Killara HIGH SCHOOL

Student Number

2016

HALF YEARLY EXAMINATION

Extension 1 Mathematics

General Instructions

- Reading time – 5 minutes
- Working tine - 2 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 70

Section I - Pages ****

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages ****

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
Total	/70

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 30% of the Higher School Certificate Course Assessment

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

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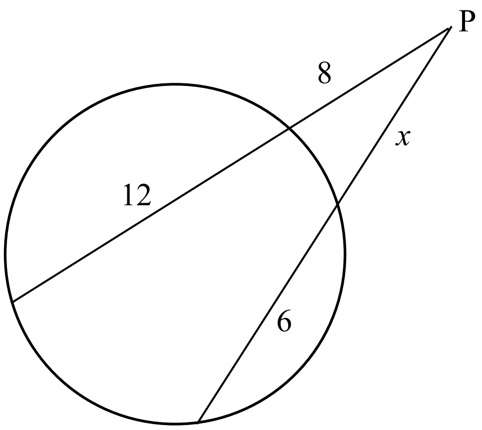
1 Two secants from point P intersect a circle as shown in the diagram.



NOT TO SCALE

What is the value of x ?

(A) 4

(B) 9

(C) 10

(D) 14

2 Which of the following is an expression for $cos^6x - sin^6x$?

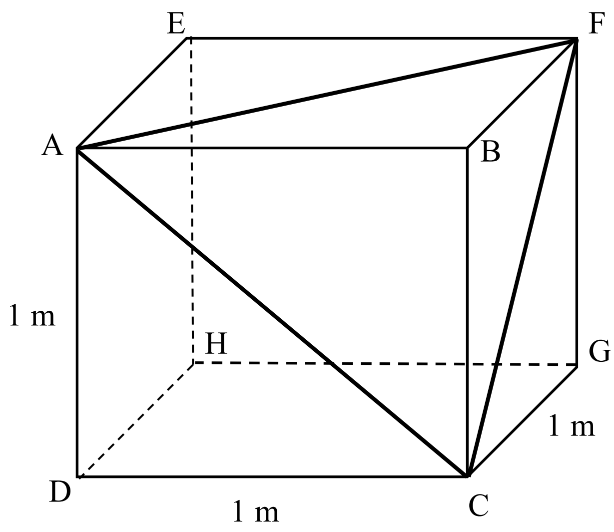
(A) $\sin 2x\ (cos^4x - cos^2x\, sin^2x + sin^4x)$

(B) $\sin 2x\ (cos^4x + cos^2x\, sin^2x + sin^4x)$

(C) $\cos 2x\ (cos^4x - cos^2x\, sin^2x + sin^4x)$

(D) $\cos 2x\ (cos^4x + cos^2x\, sin^2x + sin^4x)$

3



The length of each edge of the cube ABCDEFGH is 1 metre.

What is the exact area of Δ ACF ?

(A) $\frac{1}{2}$ m^2

(B) $\frac{1}{\sqrt{2}}$ m^2

(C) $\frac{\sqrt{3}}{2}$ m^2

(D) 1 m^2

4 Which of the following is a solution to the equation $2^{2x} = 5$?

(A) $\frac{5}{4}$

(B) $\frac{\ln 5}{2 \ln 2}$

(C) $\frac{2 \ln 5}{\ln 2}$

(D) $\frac{\ln 5}{2}$

5 After using the substitution $u = 9 - x^2$, the integral

$$\int_0^1 6x\sqrt{9 - x^2}\, dx$$

is rewritten as:

(A) $\int_0^1 -3\sqrt{u}\, du$

(B) $\int_0^1 -\frac{1}{3}\sqrt{u}\, du$

(C) $\int_9^8 -3\sqrt{u}\, du$

(D) $\int_9^8 -\frac{1}{3}\sqrt{u}\, du$

6 What is the correct solution to the inequality $\frac{x+1}{x+3} \leq -2$?

(A) $-3 \leq x \leq -\frac{7}{3}$

(B) $-3 < x \leq -\frac{7}{3}$

(C) $x \leq -\frac{7}{3}$

(D) $x \leq -\frac{7}{3}, x \neq -3$

7 What is the size of the acute angle between the lines $y = 2x - 5$ and $y = 4 - x$?

(A) $71^{o}34'$

(B) $18^{o}26'$

(C) $40^{o}36'$

(D) $33^{o}41'$

8 The interval between the 2 points A (-1, -3) and B (4, 12) is divided internally in the ratio 3:2 by the point C. The coordinates of point C are:

(A) (1, 3)

(B) $(-2\frac{2}{5}, -8\frac{2}{5})$

(C) (2, 6)

(D) $(2\frac{2}{5}, 8\frac{2}{5})$

9 The solutions to the equation $cos2\theta = sin\theta$ where $0 \leq \theta \leq \pi$ are :

(A) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

(B) $\theta = \frac{\pi}{12}, \frac{11\pi}{12}$

(C) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

(D) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

10 The correct expression of $sin2t + \sqrt{3}cos2t$ in the form $Rsin(2t + \alpha)$, where α is in radians is:

(A) $2\sin\left(2t + \frac{\pi}{3}\right)$

(B) $\sqrt{2}\sin\left(2t + \frac{\pi}{3}\right)$

(C) $2\sin\left(2t + \frac{\pi}{6}\right)$

(D) $\sqrt{2}\sin\left(2t + \frac{\pi}{6}\right)$

Section II

70 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a) Simplify $e^{\ln y}$ 1

(b) Find the range of the function $f(x) = \ln(x^2 + e^2)$. 2

(c) Differentiate $\frac{e^x}{x}$ 2

(d) Differentiate $\log_5 x^3$ with respect to x . 2

(e) A series is given as

$$S_n = sin^2x + 2sin^4x + 4sin^6x + \ldots\ldots\ldots\ldots \qquad \text{for } 0 < x < \pi$$

(i) Find the exact values of x for which the series has a limiting sum. 3

(ii) Show that the limiting sum of the series can be written in the form 2

$$\frac{sin^2x}{\cos 2x}$$

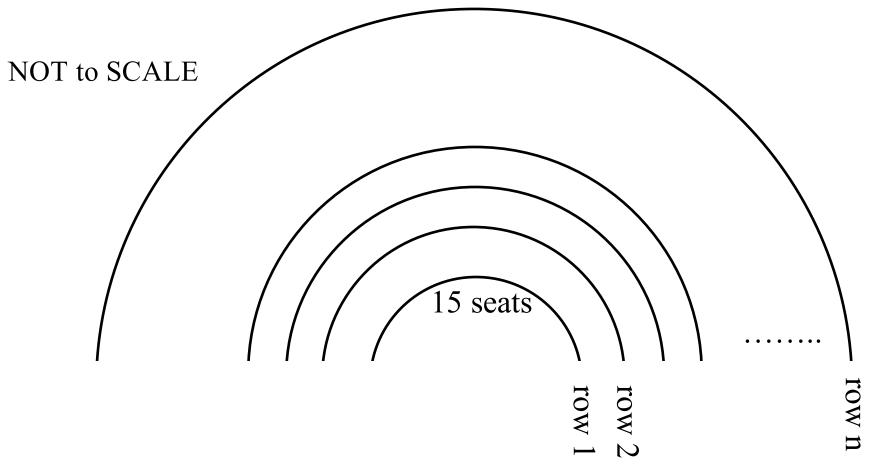
Question 11 continues on page 9

Question 11 (continued)

(f) Seats in a semi-circular shaped theatre are arranged in rows.

