Killara HIGH SCHOOL

Student Number

2017

HIGHER SCHOOL CERTIFICATE MID-COURSE EXAMINATION

Mathematics Extension 1

17th March 2017

General Instructions

- Reading time –5 minutes
- Working time – 2 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-14 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 70

Section I - Pages 2 - 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 5 - 9

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
Total	/70

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 30% of the Higher School Certificate Course Assessment

Section I

10marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which of the following is a solution of the equation $2^x = 5$?

(A) $x = \sqrt{5}$

(B) $x = \log_e 5$

(C) $x = \frac{\log_e 5}{\log_e 2}$

(D) $x = \frac{\log_e 2}{\log_e 5}$

2 The parametric equations that can be used to represent the parabola $x^2 = 10y$ are:

(A) $x = \frac{5t}{2}, \quad y = 5t^2$

(B) $x = 5t, \quad y = \frac{5t^2}{2}$

(C) $x = 2t, \quad y = t^2$

(D) $x = \frac{5t^2}{2}, \quad y = 5t$

3 The expression $\sin x - \sqrt{3}\cos x$ can be written in the form of $2\sin(x + \alpha)$. Find the value of α.

(A) $\alpha = \frac{\pi}{6}$

(B) $\alpha = -\frac{\pi}{6}$

(C) $\alpha = \frac{\pi}{3}$

(D) $\alpha = -\frac{\pi}{3}$

4 The function $f(x) = -3\tan(2\pi x)$ has a period of:

(A) $\frac{2}{\pi}$

(B) 2

(C) $\frac{1}{2}$

(D) 2π

5 Evaluate:

$$\lim_{h \to 0} \frac{\sin\frac{h}{2}}{3h}$$

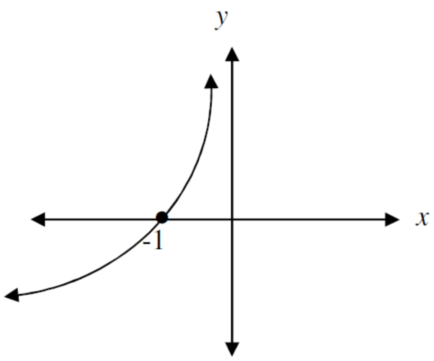
(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 6

6



The graph above could be represented by:

(A) $y = -\ln(-x)$

(B) $y = \ln(-x)$

(C) $y = -\ln(x)$

(D) $y = -e^{-x}$

7 The area enclosed between the curve $y = x^3 - 1$, the y-axis and the lines $y = 0$ and $y = 2$ is given by:

(A) $\int_0^2 x^3 - 1\, dy$

(B) $\int_0^2 \sqrt[3]{y+1}\, dy$

(C) $\int_0^2 \sqrt[3]{y} + 1\, dy$

(D) $\int_0^2 y + 1\, dy$

8 The condition for value of $ax + by + c = 0$ is a tangent to the curve $y^2 = 4ax$ is:

(A) $a^2 = b^2 = c^2$

(B) $a^2 = b$

(C) $b^2 = a$

(D) $b^2 = c$

9 If $1 + \cos x = k$, where x is acute, then the value of $\sin\frac{x}{2}$ is:

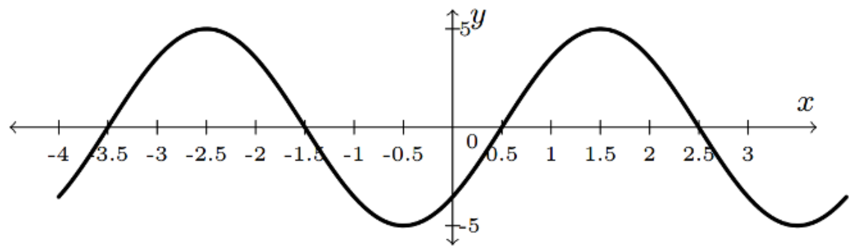
(A) $\sqrt{\frac{k}{2}}$

(B) $\sqrt{\frac{1-k}{2}}$

(C) $\sqrt{\frac{2+k}{2}}$

(D) $\sqrt{\frac{2-k}{2}}$

10 Which functions below corresponds to the given graph?



(A) $5sin\left(\frac{x}{2}-\frac{1}{4}\right)$

(B) $5sin\left(\frac{\pi x}{2}-\frac{\pi}{4}\right)$

(C) $5cos\left(\frac{\pi x}{2}-\frac{\pi}{4}\right)$

(D) $5cos\left(\frac{x}{2}-\frac{1}{4}\right)$

Section II

60marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a) Solve: **3**

$$\frac{2x+5}{x} \leq 1$$

(b) The graphs of $2x - y + 4 = 0$ and $y = x^3$ intersect at $x = 2$. **2**
Find the size of the acute angle between the line and the curve at $x = 2$.

