0$

$-54 + 9 + 3p + 6 = 0$

$3p = 39$

$p = 13$ Ⓐ

7. $\sin 36^\circ \cos 24^\circ + \cos 36^\circ \sin 24^\circ$

$= \sin(36 + 24)$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$ Ⓑ

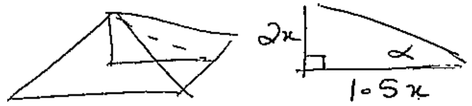
8. $\frac{1 + \cos x + \cos 2x}{\sin x + \sin 2x}$

$= \frac{1 + \cos x + 2\cos^2 x - 1}{\sin x + 2\sin x \cos x}$

$= \frac{\cos x\,\cancel{(2\cos x + 1)}}{\sin x\,\cancel{(2\cos x + 1)}}$

$= \cot x$. Ⓒ

9.



$\therefore \tan\alpha = \frac{2x}{1.5x}$

$\tan\alpha = \frac{4}{3}$

$\alpha = 53^\circ$ (nearest °) (C

10. $\int_0^{\pi/8} \sin^2 2x \cdot dx$.

using $\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$2\sin^2 2\theta = 1 - \cos 4\theta$

$\sin^2 2\theta = \frac{1}{2}(1 - \cos 4\theta$

expression becomes:

$= \frac{1}{2}\int_0^{\pi/8} 1 - \cos 4x \cdot dx$

$= \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/8}$

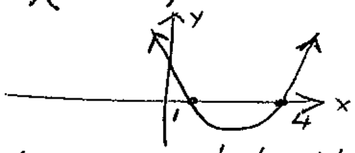
$= \frac{1}{2}\left[\frac{\pi}{8} - \frac{\sin \pi/2}{4} - 0 -\right.$

$= \frac{1}{2}\left[\frac{\pi}{8} - \frac{1}{4}\right]$

$= \frac{\pi}{16} - \frac{1}{8}$

$= \frac{1}{16}(\pi - 2)$ Ⓑ

QUESTION 11

a) $\frac{2x+1}{x-1} \geqslant 3 \quad , x \neq 1$ $\times(x-1)^2$ $(x-1)(2x+1) \geqslant 3(x-1)^2$ $2x^2-x-1 \geqslant 3x^2-6x+3$ $0 \geqslant x^2-5x+4$ $(x-4)(x-1) \leqslant 0$  $1 \leqslant x \leqslant 4$ but $x \neq 1$ ∴ $1 < x \leqslant 4$	This question could be done using 2 cases – award 1 mark for 1 case correc and then award 1 mark for correct answer 1 for correct inequality Some had difficulties in solving the quadratic inequality $1<x, x \leq 4$ not acceptable 1 for correct answer
b) i) $\frac{d}{dx} e^{6x+3} = 6e^{6x+3}$ ii) $\frac{d}{dx} e^{2x^2} + \log 3x = \overset{4}{\textcircled{2}}xe^{2x^2} + \frac{3}{3x}$ $= \overset{4}{\cancel{2}}xe^{2x^2} + \frac{1}{x}$	1 mark correct answer – some did not differentiate log 3x properly. 1 mark for $\overset{4}{\cancel{2}}xe^{2x^2}$ 1 mark for $\frac{1}{x}$
c) $\int \frac{6x+12}{x^2+4x} \cdot dx = \int \frac{3(2x+4)}{x^2+4x} \cdot dx$ $= 3\ln(x^2+4x) + c$ ↑ Some put $\frac{1}{3}$ instead of 3	1 mark for $\ln(x^2+4x)$ 1 mark awarded for $3\ln(x^2+4x)$ (no penalty for omitting 'c')
d) $\int \sin x . \cos^2 x . dx$ think: $\frac{d}{dx} \cos^3 x = 3\cos^2 x . -\sin x$ $\therefore \int \sin x . \cos^2 x . dx = \frac{-1}{3} \cos^3 x + C$	1 mark for $\cos^3 x$ 1 mark for correct answer,

QUESTION 11 continued.

e) $\int 5^x . dx$. using $\frac{d}{dx} 5^x = \ln 5 . 5^x$ $\therefore \int 5^x . dx = \frac{1}{\ln 5} . 5^x + C$ ↖ many did not give the correct answer	1 mark for 5^x 1 marks for correct answer (no penalty for no 'c')
f) $\int_0^3 \frac{1}{9-x^2} dx = \left[\tan^{-1}\frac{x}{3}\right]_0^3$ wrong deleted $= \tan^{-1}\frac{3}{3} - \tan^{-1}\frac{0}{3}$ $= \pi/4 + C$	1 mark for correct integral 1 mark for $\pi/4$. (no penalty for no 'c')
g) $x^4 - 3x^3 + x - 7 = 0$ $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ $= \frac{\beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\gamma}{\alpha\beta\gamma\delta}$ $= \frac{-d/a}{e/a}$ $= \frac{-1/1}{-7/1}$ $= \frac{1}{7}$	1 mark for correct algebraic statement 1 mark for correct answer