The first row has 15 seats in it. Each row after the first row has 4 more seats that the row directly in front of it.

There are a total of n rows of seats in the theatre.



(i) Find an expression for the total number of seats in the theatre. 1

(ii) Harriet notices that there is an average of 43 seats per row.

How many seats are there in the last row of the theatre? 2

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Using the substitution $u = lnx$, or otherwise, find $\int \frac{1}{x(lnx)^4} dx$, 3

(b) By using the substitution $x = sint$

(i) Show that $\int_0^{\frac{1}{2}} \sqrt{1-x^2}\, dx$ becomes $\int_0^{\frac{\pi}{6}} cos^2 t\, dt$ 2

(ii) Hence evaluate $\int_0^{\frac{1}{2}} \sqrt{1-x^2}\, dx$ 3

(c)

(i) Starting from $\cos 3\theta = \cos(\theta + 2\theta)$ show that 2

$$cos3\theta = 4cos^3\theta - 3cos\theta$$

(ii) Hence find $\int 12cos^3\theta - 9cos\theta \; d\theta$ 2

(d) Using the substitution $u = 1 - 3x$, find $\int x\sqrt{1-3x} \; dx$. You can assume $x \le \frac{1}{3}$. 3

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Consider the function $f(x) = \frac{2x^2}{9-x^2}$

(i) Write down the equations of the asymptotes of $f(x)$. 2

(ii) Show that $f(x)$ is even. 1

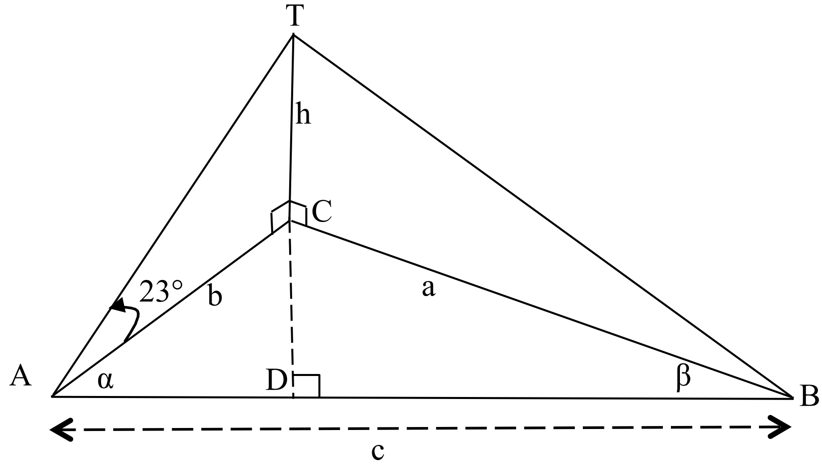
(iii) Find any stationary points and determine their nature. 2

(iv) Sketch a graph of $f(x)$, showing all important features. The graph must be at least one third of a page in size. 2

Question 13 continues on page 12

Question 13 (continued)

(b)



Triangle Δ ABC lies on flat ground. At point C, a tower rises to a height of h metres. Angle ∠CDB = 90°.

Let ∠CAD = α and ∠CBD = β.

(i) Show that $a\ sin\beta = b \sin\alpha$ 1

(ii) Show that $c\ = a\ cos\beta + b\ cos\alpha$ 1

(iii) Given that $c^2 = 4ab\cos\alpha\cos\beta$, show that $a = b$ 2

(iv) Given that Point D is due South of the base of the tower and that the bearing of the tower from point B is 320°, explain why ∠ACB = 80°. 1

(v) Given that at point A, the angle of elevation of the top of the tower, T, is 23° and that distance AB = 85 metres, calculate the height, h, of the tower. Answer correct to 1 decimal place. 3

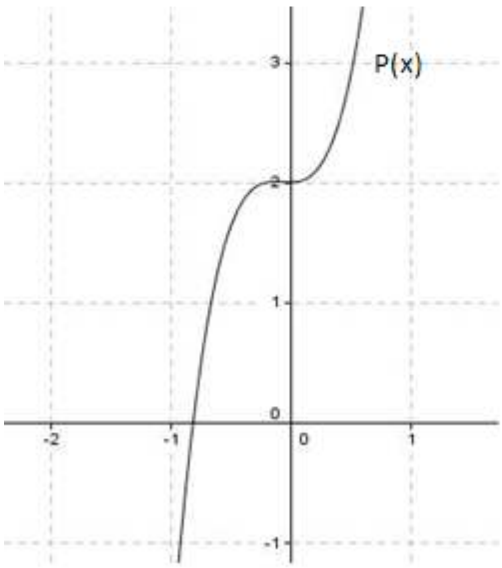
End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a)

(i) The polynomial $P(x) = 5x^3 + x^2 + 2$ has one real root in the interval $-1 < x < 1$. By halving the interval show that the root lies between -1 and -0.75. 2

(ii) Given the graph as shown below of the polynomial $P(x) = 5x^3 + x^2 + 2$, briefly justify why Newton's method with an initial approximation of $x = 0$, would be ineffective in improving the approximation of the root. 1



(b) The polynomial $P(x) = x^3 - 3x^2 + kx + 6$ has roots α, β, γ.

(i) Find the value of $\alpha + \beta + \gamma$ 1

(ii) Find the value of $\alpha\beta\gamma$ 1

(iii) It is known that two of the roots are equal in magnitude but opposite in sign. Find the third root and hence find the value of k. 2

Question 14 continues on page 14

Question 14 (continued)

(c) The two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are on the parabola $x^2 = 4ay$.