(c) (i) Show that $x = 3$ is a root of $P(x) = x^3 - 4x^2 + x + 6$. **1**

(ii) Hence, or otherwise, factorise $P(x) = x^3 - 4x^2 + x + 6$. **2**

(d) A and B are the points $(5, 4)$ and $(-1, 8)$ respectively. **2**
Find the point P which divides AB externally in the ratio $5:3$.

(e) Solve the inequality $(x-2)^2(x+1) \geq 0$ **2**

(f) Let $f(x) = cosx - x^2$.

(i) Taking x = 1 as the first approximation to $f(x) = 0$, use one application of Newton's method to find a better approximation. **2**

(ii) What other solutions does $f(x) = 0$ have? Give reasons. **1**

End of Question 11

Question 12(15 marks) Use a SEPARATE writing booklet

(a) The cubic equation $2x^3 - 2x^2 + x - 1 = 0$ has roots α, β and γ. Evaluate:

(i) $\alpha\beta + \beta\gamma + \alpha\gamma$. **1**

(ii) $\alpha\beta\gamma$ **1**

(iii) $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$ **2**

(b) The equation $2\cos^3\theta - 2\cos^2\theta + \cos\theta - 1 = 0$ has roots $\cos a$, $\cos b$ and $\cos c$. Using appropriate information from (a) above prove that $\sec a + \sec b + \sec c = 1$ **2**

(c) Find the general solution of: **3**

$$2\sin 2x cos x = \sqrt{3}\sin 2x$$

(d) Using the substitution $u = 2x + 1$, evaluate the integral **3**

$$I = \int_0^1 \frac{2x}{(2x+1)^2}\,dx$$

(e) (i) Differentiate $x \tan x$ with respect to x. **1**

(ii) Hence, find $\int x \sec^2 x \, dx$. **2**

End of Question 12

Question 13(15 marks) Use a SEPARATE writing booklet

(a) Consider the function $f(x) = \frac{e^x}{e^x+3}$

(i) Show that $f(x)$ has no stationary points. **2**

(ii) Find the coordinates of the point of inflection, given that **2**

$$f''(x) = \frac{3e^x(3-e^x)}{e^x+3}$$

(iii) Show that $0 < f(x) < 1$ for all x. **2**

(iv) Describe the behaviour of $f(x)$ for very large positive and very large negative values of x, giving reasons. (ie. as $x \to \infty$ and $x \to -\infty$) **2**

