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Student Number

2016										
Mathematics Extension 1										
HSC Course Assessment Task 3										
WEEK 7A – Wed 8th June 2016										
General Instructions • Reading time – 5 minutes • Working time 60 minutes • Write using blue or black pen Black pen is preferred • Approved calculators may be used • In Questions 6 and 7 show relevant mathematical reasoning and/or calculations • Answer each question in a separate writing booklet • This paper must not be removed from the examination room • A reference sheet is provided at the back of this paper • Diagrams are NOT to scale	Total Marks – 35 Section I - Pages 2 - 3 5 marks • Attempt Questions 1-5 • Allow about 8 minutes for this section Section II - Pages 4 - 6 30 marks • Attempt Questions 6 and 7 • Allow about 50 minutes for this section									
	<table border="1"> <tr> <td>Multiple Choice</td> <td>/5</td> </tr> <tr> <td>Question 6</td> <td>/15</td> </tr> <tr> <td>Question 7</td> <td>/15</td> </tr> <tr> <td>Total</td> <td>/35</td> </tr> </table>	Multiple Choice	/5	Question 6	/15	Question 7	/15	Total	/35	
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Question 7	/15									
Total	/35									

Article III. THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 20% of the Preliminary / HSC Course Assessment

Section I

5 marks

Attempt Questions 1-5

Allow about 8 minutes for this section

Use the multiple-choice answer sheet for questions 1-5 (Detach from paper)

1	Evaluate $\int_0^{3\sqrt{3}} \frac{3}{x^2+9} dx$. (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$
2	$\cos^{-1}x - \cos^{-1}(-x) =$ (A) π (B) $2\cos^{-1}x - \pi$ (C) $2\cos^{-1}x$ (D) 0
3	The derivative of $\sin^{-1}3x$ is: (A) $\frac{1}{\sqrt{9-x^2}}$ (B) $\frac{1}{\sqrt{1-9x^2}}$ (C) $\frac{3}{\sqrt{1-9x^2}}$ (D) $\frac{3}{\sqrt{9-x^2}}$

4	Seven cubes are identical in shape. Four are red and three are black, are arranged in a row. How many different arrangements are possible? (A) 210 (B) 35 (C) 840 (D) 144
5	From seven teachers and five students, a committee of seven is formed. How many different committees can be selected if teachers and students are both represented and the teachers are a majority? (A) 345 (B) 420 (C) 595 (D) 560

Section II

30 marks

Attempt Questions 6 and 7

Allow about 50 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 6 and 7, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (15 marks) Use a SEPARATE writing booklet

- (a) There is a row of five coloured lights, each with its own switch. When switched on, each light is equally likely to show green, red or amber.
- (i) If all the lights are switched on:
- α) How many different colour patterns are possible? 1
 - β) What is the probability that the first three lights from the left are green, but not the fourth or fifth? 1
 - γ) What is the probability that the first three lights from the left are the same colour, but not the fourth or fifth? 1
 - δ) Exactly three of the lights are green? 2
- ~~(ii)~~ If all the lights are switched on five times, find as a decimal correct to three places, the probability that exactly three lights will be the same colour on two or three occasions. 2
- (b) Statistics show that, of motorists tested for drink-driving, 3% are found to be over the limit. Find, as decimals to three places, the probability that, in a group of thirty drivers tested:
- (i) None will be over the limit. 1
 - (ii) Exactly one will be over the limit. 1
 - (iii) At least two will be over the limit. 1

- (c) Asha, Bianca, Charlie, Diana and Edward are five friends going to the movies. They all sit on the same row.
- i) How many different seating arrangement are there? 1
 - ii) Charlie and Diana are a couple and have to sit next to each other. How many different arrangements are there? 1
 - iii) Asha and Edward have had a fight and want to sit at the ends as far away from each other as possible. How many arrangements are there now? Assume that Charlie and Diana don't need to sit next to each other. 1
 - iv) The five friends go out for dinner after the movies and sit at a circular table. How many different seating arrangements are there. Assume Asha and Edward are friends again. 1
 - v) Charlie and Diana are still inseparable and want to sit next to each other. How many different arrangements are there at the circular table.? 1