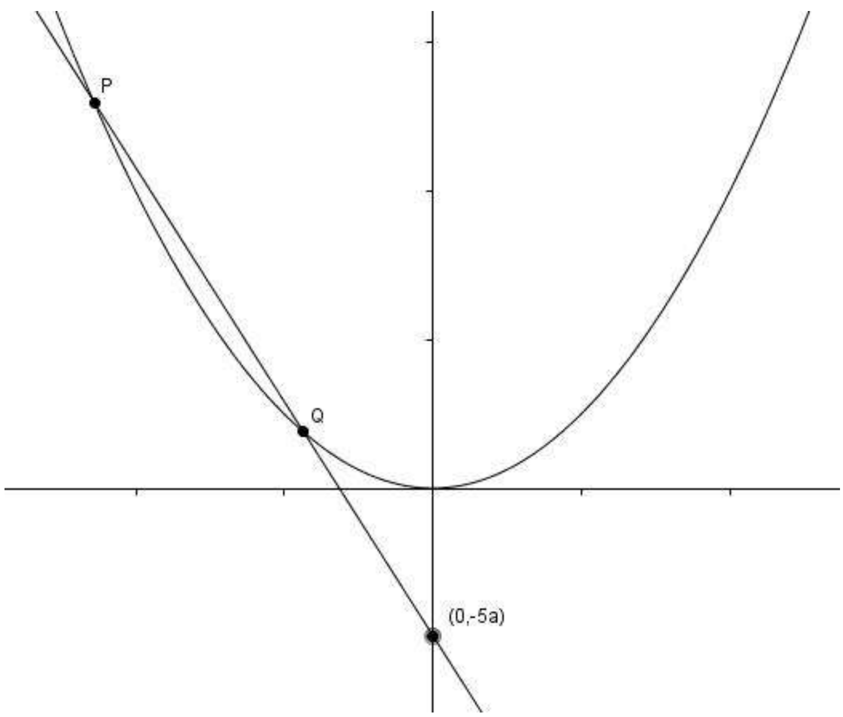
(i) The chord PQ varies in such a way as to always pass through the point $(0, -5a)$ as shown on the diagram below. The equation of the chord PQ is $y - \frac{1}{2}(p+q)x + apq = 0$ (Do NOT prove this). 1

Show that $pq = 5$.

(ii) The equation of the tangent to $x^2 = 4ay$ at an arbitrary point $(2at, at^2)$ on the parabola is $y = tx - at^2$. (Do NOT prove this). 3

Show that the tangents at the points P and Q meet at R, where R is the point $(a(p+q), apq)$.

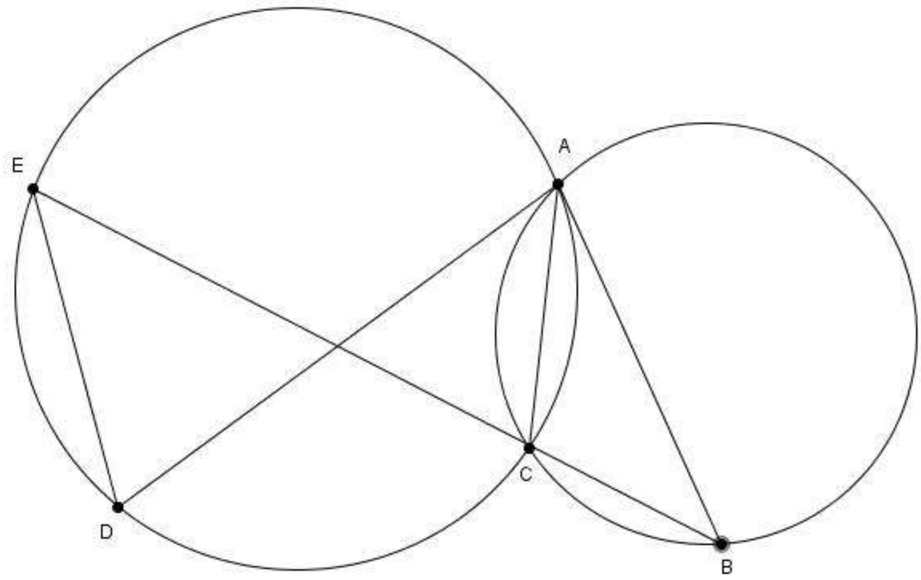
(iii) Hence find the equation of the locus of R. 1



Question 14 continues on page 15

Question 14 (continued)

(d) AD is a tangent to the circle with points A, B and C on its circumference as shown in the diagram below. Copy this diagram. Prove $AB \parallel DE$. 3



End of Question 14

End of Paper ☺

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Student Number

Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ☒ B ☒ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

Marking scheme.

Extension 1 - 2016
Half Yearly

Killara HIGH SCHOOL

A	N	S	W	E	R	S		☺

Student Number

Mathematics

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A ● B ~~●~~ C ○ D ○

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correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here →

	A	B	C	D
1.	~~●~~	○	●	○
2.	○	○	○	●
3.	○	○	●	○
4.	○	●	○	○
5.	○	○	●	○
6.	○	●	○	○
7.	●	○	○	○
8.	○	○	●	○
9.	○	○	○	●
10.	●	○	○	○

Mutiple Choice

1) $\frac{8}{20} = \frac{x}{x+6}$

$8(x+6) = 20x$

$8x + 48 = 20x$

$48 = 12x$

$x = 4$

(A)

2.) $\cos^6 x - \sin^6 x$

$= (\cos^2 x)^3 - (\sin^2 x)^3$

$= (\cos^2 x - \sin^2 x)(\cos^4 x + \cos^2 x \sin^2 x + \sin^4 x)$

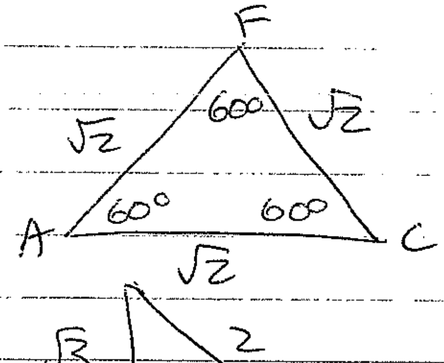
$= \cos 2x(\cos^4 x + \cos^2 x \sin^2 x + \sin^4 x)$

(D)

3.) By Pythagoras' Theorem,

$AC = CF = AF = \sqrt{2}$

$\triangle ACF$ is equilateral $\therefore \angle AFC = 60°$



$A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times \sin 60°$

$= \frac{1}{2} \times 2 \times \frac{\sqrt{3}}{2}$

$\therefore A = \frac{\sqrt{3}}{2} m^2$

(C)

4.)
$$2^{2x} = 5$$
$$\ln 2^{2x} = \ln 5$$
$$2x \ln 2 = \ln 5$$
$$2x = \frac{\ln 5}{\ln 2}$$
$$x = \frac{\ln 5}{2 \ln 2}$$
B

or substitute alternatives into original equation.