(v) Sketch the curve $y = f(x)$. **2**

(b) Evaluate: **3**

$$\int_0^1 2^{lnx}\,dx$$

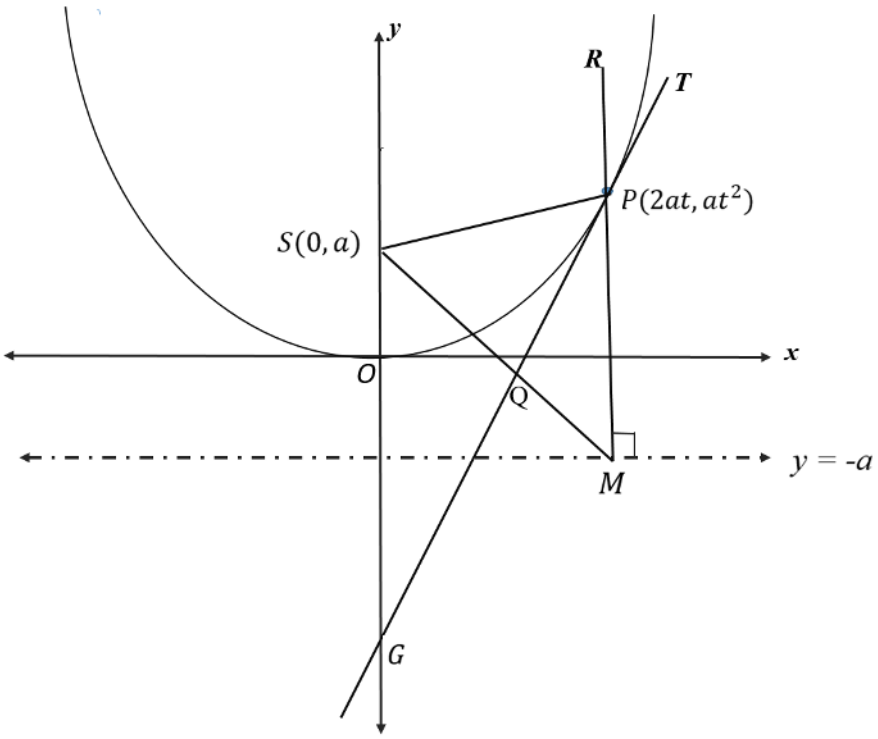
(c) Evaluate: **2**

$$\lim_{x\to\infty} \frac{x^2 sin\frac{1}{x}}{x+1}$$

End of Question 13

Question 14(15 marks) Use a SEPARATE writing booklet

(a) The point $P(2at, at^2)$ lies on the parabola $x^2 = 4ay$. The foot of the perpendicular from P intersects the directrix at M. SM intersects the tangent TG at Q.



(i) Explain why $SP = PM$. **1**

(ii) The equation of the tangent at P is given by $tx - y = at^2$. Show that $PQ \perp SM$. **2**

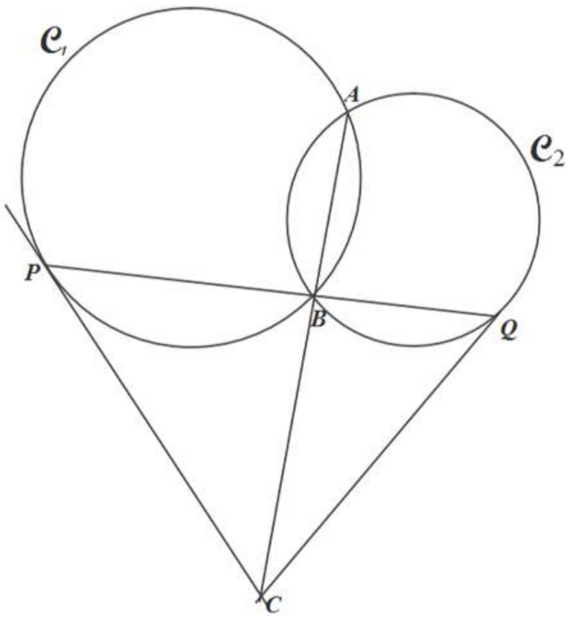
(iii) Show that $\angle RPT = \angle SPQ$. **2**

(iv) G is the y-intercept of tangent at P. Show that $SP = SG$. **2**

(v) From O the vertex of the parabola, a perpendicular is drawn to cut the tangent PG at Z. Find the locus of Z. **2**

Question 14 continued on page 9

(b) Two circles intersect at A and B. AB is produced to C such that CP and CQ are tangents to the circles C_1 and C_2 respectively. PBQ is a straight line.



(i) Prove that $CP = CQ$. **1**

(ii) Show that A, P, C and Q are concyclic. **3**

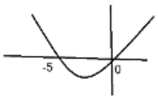
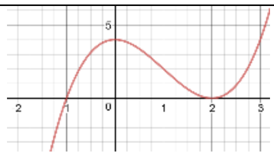
(iii) QA is produced to meet the circle C_1 at D. Show that PB bisects $\angle CPD$. **2**

End of paper

2017 KHS Extension 1 half-yearly marking scheme

1.	C	C	
2.	4a = 10 $a = \frac{5}{2}$ $(2at, at^2) = (5t, \frac{5t^2}{2})$	B	
3.	$\sin x - \sqrt{3}\cos x = 2\sin(x+\alpha)$ $2\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right)$ $= 2(\sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3})$ $= 2\sin\left(x - \frac{\pi}{3}\right)$	D	
4.	Period $= \frac{\pi}{n} = \frac{\pi}{2\pi} = \frac{1}{2}$	C	
5.	$\lim_{h\to 0} \frac{sin\frac{h}{2}}{3h}$ $=$ $\frac{1}{3}\lim_{h\to 0} \frac{sin\frac{h}{2}}{2\times\frac{h}{2}} = \frac{1}{3\times 2} = \frac{1}{6}$	A	
6.	$y = -\ln(-x)$	A	
7.	Area = $\int_0^2 x\, dy$ $y = x^3 - 1$ $x^3 = y + 1$ $x = \sqrt[3]{y+1}$ Area = $\int_0^2 \sqrt[3]{y+1}\, dy$	B	
8.	$ax + by + c = 0$ $\therefore x = -\frac{c+by}{a}$ $y^2 = -4a\times\frac{c+by}{a}$	D	