Question 7 (15 marks) Use a SEPARATE writing booklet

- a) (i) State the domain and range of the function given by: $y = \sin^{-1} 2x$. 2
- (ii) Sketch the graph of the function given by: $y = \sin^{-1}(2x)$. 1
- (iii) Find the slope of the curve at the point where it cuts the y -axis. 2
- b) The function $f(x) = \operatorname{cosec} x$ for $0 \leq x \leq \frac{\pi}{2}$, and is not defined anywhere else.
- (i) Show that $f^{-1}(x) = \sin^{-1} \frac{1}{x}$. 1
- (ii) Hence find $\frac{d}{dx} [f^{-1}(x)]$. 2
- c) Find $\frac{d}{dx} \left(\sin^{-1} \left(\frac{x-4}{4} \right) \right)$ and hence evaluate $\int_4^8 \frac{dx}{\sqrt{8x-x^2}}$. 2
- d) (i) Show that $\sin(\sin^{-1} x - \cos^{-1} x) = 2x^2 - 1$ for $-1 \leq x \leq 1$. 2
- (ii) Show that if $-1 \leq x \leq 1$ then $-1 \leq 2x^2 - 1 \leq 1$. 1
- (iii) Hence solve $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(5x - 4)$. 2

End of Paper

Q6 (a)

most students got full mark

i) $\alpha) 3^5 = 243$

$\beta) \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{243}$

$\gamma) 3 \times \frac{4}{243} = \frac{4}{81}$

GGGRR
GGGRA
GGGAR
GGGAA

$4 \times \left(\frac{1}{3}\right)^5$

or $\underline{1} \underline{1} \underline{1} \underline{2} \underline{2} \quad \frac{4}{5!} = \frac{4}{243}$

$\delta) {}^5C_3 \left(\frac{1}{3}\right)^3 \times \left(\frac{2}{3}\right)^2 = \frac{40}{243}$

(2) marks

students as difficulty in understanding the question.

ii) $P(3 \text{ same colours}) = 3 \times \frac{40}{243} = \frac{40}{81}$

$P = {}^5C_3 \left(\frac{40}{81}\right)^3 \cdot \left(\frac{41}{81}\right)^2 + {}^5C_2 \left(\frac{40}{81}\right)^2 \cdot \left(\frac{41}{81}\right)^3 = 0.625$

(2) marks

b) $P(D) = 0.03, P(\bar{D}) = 0.97$

i) $P(\bar{D} \text{ 's } 30) = (0.97)^{30} = 0.401$

ii) $30C_1 \times 0.03 \times (0.97)^{29} = 0.372$ ← quite a few made mistake in this part.

iii) $P(\text{at least } 2) = 1 - 0.401 - 0.372 = 0.227$

almost every one got this question.

c)

i) $5! = 120$

ii) $2 \times 4! = 48$

iii) $2 \times 3! = 12$

iv) $4! = 24$

v) $3! \times 2 = 12$

almost every one got full marks.

Start here

OR.

$$\sin(\sin^{-1}x - \cos^{-1}x) = 2x^2 - 1$$

from (1).

$$\text{Let } \theta = \sin^{-1}x - \cos^{-1}x.$$

$$-1 \leq \sin \theta \leq 1.$$

$$\text{Hence } -1 \leq 2x^2 - 1 \leq 1.$$

$$(14) \sin^{-1}x - \cos^{-1}x = \sin^{-1}(5x - 4).$$

$$\text{Hence } \sin(\sin^{-1}x - \cos^{-1}x) = 5x - 4.$$

Using (1),

$$2x^2 - 1 = 5x - 4. \checkmark$$

$$2x^2 - 5x + 3 = 0.$$

$$(2x - 3)(x - 1) = 0$$

$$x = 3/2 \quad x = 1.$$

$$-1 \leq x \leq 1. \text{ Hence reject } x = 3/2$$

$$\therefore x = 1. \checkmark$$

(Very badly done)

1 mark: With correct reasoning establishes

1 mark: Solves $x = 1$ after reject $x = 3/2$ with

Correct reasoning.

Well done.

Additional writing space on back page.

Start here

Question 7.

a) $y = \sin^{-1} 2x$

(i) $D: -1 \leq 2x \leq 1$

$$\therefore -\frac{1}{2} \leq x \leq \frac{1}{2}$$

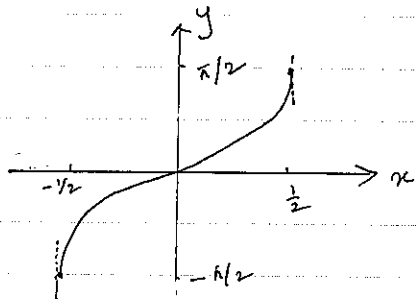
$$R: \frac{\pi}{2} \leq y \leq \frac{3\pi}{2}$$

1 mark for domain

1 mark for correct range

well done.

(ii)



1 mark: Correct graph

Note: Vertical tangent at $x = \pm \frac{1}{2}$; and No zero gradient at $x = 0$.