5.)
$$\int_0^1 6x\sqrt{9-x^2}\,dx$$
$$= \int_0^1 -3\sqrt{9-x^2} \times -2x\,dx$$
$$= \int_9^8 -3\sqrt{u}\,du$$

$u = 9-x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

when $x=0$,

$u = 9$

$x = 1$,

$u = 9-1$

$= 8$

C

Multiple Choice

Q6

$$(x+3)^{\cancel{2}} \times \frac{x+1}{\cancel{x+3}} \le -2 \times (x+3)^2$$

Note $x \neq -3$

$$(x+3)(x+1) \le -2(x+3)^2$$

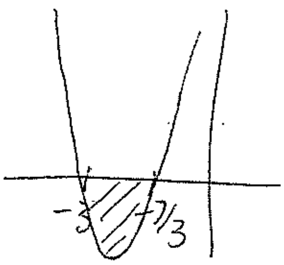
$$(x+3)(x+1) + 2(x+3)^2 \le 0$$

$$(x+3)\left[(x+1) + 2(x+3)\right] \le 0$$

$$(x+3)(x+1+2x+6) \le 0$$

$$(x+3)(3x+7) \le 0$$

$\therefore$ x-int's of parabola at $x=-3$ (impossible) and $x = \frac{-7}{3}$



$\therefore \quad 3 < x \le \frac{-7}{3}$ (B)

Q7

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad m_1 = 2,\ m_2 = -1$$

$$= \left|\frac{2 - -1}{1 + 2 \cdot -1}\right|$$

$$= \left|\frac{3}{-1}\right| = |-3| = 3$$

$\therefore \theta = \tan^{-1}(3)$

$\therefore \theta = 71°34'$ (A)

Q8 $A\ (-1, 3)$ $\quad B\ (4, 12)$ $\quad 3 : 2$

$x_1, y_1 \quad x_2, y_2 \quad m : n$

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$= \left(\frac{3\times 4 + 2\times -1}{3+2}, \frac{3\times 12 + 2\times -3}{3+5} \right)$$

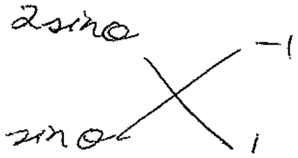
$$= \left(\frac{12-2}{5}, \frac{36-6}{5} \right) = \left(\frac{10}{5}, \frac{30}{5} \right) = (2, 6) \quad \text{C}$$

Q9 $\cos 2\theta = \sin\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

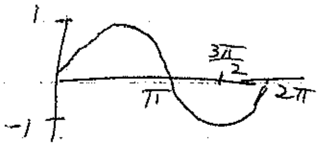
$1 - 2\sin^2\theta = \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

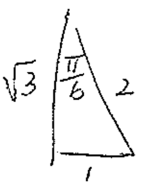


$\therefore (2\sin\theta - 1)(\sin\theta + 1) = 0$

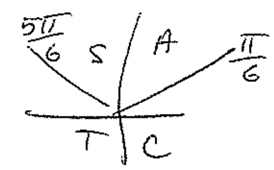
$\therefore \sin\theta = \frac{1}{2}, \ \sin\theta = -1$



$\sin\theta = -1$ is not a solution in domain $0 \le \theta \le \pi$.



$\sin\theta = \frac{1}{2}$



$\therefore$ solutions are

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ D

Q10

$$a\sin\theta + b\cos\theta = R\sin(2t+\alpha)$$

$$1\sin 2t + \sqrt{3}\cos 2t = R\sin(2t+\alpha)$$

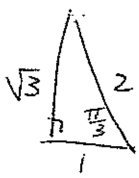
$$R = \sqrt{1^2 + (\sqrt{3})^2} = 2$$

$$\therefore\ 1\sin 2t + \sqrt{3}\cos 2t = 2\sin(2t+\alpha)$$

need α ...

fast way $\tan\alpha = \frac{b}{a} = \frac{\sqrt{3}}{1}$

$$\alpha = \tan^{-1}\sqrt{3}$$



$$\therefore\ \alpha = \frac{\pi}{3}$$

$$\therefore\ 1\sin 2t + \sqrt{3}\cos 2t = 2\sin\left(2t + \frac{\pi}{3}\right) \quad (A)$$

OR long way

$$1\sin 2t + \sqrt{3}\cos 2t$$

$$= 2\left(\frac{1}{2}\sin 2t + \frac{\sqrt{3}}{2}\cos 2t\right)$$

$$= 2\left(\sin 2t\,\frac{1}{2} + \cos 2t\,\frac{\sqrt{3}}{2}\right)$$

$$= 2(\sin 2t \cdot \cos\alpha + \cos 2t \cdot \sin\alpha)$$

$$= 2(\sin 2t + \alpha)$$

$$\cos\alpha = \frac{1}{2}, \quad \sin\alpha = \frac{\sqrt{3}}{2}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\therefore\ \alpha = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

$$= 2\left(\sin 2t + \frac{\pi}{3}\right) \quad (A)$$

Question 11

a) $e^{\ln y} = y$

Done poorly. Many students do not realise $e^{\dots}$ and $\ln(\dots)$ are opposite operations.

1 mark correct answer.

b.) $f(x) = \ln(x^2 + e^2)$

$x^2 \geq 0$

$\therefore x^2 + e^2 \geq e^2$

Many could not see this.

1 mark $x^2 + e^2 \geq e^2$

$f(x) \geq \ln e^2$

$\geq 2 \ln e$

≥ 2

$\therefore$ range is $f(x) \geq 2$

Simply $y \geq 2$ with no working received 1 mark.

1 mark correct range.

c.) $\frac{d}{dx} \frac{e^x}{x} = \frac{u}{v}$

$= \frac{xe^x - e^x}{x^2}$

Well done.

2 marks: Correct derivative.

1 mark:

d.) $\frac{d}{dx} \log_5 x^3$

$= \frac{d}{dx} \frac{\ln x^3}{\ln 5}$

1 mark change of base

$= \frac{d}{dx} \frac{3}{\ln 5} \ln x$

$= \frac{3}{\ln 5} \times \frac{1}{x}$

$= \frac{3}{x \ln 5}$

Those who realised $\frac{3}{\ln 5}$ is a constant, easily got to the final answer. Many students attempted quotient rule unsuccessfully.