	$y^2 = -4c - 4by$ $y^2 + 4by + 4c = 0$ For the line to be a tangent, $\Delta = 0$ $\Delta = 16b^2 - 16c = 0$ $b^2 - c$		
9.	$1 + \cos x = k$ $1 + 1 - 2sin^2\frac{x}{2} = k$ $2\left(1 - sin^2\frac{x}{2}\right) = k$ $sin^2\frac{x}{2} = 1 - \frac{k}{2} = \frac{2-k}{2}$ $sin\frac{x}{2} = \pm\sqrt{\frac{2-k}{2}}$ x is acute, and hence $sin\frac{x}{2} = \sqrt{\frac{2-k}{2}}$	D	
10.	From the graph period $= 4$. Amplitude $= 5$ It is could be sine graph shifted to right by 0.5. Option B: $5\sin\left(\frac{\pi x}{2} - \frac{\pi}{4}\right)$: Amplitude $= 5$, Period $\frac{2\pi}{\frac{\pi}{2}} = 4$, Shift $-\frac{\pi/4}{\pi/2} = 0.5$ to the right. (B)		

Question 11			
a)	$\frac{2x+5}{x} \le 1$ $\frac{x^2(2x+5)}{x} \le 1\times x^2$ $x(2x+5) \le 1\times x^2$ $x(2x+5-x) \le 0$ $x(x+5) \le 0$ $-5 \le x \le 0$ But $x \ne 0$ Hence, $-5 \le x < 0$  	1 mark: multiplies by x^2 on both sides 1 mark: Solves the inequality 1 mark: Applies $x \ne 0$ and gives the correct solution	Done well. Some students neglected to exclude $x=0$
b)	$2x - y + 4 = 0$ Gradient = 2 $y = x^3$ $\frac{dy}{dx} = 3x^2$ At $x = 2$, $\frac{dy}{dx} = 12$ Angle between the lines $tan\theta = \left\lvert\frac{12-2}{1+12\times2}\right\rvert = \frac{2}{5}$ $\theta = tan^{-1}\left(\frac{2}{5}\right) = 21°48'$	2 marks: correct answer from correct working 1 mark: calculates the two gradients correctly OR correct working using their gradients to find the angle	Done well. Some students neglected to find $\frac{dy}{dx}$ + gradient=12
c)	(*i*) $P(x) = x^3 - 4x^2 + x + 6$. X = 3 is a root. P(3) $=3^3 - 4\times3^2 + 3 + 6 = 0$ *Hence*, $x - 3$ is a factor. (ii) $x^2 - x - 2$ $x-3\,)\,x^3 - 4x^2 + x + 6$ $x^3 - 3x^2$ $-x^2 + x$ $-x^2 + 3x$ $-2x + 6$ $-2x + 6$ 0 $x^2 - x - 6 = (x-2)(x+1)$ $x^3 - 4x^2 + x + 6 = (x-2)(x-2)(x+1)$	1 mark: Show by substitution P(3) = 0 (ii) 1 mark: Correctly divides P(x) by x – 3. 1 mark: Expresses P(x) as product of linear factors using their Q(x).	Done well Many students neglected to factorise (x^2-x-2) into linear factors
d)	$(5,4)$ $(-1,8)$ 5: 3 The point is $\left(\frac{5\times(-1,8)-1-3\times5}{5-3} - \frac{5\times8-3\times4}{5-3}\right)$ $= (-10, 14)$	2 marks: correct answer from correct working 1 mark: substitutes into the correct formula with minor error in calculations	Many received $\frac{1}{2}$ marks for minor errors
e)	 	2 marks: correct answer from correct working (algebraic or graphic) 1 mark: Incorrect graph, but correct	Done well. Needed working or graph.

	$(x-2)^2(x+1) \geq 0$ $x \geq -1$	answer from their graph	
f) (i)	$f(x) = cosx - x^2$ $f'(x) = -sinx - 2x$ $x_0 = 1$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= 1 - \frac{cos1-1}{-sin1-2}$ $=$ 0.84 (2 d.p.)	2 marks: correct answer from correct working 1 mark: correct differentiation and substitutes into correct formula (uses degrees)	Done well Some neglected to use radians
(ii)	$f(x)$ is even, so $x = \alpha$, so $x = -\alpha$ is also an approximation to the root x = -0.84	1 mark: correct answer with correct reasoning	Poorly done. Insufficient reasoning.