(iii) $x = 0$

$$y = \sin^{-1} 2x$$

$$\frac{dy}{dx} = \frac{1 \times 2}{\sqrt{1-4x^2}} \quad (1 \text{ mark})$$

$$x=0, \quad \frac{dy}{dx} = \frac{2}{1} = 2. \quad (1 \text{ mark})$$

1 mark: applies chain rule

Done well

1 mark: Correctly calculate gradient from $\frac{dy}{dx}$

b) $f(x) = \operatorname{cosec} x$

let $y = \operatorname{cosec} x$

$$x = \operatorname{cosec}^{-1} y \text{ is the inverse}$$

$$= \frac{1}{\sin y}$$

$$\sin y = \frac{1}{x}$$

$$\therefore y = \sin^{-1} \left(\frac{1}{x} \right)$$

1 mark: Correct proof.

(Must show every line of working).

Many students struggled with the $f(x)$. Maybe it is easier to make it to y .

(ii) $\frac{d}{dx} \left(\sin^{-1} \frac{1}{x} \right)$

$$= \frac{1}{\sqrt{1-\frac{1}{x^2}}} \times \frac{-1}{x^2}$$

$$= \frac{-1}{x^2} \times x$$

$$= -\frac{1}{x\sqrt{x^2-1}}$$

2 marks: Correct answer from correct working.

1 mark: Significant progress towards the solution.

Must leave the answer in fractional form with positive indices.

c) $\frac{d}{dx} \sin^{-1} \left(\frac{x-4}{4} \right)$

$$u = \frac{x-4}{4}$$

$$u' = \frac{1}{4}$$

$$= \frac{1}{\sqrt{1-\left(\frac{x-4}{4}\right)^2}} \times \frac{1}{4} \quad \checkmark$$

$$= \frac{1 \times 4 \times \frac{1}{4}}{\sqrt{16-(x-4)^2}}$$

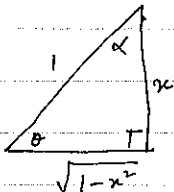
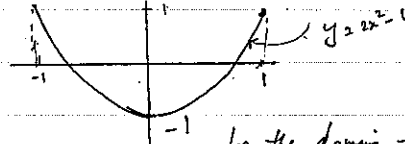
$$= \frac{1}{\sqrt{16-(x-4)^2}}$$

$$= \frac{1}{\sqrt{16-16+8x-x^2}}$$

$$= \frac{1}{\sqrt{8x-x^2}} \quad \checkmark$$

1 mark: must show the multiplication by u' 1 mark: Correct simplification to get $\frac{1}{\sqrt{8x-x^2}}$ Many students went from $\frac{1}{\sqrt{16-(x-4)^2}}$ and then simplified to the answer.

Additional writing space on back page.

$\int_4^8 \frac{dx}{\sqrt{8x-x^2}}$ $= \left[\sin^{-1} \frac{x-4}{4} \right]_4^8$ $= \frac{\pi}{2} - 0$ $= \frac{\pi}{2} \quad \checkmark$	1 mark: Correct value from working. Well done
d) Let $\sin^{-1} x = \theta$ (i) $\cos^{-1} x = \alpha$ $\sin(\theta - \alpha)$ $= \sin\theta \cos\alpha - \cos\theta \sin\alpha \quad \checkmark$ $= x \times x - \sqrt{1-x^2} \sqrt{1-x^2}$ $= x^2 - (1-x^2)$ $= 2x^2 - 1$ 	1 mark: Uses $\sin(\theta - \alpha)$ formula. 1 mark: Correct proof. Many students got the 1st mark and could not represent it using a triangle
(ii) $-1 \leq x \leq 1$ $0 \leq x^2 \leq 1$ $0 \leq 2x^2 \leq 2$ $-1 \leq 2x^2 - 1 \leq 1$ OR. graph of $y = 2x^2 - 1$  in the domain $-1 \leq x \leq 1$, $-1 \leq 2x^2 - 1 \leq 1$. Tick this box if you have continued this answer in another writing booklet.	1 mark: Correct proof.



Killara
HIGH SCHOOL

Marking Scheme EXT1.

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Centre Number

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Student Number

Examination Writing Booklet

Examination

Date

Instructions

- Write the question number you are answering in this box. →

- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3).

⇒
this
booklet

of

number of
booklets for
this question

- Write your Centre Number and Student Number at the top of this page.
- Write using black or blue pen. (black pen is recommended.)
- You may ask for an extra writing booklet if you need more space.
- If you have not attempted the question(s), you must still hand in the writing booklet, with the words 'NOT ATTEMPTED' written clearly on the front cover.
- You may NOT take any Writing Booklets, used or unused, from the examination room.