1 mark correct derivative

Question 11 (continued)

e.) Limiting sum when $|r| < 1$

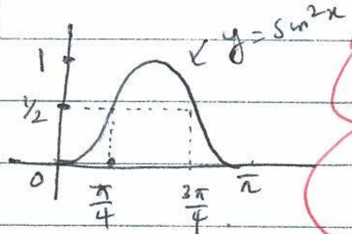
(i)

$r = 2\sin^2 x \qquad 0 \le x \le \pi$

$-1 < 2\sin^2 x < 1$ ✓

1 mark for common ratio between -1 and 1

$-\frac{1}{2} < \sin^2 x < \frac{1}{2}$



$0 \le \sin^2 x < \frac{1}{2}$

since $\sin^2 x \ge 0$

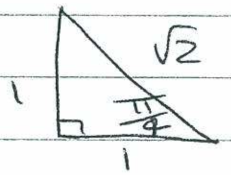
Drawing a diagram would have been the easiest way to solve this inequality

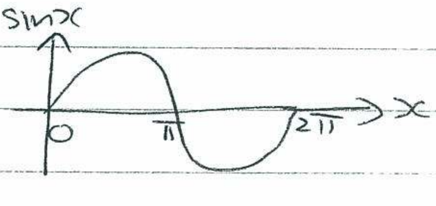
$-\frac{1}{\sqrt{2}} < \sin x < \frac{1}{\sqrt{2}}$

for $0 \le x \le \pi$, $\sin x \ge 0$

$\therefore 0 < \sin x < \frac{1}{\sqrt{2}}$ ✓ $\quad 0 \le x \le \pi$

2 marks: Correct solutions from correct working.





$\sin^{-1} 0 < x < \sin^{-1} \frac{1}{\sqrt{2}}$

Award 1 mark if one of the solutions is given $0 < x < \pi/4$ or the other.

$0 \le x < \frac{\pi}{4}$

$\therefore$ limiting sum exists for

$$\left. \begin{array}{l} 0 \le x < \frac{\pi}{4} \\ \frac{3\pi}{4} < x < \pi \end{array} \right\}.$$

Many students solved the equation $\sin x = \pm\frac{1}{\sqrt{2}}$ and giving the answer $-\pi/4 \le x \le \pi/4$ or $\frac{\pi}{4} \le x \le 3\pi/4$. No marks

Question 11 (cont.)

e.)

(ii)

$$S_\infty = \frac{a}{1-r}$$

$a = \sin^2 x$

Well done

$r = 2\sin^2 x$

1 mark expression for S_∞

$$S_\infty = \frac{\sin^2 x}{1-2\sin^2 x}$$

$$= \frac{\sin^2 x}{1-\sin^2 x - \sin^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x - \sin^2 x}$$

1 mark final answer

$$= \frac{\sin^2 x}{\cos 2x}$$

Question 11 (cont.)

f.)

(i) 15, 19, 23, 27, . . .

An AP with $a = 15$, $d = 4$

Well done

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(30 + 4(n-1))$$

$$= \frac{n}{2}(30 + 4n - 4)$$

$$= \frac{n}{2}(26 + 4n)$$

$$= n(13 + 2n)$$

$$S_n = 2n^2 + 13n$$

1 mark correct expression in any form.

Some Students found T_n instead of S_n

(ii) $$\frac{S_n}{n} = 43$$

$$\frac{n(13+2n)}{n} = 43$$

$$13 + 2n = 43$$

$$2n = 30$$

$$n = 15$$

1 mark $n = 15$ rows in the theatre.

$$T_n = a + (n-1)d$$

$$T_{15} = 15 + 14 \times 4$$

$$= 71 \text{ seats}$$

Well done

There are 71 seats in the last row of the theatre.

1 mark 71 seats in last row.

Q12 a $\quad u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\therefore du = \frac{1}{x}\,dx$$

$$\therefore \int \frac{1}{x(\ln x)^4}\,dx = \int \frac{1}{(\ln x)^4}\cdot\frac{1}{x}\,dx$$

$$= \int \frac{1}{u^4}\cdot du$$ (1 mark correct integral in terms of u)

$$= \int u^{-4}\,du$$

$$= \frac{u^{-3}}{-3} + c$$ (1 mark correct integration)

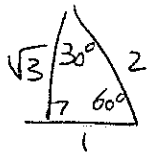
$$= \frac{(\ln x)^{-3}}{-3} + c$$

$$= \frac{-1(\ln x)^{-3}}{3} + c$$

$$= \frac{-1}{3(\ln x)^3} + c$$

(1 final answer in either form)

b $\int_0^{\frac{1}{2}} \sqrt{1-x^2}\,dx$ let $x = \sin t$

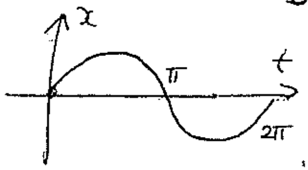


limits $\frac{1}{2} = \sin t$

$t = \sin^{-1}\frac{1}{2}$

$t = \frac{\pi}{6}$

(1 mark correct limits)



$x = \sin t$

$0 = \sin t$

$\therefore t = 0.$

$\therefore \int_0^{\frac{1}{2}} \sqrt{1-x^2}\,dx$

if $x = \sin t$

$\frac{dx}{dt} = \cos t$

$\therefore dx = \cos t\,dt$

$= \int_0^{\frac{\pi}{6}} \sqrt{1-(\sin t)^2}\,.dt.\cos t$

$= \int_0^{\frac{\pi}{6}} \sqrt{\cos^2 t}\,.\cos t.dt$

$= \int_0^{\frac{\pi}{6}} \cos t.\cos t.dt$

(1 mark) $= \int_0^{\frac{\pi}{6}} \cos^2 t\,dt$

$\cos 2t = 2\cos^2 t - 1$

$2\cos^2 t = \cos 2t + 1$

$\cos^2 t = \frac{1}{2}(\cos 2t + 1)$

(1 mark) $= \int_0^{\frac{\pi}{6}} \frac{1}{2}(\cos 2t + 1)\,dt$

$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \cos 2t + 1\,dt$

(1 mark) $= \frac{1}{2}\left[\frac{1}{2}\sin 2t + t\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left[\left(\frac{1}{2}\sin\frac{\pi}{3} + \frac{\pi}{6}\right) - \left(\frac{1}{2}\sin 0 + 0\right)\right] = \frac{1}{2}\left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\pi}{6}\right)$

(1 mark) $= \frac{\sqrt{3}}{8} + \frac{\pi}{12}$

c i $\cos(\theta+2\theta) = \cos\theta . \cos 2\theta - \sin\theta . \sin 2\theta$

Note: $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

sub into $\cos\theta . \cos 2\theta - \sin\theta . \sin 2\theta$

$= \cos\theta(2\cos^2\theta - 1) - \sin\theta . 2\sin\theta . \cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta . \cos\theta$ (1 mark)

Note: $\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta) . \cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

$\therefore$ as $\cos(\theta+2\theta) = \cos 3\theta$
$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (1 mark)

ii $\int 12\cos^3\theta - 9\cos\theta \, d\theta$

$= \int 3(4\cos^3\theta - 3\cos\theta)\, d\theta$

$= 3\int 4\cos^3\theta - 3\cos\theta \, d\theta$

$= 3\int \cos 3\theta \, d\theta$ 1 mark.