QUESTION 12

⑤

a) $u = 1+t$

$\int \frac{t}{\sqrt{1+t}}\, dt.$

$u = 1+t$

$\frac{du}{dt} = 1$

$du = dt$

Expression becomes

$\int \frac{u-1}{\sqrt{u}}\, du$

1 for correct. substituted expression.

$= \int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\, du$

$= \int u^{\frac{1}{2}} - u^{-\frac{1}{2}}\, du$

$= \frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + c$ many student's left their solution in terms of u

1 for correct integration

$= \frac{2}{3}(1+t)^{\frac{3}{2}} - 2\sqrt{1+t} + c$

1 for correct expression in 't'

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b) $\int_1^2 x\sqrt{2-x}\,.dx$

$u = 2-x,$

$\frac{du}{dx} = -1$

$dx = -du$

also $x=1, u=1$

$x=2, u=0$

Expression becomes

$= -\int_1^0 (2-u)\sqrt{u}\,.du$

1 for correct expression in terms of 'u'

$= -\int_1^0 2u^{\frac{1}{2}} - u^{3/2}\,.du$

$= -\left[\frac{4}{3}u^{\frac{3}{2}} - \frac{2u^{\frac{5}{2}}}{5}\right]_1^0$ many got bounds around the wrong way

1 for correct integration

$= -\left(0 - \left(\frac{4}{3} - \frac{2}{5}\right)\right)$

$= - -\frac{14}{15}$

$= \frac{14}{15}$

1 for correct answer/value.

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12 c) $\frac{x+7}{x-1} = \frac{x-1+8}{x-1}$

$= \frac{x-1}{x-1} + \frac{8}{x-1}$

$= 1 + \frac{8}{x-1}$

1 for showing

$\therefore \int_2^{e+1} \frac{x+7}{x-1}\,.dx = \int_2^{e+1} 1\,.dx + \int_2^{e+1} \frac{8}{x-1}\,.dx$

$= \left[x\right]_2^{e+1} + \left[8\ln(x-1)\right]_2^{e+1}$

1 mark for correct primitive

$= (e+1-2) + (8\ln(e) - 8\ln(1))$

$= e-1+8$

$= e+7$

$= RHS$

1 mark for correct substitution

12d) $\lim_{x\to0} \frac{1-\cos 2x}{x^2} = \lim_{x\to0} \frac{2\sin^2 x}{x^2}$

$\left[\begin{array}{l}\text{Since}\\ \cos 2x = 1-2\sin^2 x\\ \therefore 2\sin^2 x = 1-\cos 2x\end{array}\right]$

1 mark for trig substitution

$= 2\lim_{x\to0} \frac{\sin x}{x} \cdot \frac{\sin x}{x}$

1 mark for separating into standard limit

$= 2 \times 1$

$= 2$

1 mark correct evaluation

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12 e) $h(x) = \sin^{-1}(x) + \cos^{-1}(x)$

i) $\therefore h'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$

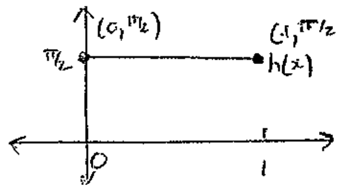
$= 0$

1 mark correct evalution of $h'(x)$

ii) Since $h'(x) = 0 \rightarrow h(x)$ is a horizontal line

$h(0) = \pi/2$, $h(1) = \pi/2$

1 mark working leading to conclusion that it a horizontal line



1 mark correct graph with end points marked

$y = 1 + \sin x$

$y^2 = 1 + 2\sin x + \sin^2 x$

$y^2 = 1 + 2\sin x + \frac{1}{2} - \frac{1}{2}\cos 2x$

$= \frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x$

$\therefore V = \pi \int_0^{2\pi} y^2 . dx$

$= \pi \int_0^{2\pi} \frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x \ . dx$