Question 12			
a)	$2x^3-2x^2+x-1=0$		
	(i) $\alpha\beta+\beta\gamma+\alpha\gamma = \frac{1}{2} = \frac{1}{2}$	1 mark: correct answer	
	(ii) $\alpha\beta\gamma = \frac{-(-1)}{2} = \frac{1}{2}$		
	(iii) $\alpha+\beta+\gamma = \frac{-(-2)}{2} = 1$ $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma} =$ $\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{1}{1/2} = 2$	1 mark: Correct $\alpha+\beta+\gamma$ value 1 mark: correct value using their answer in (ii)	
b)	$2\cos^3\theta-2\cos^2\theta+\cos\theta-1=0$ Let $\alpha = \cos a,\ \beta = \cos b$ and $\gamma = \cos c$ $\mathrm{Sec}a+\sec b+\sec c =$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\alpha} =$ **1 mark** $\frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha\beta\gamma} = \frac{1/2}{1/2} = 1$	1 mark: writes $\mathrm{Sec}a+\sec b+\sec c$ as sum of reciprocals 1 mark: uses their values from (a)(i), (ii) to prove the result	
c)	$2\sin 2x\cos x = \sqrt{3}\sin 2x$ $2\sin 2x\cos x - \sqrt{3}\sin 2x = 0$ $\sin 2x(2\cos x-\sqrt{3}) = 0$ $\mathrm{Sin}2x = 0$ $\therefore 2x = n\pi$ $\therefore x = \frac{n\pi}{2}$ $2\cos x-\sqrt{3} = 0$ $\cos x = \frac{\sqrt{3}}{2}$ $x = 2n\pi \pm \frac{\pi}{6}$	1 mark: factorises to give $\mathrm{Sin}2x = 0$ and $\cos x = \frac{\sqrt{3}}{2}$ 1 mark each: for the general solutions.	
d)	$I = \int_0^1 \frac{2x}{(2x+1)^2}dx$ $u = 2x+1$ $x = 0,\ u = 1$ $du = 2dx$ $x = 1,\ u = 3$ **1 mark** $x = \frac{u-1}{2}$ $I = \int_0^1 \frac{2x}{(2x+1)^2}dx =$ $= \int_1^3 \frac{u-1}{2u^2}du$ $= \frac{1}{2}\int_1^3 \frac{1}{u} - u^{-2}du$ $= \frac{1}{2}\left[\ln u + \frac{1}{u}\right]_1^3$ $= \frac{1}{2}\left[\ln 3 + \frac{1}{3} - 0 - 1\right]$ $= \frac{1}{2}\left[\ln 3 - \frac{2}{3}\right]$	1 mark: correctly converts the integrand and the limits 1 mark: correctly expresses I in terms of u. 1 mark: correctly integrates and evaluates	
e) (i)	$\frac{d}{dx}(x\tan x) = x\sec^2 x + \tan x$	1 mark: correctly differentiates	

(ii)	$\int xsec^2x + tanx\,dx = xtanx$ **1 mark** $\int xsec^2xdx + \int tanxdx = xtanx + C$ $\int xsec^2xdx - \ln(cosx) = xtanx + C$ $\int xsec^2xdx = \ln(cosx) + xtanx + C$	2 marks: correct answer from correct working 1 mark: minor error in the integration of tanx.	

12(a)(i) and (a)(ii) too straight forward- most, if not all, got it right

12a(iii) a few students had difficulty in expressing it in terms of sum and product of roots.

12(b) some students expressed the sum of the reciprocals as the reciprocal of the sum- no marks awarded though the values of both happened to be same.

12(c) many students cancelled $\sin 2\,x$ and lost a root.

12(d) some failed to change the limits of integration. Just a few did not know to integrate $\frac{1}{u}$ with respect to u.

12c(i) most got it

12c(ii) many lost a mark for not integrating $\tan x$.

<table>
<tr><td colspan="4">Question 13</td></tr>
<tr><td>a)</td><td>

$\lim_{x\to\infty} \frac{x^2 sin\frac{1}{x}}{x+1}$

$\lim_{x\to\infty} \frac{sin\frac{1}{x}}{\frac{1}{x^2}(x+1)}$

$\lim_{x\to\infty} \frac{sin\frac{1}{x}}{\frac{1}{x}\frac{(x+1)}{x}}$

As $x \to \infty$, $\frac{1}{x} \to 0$.