$= 3 \cdot \frac{1}{3}\sin 3\theta + C$

$= \sin 3\theta + C$ 1 mark

Q12d

$\int x\sqrt{1-3x}\,dx$

$x \le \frac{1}{3}$

let $u = 1-3x$

$\frac{du}{dx} = -3$

$du = -3dx$

$dx = \frac{du}{-3}$

$u = 1-3x$

$3x = 1-u$

$x = \frac{1-u}{3} = \frac{1}{3}(1-u)$

$\therefore \int x\sqrt{1-3x}\,dx$

$= \int \frac{1}{3}(1-u)\cdot\sqrt{u}\cdot\frac{du}{-3}$

$= \int \frac{-1}{9}(1-u)\cdot u^{1/2}\,du$ (1 mark)

$= \frac{-1}{9}\int u^{1/2} - u^{3/2}\,du$

$= -\frac{1}{9}\left(\frac{2u^{3/2}}{3} \quad \frac{-2u^{5/2}}{5}\right) + c$ (1 mark)

$= \frac{-2u^{3/2}}{27} + \frac{2u^{5/2}}{45} + c \quad = \frac{-2(1-3x)^{3/2}}{27} + \frac{2(1-3x)^{5/2}}{45} + c$

(1 mark)

Question 13

(i) $f(x) = \dfrac{2x^2}{9-x^2}$

$= \dfrac{2x^2}{(3+x)(3-x)}$

vertical asymptotes at

$x = -3, \quad x = 3$

1 mark vertical asymptotes.

$f(x) = \dfrac{2x^2}{9-x^2}$

$= \dfrac{\frac{2x^2}{x^2}}{\frac{9}{x^2} - 1} = \dfrac{2}{\frac{9}{x^2} - 1}$

as $x \to \infty$,

$f(x) \to \dfrac{2}{-1} = -2$

1 mark horizontal asymptote.

horizontal asymptote at $y = -2$

(ii) $f(x) = \dfrac{2x^2}{9-x^2}$

1 mark

$f(-x) = \dfrac{2(-x)^2}{9-(-x)^2}$

showing $f(x)$ is even

$= \dfrac{2x^2}{9-x^2}$

$= f(x)$

$\therefore f(x)$ is even

Question 13a (cont.)

(iii) $f(x) = \dfrac{2x^2}{9-x^2} = \dfrac{u}{v}$

Some students wrote: horizontal asymptotes $x \neq \pm 3$ which should be $x = \pm 3$

$f'(x) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \dfrac{(9-x^2)(4x) - 2x^2(-2x)}{(9-x^2)^2}$

$= \dfrac{36x - 4x^3 + 4x^3}{(9-x^2)^2}$

generally well done

$= \dfrac{36x}{(9-x^2)^2}$

Stationary points at $f'(x) = 0$

$0 = \dfrac{36x}{(9-x^2)^2}$

$36x = 0$

$x = 0$

1 mark stationary point at $(0,0)$

at $x = 0$, $y = \dfrac{0}{9-0} = 0$

$\therefore$ stationary point at $(0,0)$

test its nature.

x	-1	0	1
$f'(x)$	-0.5625	0	0.5625
	\	—	/

quite a few did not give values.

at $x = -1$, $f'(x) = \dfrac{-36}{8^2} = -0.5625$

at $x = 1$, $f'(x) = \dfrac{36}{8^2} = 0.5625$

1 mark prove point is a minimum.

$\therefore$ A minimum at $(0,0)$

There is a minimum stationary point at $(0,0)$.

Question 13a (cont.)