$= \pi \left[\frac{3}{2}x - 2\cos x + \frac{1}{4}\sin 2x \right]_0^{2\pi}$

$= \pi\left([3\pi - 2 + 0] - [0 - 2 + 0] \right)$

$= \pi(3\pi)$

$= 3\pi^2$

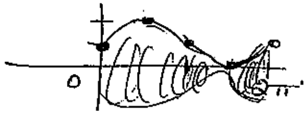
$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

QUESTION 13.

a) $y = 1 + \sin x \quad 0 \le x \le 2\pi$

$V = \pi \int y^2 . dx$



$= \pi \int_0^{2\pi} (1 + \sin x)^2 dx$

$= \pi \int_0^{2\pi} 1 + 2\sin x + \sin^2 x . dx$

1 for correct expression /integr

$= \pi \int_0^{2\pi} 1 + 2\sin x + \frac{1}{2}(1 - \cos 2x) dx.$

1 for correct substitution for $\sin^2 x$

$= \pi \left[x - 2\cos x + \frac{1}{2}\left(x - \frac{\sin 2x}{2} \right) \right]_0^{2\pi}$

1 for correct integration.

$= \pi \left[2\pi - 2\cos 2\pi + \frac{1}{2}\left[2\pi - \frac{\sin 2 \times 2\pi}{2} \right] - \left[0 - 2\cos 0 + \frac{1}{2}\left(0 - \frac{\sin 0}{2} \right) \right] \right]$

$= \pi \left[2\pi - 2 + \pi + 0 - 0 + 2 + 0 \right]$

$= \pi \times 3\pi$

$= 3\pi^2 \text{ units}^3$

1 for correct answe

b) $\int_0^{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{4 - 2x^2}} dx = \int_0^{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2(2 - x^2)}} dx$

1 for working

$= \int_0^{\sqrt{2}} \frac{1}{\sqrt{2 - x^2}} dx$

$= \left[\sin^{-1} \frac{x}{\sqrt{2}} \right]_0^{\sqrt{2}}$

1 for correct integration express

$= \sin^{-1} \frac{\sqrt{2}}{\sqrt{2}} - \sin^{-1} \frac{0}{\sqrt{2}}$

$= \frac{\pi}{2}$

1 for correct answ

QUESTION 13 continued.

d) $\sqrt{3}\sin x - \cos x \rightarrow r\sin(x-\alpha)$

$r = \sqrt{(\sqrt{3})^2+(-1)^2} = 2 \qquad \tan\alpha = \frac{1}{\sqrt{3}}$

$\therefore \alpha = \pi/6$

exp. becomes $2\sin(x-\pi/6)$

2 for expression

$\therefore 2\sin\left(x-\frac{\pi}{6}\right) = 1 \qquad 0 \le x \le 2\pi$

$\therefore -\pi/6 \le x-\pi/6 \le 11\pi/6$

$\sin(x-\pi/6) = \frac{1}{2}$

$\therefore x-\frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}.$

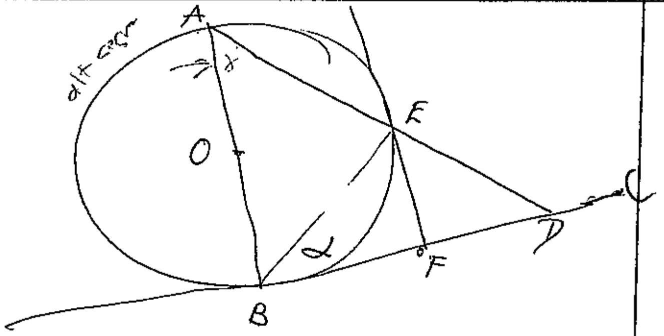
1 for $x-\frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore x = \frac{2\pi}{6}, \frac{6\pi}{6}$

$= \frac{\pi}{3}, \pi \qquad 0 \le x \le 2\pi$

1 for correct answers

d)



i) Aim: To prove $F\hat{E}D = \pi/2 - \alpha$

Proof: $A\hat{E}B = \frac{\pi}{2}$ (angle in a semi-circle)

$\therefore B\hat{E}D = \pi/2$ (supplementary angles / straight line)

and $B\hat{E}F = \alpha$ (BF = EF, tangents from external point are equal)

$\therefore F\hat{E}D = \frac{\pi}{2} - \alpha$

1 mark for 1 correct statement with reason.

②

~~1 for correctly~~ showing result.

ii) Now $E\hat{D}F = \pi - \frac{\pi}{2} - \alpha$ (angle sum of $\triangle$)

$= \frac{\pi}{2} - \alpha$

So $F\hat{E}D = E\hat{D}F$ hence FD = EF

②

1 mark for showing $E\hat{D}F =$

QUESTION 13 continued.

and BF = EF (from part (i))

so BF = FD (both equal EF)

1 mark for correct conclusion i.e. with reason.