$\lim_{x\to\infty} \frac{sin\frac{1}{x}}{\frac{1}{x}\frac{(x+1)}{x}} =$

$\lim_{\frac{1}{x}\to 0} \frac{sin\frac{1}{x}}{\frac{1}{x}} \times \lim_{\frac{1}{x}\to 0} \frac{1}{\frac{(x+1)}{x}}$

$= \lim_{\frac{1}{x}\to 0} \frac{sin\frac{1}{x}}{\frac{1}{x}} \times \lim_{\frac{1}{x}\to 0} \frac{1}{1+\frac{1}{x}}$

$1 \times \lim_{\frac{1}{x}\to 0} \frac{1}{1+0}$

$= 1$

</td><td>

2 marks: correct answer from correct working

1 mark: explains as $x \to \infty$, $\frac{1}{x} \to 0$ and attempts to separate functions into $\frac{sin\frac{1}{x}}{\frac{1}{x}}$ form.

</td><td></td></tr>
<tr><td>a) (i)</td><td>

$f(x) = \frac{e^x}{e^x+3}$

$f'(x) = \frac{(e^x+3)e^x - e^x.e^x}{(e^x+3)^2}$

$= \frac{e^x(e^x+3-e^x)}{(e^x+3)^2}$

$= \frac{e^x.3}{(e^x+3)^2}$

$e^x > 0$ for all x.

hence $f'(x) > 0$ for all x.

</td><td>

1 mark: writes $\sec a + \sec b + \sec c$ as sum of reciprocals

1 mark: uses their values from (a)(i), (ii) to prove the result

</td><td></td></tr>
<tr><td>(ii)</td><td>

$f''(x) = \frac{3e^x(3-e^x)}{e^x+3} = 0$ for points of inflection.

$e^x > 0$ for all x, hence $e^x = 3$

$x = ln\ 3$

x	1	$ln\ 3$	2
$\frac{3e^x(3-e^x)}{e^x+3}$	0.401	0	-9.364

The concavity changes at $x = ln\ 3$, hence point of inflection

At $x = ln\ 3$, $y = \frac{1}{2}$

Point of inflection at $\left(ln3, \frac{1}{2}\right)$.

</td><td>

2 marks: correct answer from correct working

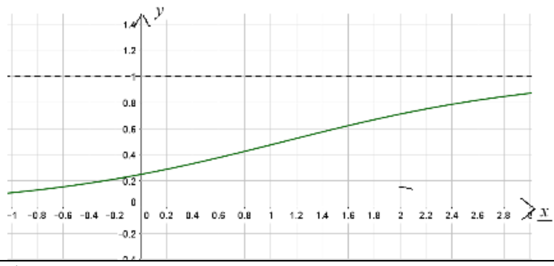
1 mark: all working except the concavity test is performed.

</td><td></td></tr>
<tr><td>(iii)</td><td>

$f(x) = \frac{e^x}{e^x+3}$

$e^x > 0$, hence, $\frac{e^x}{e^x+3} > 0$. **1 mark**

</td><td>1 mark: proves $f(x) > 0$</td><td></td></tr>
</table>

	$= \frac{e^x+3-3}{e^x+3}$ $= 1 - \frac{3}{e^x+3}$ $\frac{3}{e^x+3} > 0$, hence, $1 - \frac{3}{e^x+3} < 1$ Hence, $0 < f(x) < 1$	1 mark: proves $f(x) < 1$	
(iv)	When $x \to \infty$, $\frac{3}{e^x+3} \to 0$ then $1 - \frac{3}{e^x+3} \to 1^-$. **1 mark** When $x \to -\infty$, $\frac{3}{e^x+3} \to 1$ then $1 - \frac{3}{e^x+3} \to 0$. **1 mark**	1 mark: for each proof	
(v)		1 mark: correct shape 1 mark: shows the asymptotes and y - intercept	
b)	$\int_0^1 2^{lnx} dx$ $\int_0^1 (e^{ln2})^{lnx} dx$ $\int_0^1 (e^{lnx})^{ln2} dx$ $\int_0^1 x^{ln2} dx$ $\left[\frac{x^{ln2+1}}{ln2+1}\right]_0^1 = \frac{1}{ln2+1} - 0 = \frac{1}{ln2+1}$	2 mark: correct answer from correct working 1 mark: expresses 2 as e^{ln2}.	
c)	$\lim_{x\to\infty} \frac{x^2 sin\frac{1}{x}}{x+1}$ $\lim_{x\to\infty} \frac{sin\frac{1}{x}}{\frac{1}{x^2}(x+1)}$ $\lim_{x\to\infty} \frac{sin\frac{1}{x}}{\frac{1}{x}\frac{(x+1)}{x}}$ As $x \to \infty$, $\frac{1}{x} \to 0$. $\lim_{x\to\infty} \frac{sin\frac{1}{x}}{\frac{1}{x}\frac{(x+1)}{x}} =$ $\lim_{\frac{1}{x}\to 0} \frac{sin\frac{1}{x}}{\frac{1}{x}} \times \lim_{\frac{1}{x}\to 0} \frac{1}{\frac{(x+1)}{x}}$	2 marks: correct answer from correct working 1 mark: explains as $x \to \infty$, $\frac{1}{x} \to 0$ and attempts to separate functions into $\frac{sin\frac{1}{x}}{\frac{1}{x}}$ form.	