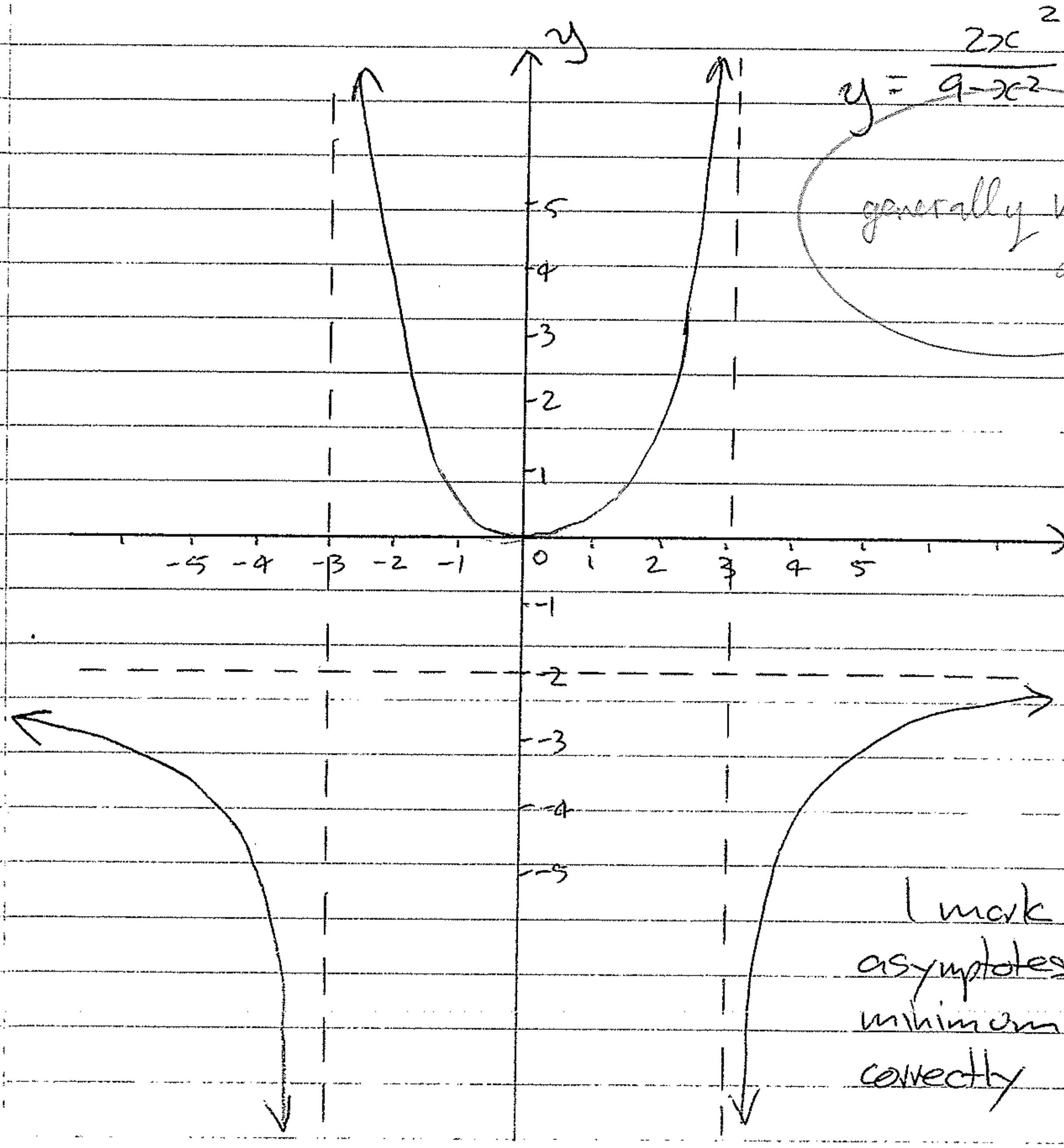
(iv) even function

asymptotes at $x=-3$, $x=3$, $y=-2$

minimum at $(0,0)$

at $x=4$, $y=\dfrac{32}{9-16} = \dfrac{-32}{7} = -4\frac{4}{7}$

$x=-4$, $y=-4\frac{4}{7}$



generally well done

1 mark asymptotes and minimum shown correctly

1 mark correct shape.

Question 13b

(i)

In $\triangle ACD$, $\sin\alpha = \frac{CD}{b}$

$CD = b\sin\alpha$

In $\triangle BCD$, $\sin\beta = \frac{CD}{a}$

$CD = a\sin\beta$

$\therefore a\sin\beta = b\sin\alpha$

1 mark showing result

(ii) In $\triangle ACD$, $\cos\alpha = \frac{AD}{b}$

$AD = b\cos\alpha$

In $\triangle BCD$, $\cos\beta = \frac{BD}{a}$

$BD = a\cos\beta$

$c = AD + BD$

$\therefore c = b\cos\alpha + a\cos\beta$

1 mark showing result

Well done

(iii) $c^2 = 4ab\cos\alpha\cos\beta$

from part (ii)

$(b\cos\alpha + a\cos\beta)^2 = 4ab\cos\alpha\cos\beta$

$b^2\cos^2\alpha + 2ab\cos\alpha\cos\beta + a^2\cos^2\beta = 4ab\cos\alpha\cos\beta$

$b^2\cos^2\alpha - 2ab\cos\alpha\cos\beta + a^2\cos^2\beta = 0$

$(b\cos\alpha - a\cos\beta)^2 = 0$

$b\cos\alpha - a\cos\beta = 0$

1 mark for result $b\cos\alpha - a\cos\beta = 0$

A few found difficulty in this part

$b\cos\alpha = a\cos\beta$

$b^2\cos^2\alpha = a^2\cos^2\beta$

$b^2(1-\sin^2\alpha) = a^2(1-\sin^2\beta)$

$b^2 - b^2\sin^2\alpha = a^2 - a^2\sin^2\beta$

from (i), $a\sin\beta = b\sin\alpha$

$\therefore b^2 - b^2\sin^2\alpha = a^2 - b^2\sin^2\alpha$

$b^2 = a^2$

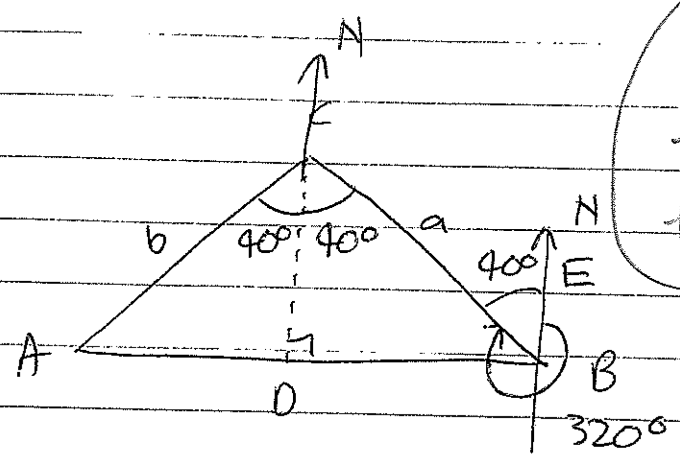
$b = a$ since $a>0, b>0$

1 mark final proof

$\therefore a = b$

Question 13b (cont.)

(iv)



quite a few assumed that A is West of B. The answer was correct but the process was incorrect

$\angle BCD = \angle EBC = 40°$ (alternate angles $EB \parallel CD$)

from part (iii) $a = b$

$\therefore \triangle ABC$ is isosceles

and CD bisects $\angle ACB$

$\therefore \angle ACD = 40°$

$\therefore \angle ACB = 80°$

1 mark for geometrical explaination may be several different ones.

v.) $AD = \frac{85}{2} = 42.5$ m

In $\triangle ACD$, $\sin 40° = \frac{42.5}{b}$

$b = \frac{42.5}{\sin 40}$

$= 66.11826\ldots$ m

1 mark calculate length of b or a

In $\triangle ATC$,

$\tan 23° = \frac{h}{b}$

$h = \tan 23 \times 66.11826\ldots$

$= 28.065537\ldots$

$= 28.1$ m (1d.p.)

1 mark calculate h

1 mark correct rounding to 1 d.p.