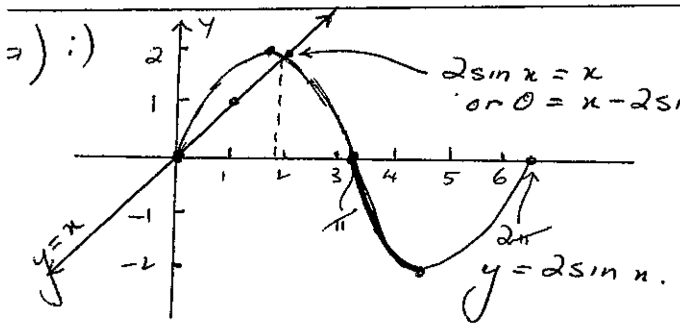
Most students did well

only a few did not memorize the formula correctly

(10)

QUESTION 14

a) i)



$2\sin x = x$

or $0 = x - 2\sin x$.

1 for both graphs

1 for suitable siz scale, labels, and showing root near $x=2$

ii) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$x_2 = 2 - \dfrac{f(2)}{f'(2)}$

$= 2 - \dfrac{(2-2\sin 2)}{(1-2\cos 2)}$

$\doteqdot 1.90$ (2 d.p).

1 for correct expression for x_2

1 for correct value.

b) $P(2ap, ap^2)$ $x^2 = 4ay$.

i) $x^2 = 4ay$

$y = \dfrac{x^2}{4a}$

$y' = \dfrac{2x}{4a} = \dfrac{x}{2a}$

at $x = 2ap$, $y' = \dfrac{2ap}{2a} = p$

$\therefore$ normal, $m = \dfrac{-1}{p}$

equation, $y - ap^2 = \dfrac{-1}{p}(x-2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3$ QED.

1 for derivative, gradient at P,

1 for correctly ~~equation~~ showing

QUESTION 14 continued....

most students di well in b (i) & (iii)

b) cont...

ii) meets y axis at Q, $x=0$

$0 + py = 2ap + ap^3$

$y = 2a + ap^2$

$\therefore Q = (0, 2a+ap^2)$

1 for correct Q coordinates.

iii) $P(2ap, ap^2)$ $Q(0, 2a+ap^2)$

2:1

$R = \left(\dfrac{2\times 0 - 1\times 2ap}{2-1}, \dfrac{2(2a+ap^2) - ap^2}{2-1}\right)$

$= \left(\dfrac{-2ap}{1}, \dfrac{4a+ap^2}{1}\right)$

$= (-2ap, 4a+ap^2)$ Q.E.D.

1 for correct expression.

1 for correctly showing result.

iv) $x = -2ap$ $y = 4a + ap^2$

$p = \dfrac{-x}{2a}$

$\therefore y = 4a + a\left(\dfrac{-x}{2a}\right)^2$

$= 4a + \dfrac{ax^2}{4a^2}$

$y = \dfrac{x^2}{4a} + 4a$, which is a parabola.

$\therefore 4ay = x^2 + 16a^2$

$x^2 = 4ay - 16a^2$

$x^2 = 4a(y-4a)$

$\therefore V = (0, 4a)$ Focus $= (0, 5a)$

a few students did n understand that lous i the Cartesian equati (in term of x & y)

1 for locus.

1 for vertex

1 for focus.

QUESTION 14 continued.

c) $3x - 5y - 1 = 0$, $bx - 2y + 2 = 0$

$m_1 = \frac{3}{5}$ $\qquad m_2 = \frac{b}{2}$

$\tan 45° = 1$

$\therefore \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = 1$

$$\left| \frac{\frac{3}{5} - \frac{b}{2}}{1 + \frac{3}{5} \cdot \frac{b}{2}} \right| = 1$$

$$\frac{\frac{6-5b}{10}}{\frac{10+3b}{10}} = \pm 1$$

$$\frac{6-5b}{10+3b} = 1 \text{ or } \frac{6-5b}{10+3b} = -1$$

$6 - 5b = 10 + 3b$ or $6 - 5b = -(10 + 3b)$

$-4 = 8b$ $\qquad 16 = 2b$

$b = \frac{-1}{2}$ $\qquad b = 8.$

Most students did well, ~~A~~ few did not memorize formula correctly

1 for correct expression

1 for $b = -\frac{1}{2}$

1 for $b = 8$.