	$= \lim\limits_{\frac{1}{x}\to 0} \frac{\sin\frac{1}{x}}{\frac{1}{x}} \times \lim\limits_{\frac{1}{x}\to 0} \frac{1}{1+\frac{1}{x}}$ $1\times \lim\limits_{\frac{1}{x}\to 0} \frac{1}{1+0}$ $= 1$		

Q13 – Markers Feedback

a/ i/ many students got full marks. Some only achieved 1 mark for using the quotient rule correctly.

Some students didn't write the quotient rule from the formula sheet correctly… Others couldn't come up with a valid reason why the derivative was $\neq 0$

ii/ many found ln3, but then worked in decimals instead of staying in exact form, some didn't go on and find the y value even though the question asked them to and lost a mark. MANY didn't check for the change in concavity and lost a mark…

iii/ many didn't use the features of e^x to prove that f(x) lay between 0 and 1. Most didn't use any clear mathematical reasoning and wrote a couple of phrases related to question iv that weren't mathematically supported. Better students showed a good understanding of the features of e^x, or used limits well.

Iv/ Many students ignored the hint in the question and didn't find the limits as requested. Instead they wrote a couple of phrases restating the question.

v/ most students who attempted this and read the question carefully got a mark. Better students clearly showed the asymptotes and y-intercept and drew good graphs ½ to a third of a page in size.

b/ Many students couldn't find a way into this question.

Quite a few got 1 mark for expressing 2 as $e^{\ln 2}$ but couldn't make further progress.

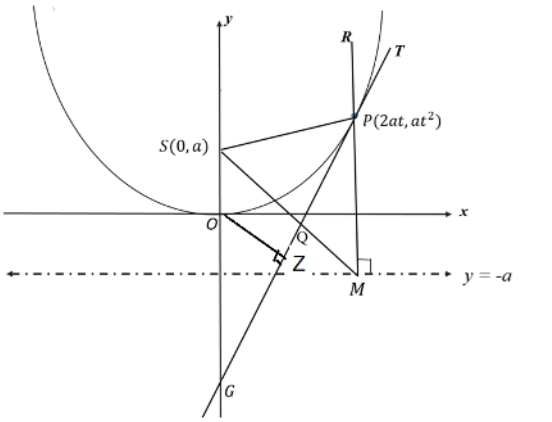
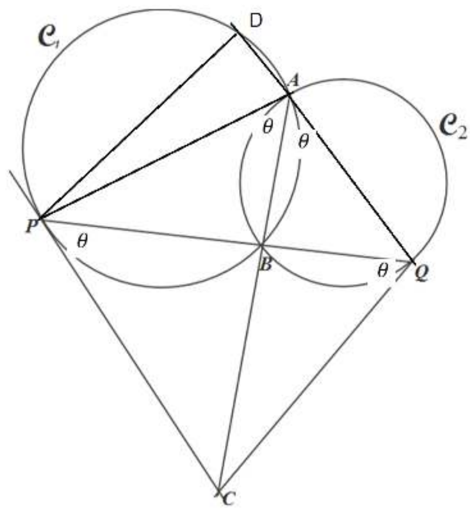
Better attempts showed good knowledge of indices and converted the integral correctly using

$\left(a^m\right)^n = \left(a^n\right)^m = a^{mn}$ .Very few achieved full marks. Students should note that x to a power can be integrated, as can e to a power, so they should have had a clear goal when doing this question.

c/ Many students didn't attempt this question or attempted it poorly. Thinking from first principles they should realise the only way to remove sine was to get $\frac{\sin\frac{1}{x}}{\frac{1}{x}}$ this is why dividing by x^2 correctly or showing sine in this form was awarded a mark.

The second part of this question required a deep knowledge of limits including splitting limits and changing what the limit was approaching in order to turn the sine part of the limit into 1. This proved too challenging for all but a few students.