Generally Well done

14a i $P(x) = 5x^3 + x^2 + 2$ 1/2

$P(-1) = 5\times(-1)^3 + (-1)^2 + 2$
$= -5 + 1 + 2$
$= -2$

Many students did not show that one value was <0 and one value >0 and thus the root lies between them

$P(1) = 5\times 1^3 + 1^2 + 2$
$= 8$

as $P(-1) < 0$ and $P(1) > 0$
root lies between -1 and 1

$P\left(\frac{-1+1}{2}\right) = P(0) = 5\times 0^3 + 0^2 + 2$
$= 2$

as $P(-1) < 0$ and $P(0) > 0$
root lies between -1 and 0

need the conclusion

$P\left(\frac{-1+0}{2}\right) = P\left(-\frac{1}{2}\right) = 5\times\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 + 2$

$= 5 \times -\frac{1}{8} + \frac{1}{4} + 2$

$= \frac{-5}{8} + \frac{2}{8} + 2$

$= \frac{-3}{8} + \frac{16}{8} = \frac{13}{8}$

(1 mark) calculate $P(-1)$, $P(1)$ $P(0)$, $P(\frac{1}{2})$ and concludes

$\therefore$ as $P\left(-\frac{1}{2}\right) > 0$ and $P(-1) < 0$
root lies between -1 and $-\frac{1}{2}$

$P\left(\frac{-1+-\frac{1}{2}}{2}\right) = P\left(\frac{-3}{4}\right) = 5\times\left(\frac{-3}{4}\right)^3 + \left(\frac{-3}{4}\right)^2 + 2$

$= 5 \times \frac{-27}{64} + \frac{9}{16} + 2 = \frac{29}{64}$

$\therefore$ as $P(-1) < 0$ and $P(0.75) > 0$ the root lies between -1 and 0.75 (1 mark) calculates $P(0.75)$ and $P(-1)$

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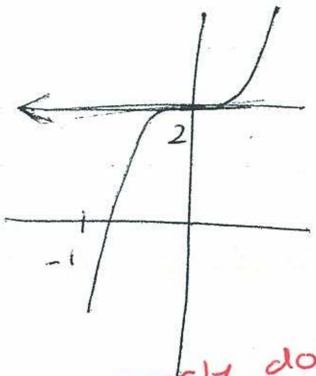
ii



The tangent at $x = 0$ is at a turning point (as shown). This will not meet the x-axis, so it will be impossible for Newton's method to work.

(other valid answers related to the tangent should gain a mark)

✓1

very poorly done
→ need to be specific

---

b i $\quad P(x) = x^3 - 3x^2 + kx + 6$

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{--3}{1}$$

$$\therefore \alpha + \beta + \gamma = 3$$

✓1

ii $\quad \alpha\beta\gamma = \frac{-d}{a} = \frac{-6}{1} = -6$

✓1

iii $\quad$ let $\beta = -\alpha$ (equal in magnitude, opposite in sign)

✓2

$$\alpha + \beta + \gamma = 3$$

$$\alpha + -\alpha + \gamma = 3$$

$$\therefore \gamma = 3 \qquad \text{(1 mark)}$$

$\alpha\beta\gamma = -6$

$\gamma = 3$

$\therefore \alpha\beta . 3 = -6$

$\alpha\beta = -2$

as $\beta = -\alpha$

$\alpha . -\alpha = -2$

$-\alpha^2 = -2$

$\alpha^2 = 2$

$\therefore \alpha = \pm\sqrt{2}$ so $\alpha = \sqrt{2}$, $\beta = -\sqrt{2}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{k}{1} = k$

as $\beta = -\alpha$

$\alpha . -\alpha + \alpha\gamma - \alpha\gamma = k$

$-\alpha^2 = k$

$-(\sqrt{2})^2 = k$

$\therefore k = -2$ (1 mark)

---

c i $y - \frac{1}{2}(p+q)x + apq = 0$

1

sub $x = 0$ $y + apq = 0$

$y = -apq$

we know y-intercept $= -5a$

$\therefore \frac{-5a}{-a} = \frac{-apq}{-a}$

$\therefore pq = 5$

Many had a very long winded approach ⇒ it was only worth 1 mark!

ii at P

tangent is $y = px - ap^2$ ①

1/3

at Q

tangent is $y = qx - aq^2$ ② (1 mark)

solve simultaneously

① − ② $0 = px - qx - ap^2 + aq^2$

$0 = x(p-q) - a(p^2 - q^2)$

$$\frac{x\cancel{(p-q)}}{\cancel{(p-q)}} = \frac{a\cancel{(p-q)}(p+q)}{\cancel{(p-q)}}$$

$\therefore x = a(p+q)$ (1 mark)

sub into $y = px - ap^2$

$y = p \cdot a(p+q) - ap^2$

$y = ap^2 + apq - ap^2$

$\therefore y = apq$ (1 mark)

$\therefore$ point of intersect R is $(a(p+q), apq)$

iii $y = apq$ (from part ii)

1/1

$pq = 5$ (from part i)

$\therefore y = a \times pq$

$= a \times 5$

$\therefore y = 5a$ (horizontal line through $5a$)

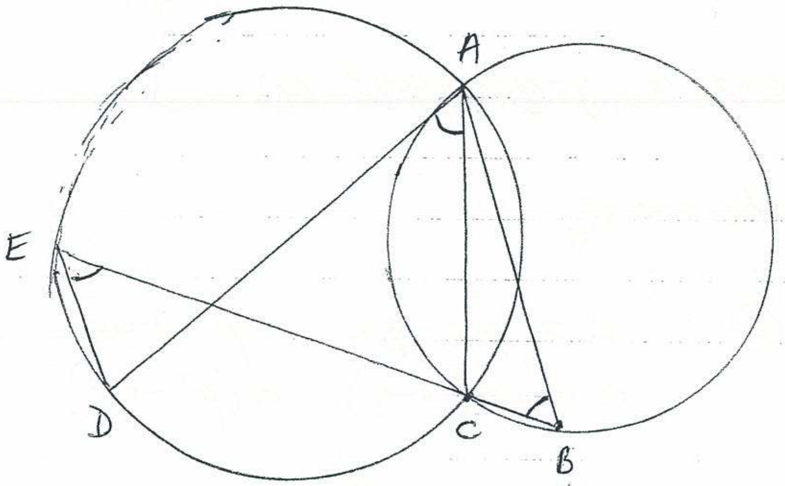
many students got this but didn't recognise it as a complete solution

(1 mark for $y = 5a$ with working)

Note: $y = 5a$ doesn't exist inside parabola, only need to state $y = 5a$ for that mark

Many students used abreviations
→ this is very risky and not advised
→ reasons need to be much better expressed

d



$\angle DAC = \angle ABC$ (angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment)

(1 mark)

$\angle DEC = \angle DAC$ (angles in the same segment of a circle are equal)

(1 mark)

$\therefore \angle DEC = \angle DAC = \angle ABC$ (from above)

$\therefore \angle DEC = \angle ABC$

note: $\angle EBA = \angle ABC$ (same angle), similarly $\angle DEC = \angle DEB$

$\therefore$ As alternate angles equal, $\angle DEB = \angle EBA$
this means the lines DE and AB cut by the transversal EB are ∥.

(1 mark)

$\therefore AB \parallel DE$