Question 14			
a) (i)	It is the focus-directrix definition of the parabola (ie. the distance of a point P(x,y) from the fixed point S(0,a) equals the distance from the fixed line y = -a, the directrix.	1 mark: correct explanation	Using the distance formula is not explaining why!
(ii)	$tx - y = at^2$. Gradient of the tangent = t S(0, a) , M(2at, -a) Gradient of SM = $\frac{2a}{0-2at} = \frac{-1}{t}$ $m_{PQ} \times m_{SM} = -1$ Hence $PQ \perp SM$	1 mark: correct answer from correct working 1 mark: calculates at least one of the gradients correctly and attempts to find the product	Some did not see the efficient way to solve this problem
(iii)	$\angle QPM = \angle QPS$ (SP = PM from (i) – (equal angles opposite equal sides in isosceles triangles) and $PQ \perp SM$. PQ is common. $\Delta PQS \equiv \Delta PQM\ (RHS)$ $\therefore \angle SPQ = \angle QPM$ $(corresponding\ angles\ in\ congruent\ triangles)$ Let $\angle RPT = \propto$ $\angle RPT = \angle QPM$ (vertically opposite angles are equal) $\therefore \angle RPT = \angle SPQ$	2 marks Proves triangles are congruent with correct reasoning and concludes ∴ $\angle RPT = \angle SPQ$ 1 mark Incomplete reasoning but still correct proof structure	Poorly done – insufficient reasoning and poor setting out
(iv)	$\angle PMS = \angle MSG = \theta$ ($PM \| SG$, alternate angles in parallel lines are equal) Hence, $\angle PSQ = \angle QSG = \theta$ Also, $PG \perp SM$(from (ii) $\angle PMS = \angle MSG = \theta$ **1 mark** Vertices are bisected and diagonals are perpendicular to each other. Hence, SPMG is a rhombus. SP = PG Alternatively, SP = $\sqrt{(2at)^2 + (at^2 - a)^2}$ $= a\sqrt{4t^2 + t^4 - 2t^2 + 1}$ $= a\sqrt{(t^2+1)^2}$ $= a(t^2+1)$ $tx - y = at^2$. Let $x = 0$, Hence $y = at^2$ SG = $a + at^2 = a(t^2+1)$ SP = SG	1 mark: proves that SQ bisects $\angle PSG$ 1 mark: establishes that SPMG is a rhombus. Hence , SP = PG OR 1 mark each for finding the lengths of SP and SG	Quite well done – most chose to use the distance formula

(iv)	 Equation of OZ is $y = \frac{-1}{t}x \implies t = \frac{-x}{y}$ **1 mark** Equation of PG is $tx - y = at^2$ Solving simultaneously, $\left(\frac{-x}{y}\right)x - y = a\left(\frac{-x}{y}\right)^2$ Ie. $-x^2y - y^3 = ax^2$ $\therefore\ y^3 + x^2y + ax^2 = 0$ is the locus of *Z*.	2 marks: correct simplification 1 mark: makes *t* the subject and significant progress towards simplifying	Very poorly done – many students went in circles Find locus means eliminate *t*
b) (i)	 $CP^2 = CB \times CA$ (square of the length of the tangent = product of secants. Similarly, $CQ^2 = CB \times CA$ Hence, $CP^2 = CQ^2$ Ie. CP = CQ	1 mark: states the theorem of product of secants and proves the result.	Very few remembered this theorem! Many suggested that they were equal because the y were tangents – despite the fact that it was to two different circles!
(ii)	Let $\angle CPB = \theta$ $\therefore \angle CPB = \angle CQB = \theta$ (CP = CQ from (i)) **1 mark** $\angle CPB = \angle PAB = \theta$ (angle between tangent and radius equals angle in the alternate segment) **1 mark** $ie. \angle PAC = \angle PQC = \theta$ Angles in the same segment are equal, hence C, P, A, Q are concyclic. **1 mark**	1 mark: for each of the results as shown	Very poorly set out – all students should revise circle geometry proofs!

(iii)	1. $\angle CQB = \angle CAB = \theta$ (angle between tangent and radius equals angle in the alternate segment) 2. $\angle QAB = \angle DPB = \theta$ (BADP is a cyclic quadrilateral. Exterior angle equals opposite interior angle) $ie. \angle DPB = \angle BPC = \theta$. PB bisects $\angle DPC$	2 marks: correct proof Award 1 mark: reasons statement 1 or 2.	Very poorly set out – all students should revise circle geometry proofs!