2019

Topics
Integral calculus (2U)
Trigonometry (2U)
Polynomials (3U)
Integration by substitution (3U)

Centre Number

Student Number

Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Year 12 Mathematics Ext 1

26/2/2019

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations in the examination books provided

Total Marks:40

	Question 1	/12
	Question 2	/14
	Question 3	/14
	Total	**/40**

This question paper must not be removed from the examination room.
This assessment task constitutes 20% of the course.

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Question 1 (12 marks) Use the Question 1 Writing Booklet.

(a) If α, β and γ are the roots of the equation $2x^3 - 3x + 1 = 0$.
What is the value of:

(i) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$ **2**

(ii) $\alpha^2+\beta^2+\gamma^2$ **2**

(b) Taking $x = 0.5$ radians as the first approximation, use Newton's method to find a second approximation to the root of $2x - 4\cos 3x = 0$. Give your answer to 2 decimal places. **3**

(c) Evaluate **3**

$$\int_0^{\frac{\pi}{4}} \sin x \cos^3 x \; dx$$

(d) Evaluate **2**

$$\lim_{x\to 0} \frac{\tan 3x}{x}$$

End of Question 1

Question 2 (14 marks) Use the Question 2 Writing Booklet.

(a) Evaluate $\int_0^{\frac{\pi}{3}} \sin^2 2x\, dx$ **3**

(b) Find $\int x\sqrt{4-x}\ dx$ using the substitution $u = 4 - x$ **3**

(c) Use the substitution $x = 4tan\theta$ to find $\int \frac{dx}{\left(x^2+16\right)^{\frac{3}{2}}}$ **4**

(d) (i) Differentiate $x\sin x$ **2**

(ii) Hence find $\int x\cos x\, dx$ **2**

End of Question 2

Question 3 (14 marks) Use the Question 3 Writing Booklet.

(a) Consider the curves $y = \sin x$ and $y = \cos 2x$ for $-\pi \le x \le \pi$.

(i) Show that $x = \dfrac{\pi}{6}$ is a root of $\sin x = \cos 2x$ **1**

(ii) Calculate the exact area of the region bounded by the curves **3**

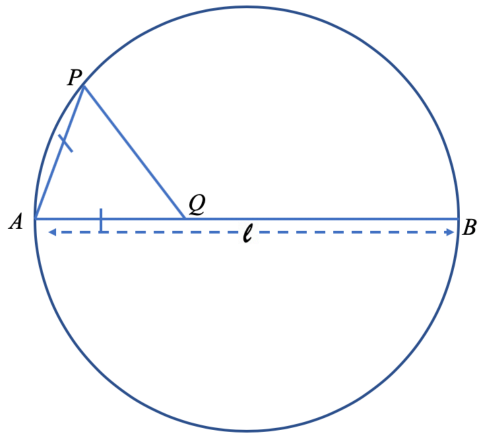
$y = sinx$ and $y = \cos 2x$ for $0 \le x \le \dfrac{\pi}{2}$

(b) (i) Divide the polynomial $f(x) = 2x^4 - 10x^3 + 12x^2 + 2x - 3$ by $g(x) = x^2 - 3x + 1$. **2**

(ii) Hence write $f(x) = g(x)q(x) + r(x)$, where $q(x)$ and $r(x)$ are polynomials and $r(x)$ has degree less than 2. **1**

(iii) Hence, deduce that $f(x)$ and $g(x)$ have no zeros in common. **2**

(c)



In the diagram above, P is a point on the circle with diameter $AB = \ell$. The point Q lies on the diameter such that $AP = AQ$. Let $\angle PAQ = \theta$ and let S be the area of ΔPQA.

(i) Show that **2**

$$S = \frac{\ell^2}{2}\cos^2\theta \sin\theta$$

(ii) Find the maximum area of ΔPQA as P moves along the circumference of the circle. **3**

END OF EXAMINATION

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End of Question 2

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$y = sinx$ and $y = \cos 2x$ for $0 \le x \le \dfrac{\pi}{2}$

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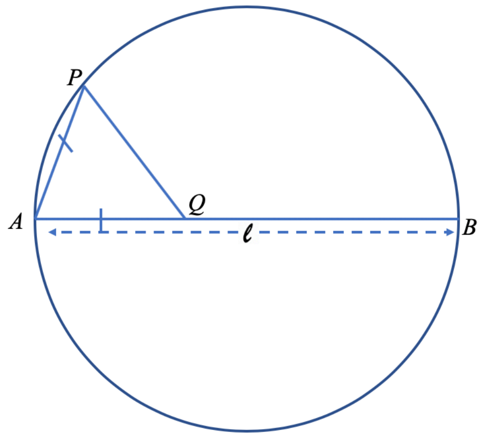

In the diagram above, P is a point on the circle with diameter $AB = \ell$. The point Q lies on the diameter such that $AP = AQ$. Let $\angle PAQ = \theta$ and let S be the area of ΔPQA.

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(ii) Find the maximum area of ΔPQA as P moves along the circumference of the circle. **3**

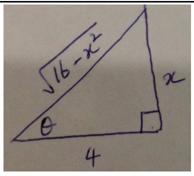
END OF EXAMINATION

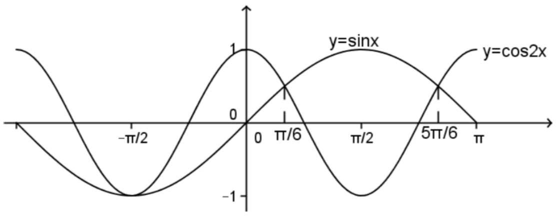
2019 Extension 1 Maths Solutions Term 2 Exam (Feb)

QUESTION 1

(a)(i)	$2x^3 - 3x + 1 = 0$ $\alpha + \beta + \gamma = 0.$ (Coefficient of $x^2 = 0$) $\alpha\beta + \beta\gamma + \alpha\gamma = -\dfrac{3}{2}$ $\alpha\beta\gamma = -\dfrac{1}{2}$ $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ $= \frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha\beta\gamma}$ =. $\frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$	1 mark for relation between roots and coefficients 1 mark: Manipulates the expression and evaluates correctly	
(a)(ii)	$\alpha^2 + \beta^2 + \gamma^2$ $= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$ $= 0 - 2 \times -\frac{3}{2} = 3$	2 marks: Correct answer from correct working 1 mark: correct formula, but minor error in evaluation	
(b)	$f(x) = 2x - 4\,cos3x$ $f'(x) = 2 + 12sin3x$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ $x_1 = 0.5 - \dfrac{2(0.5) - 4\cos(3 \times 0.5)}{2 + 12sin(3 \times 0.5)}$ = 0.44867…. = 0.45 (2d.p)	2 marks: correct answer from correct working 1 mark: correct formula with minor error in derivative or evaluation	
(c)	$\int_0^{\frac{\pi}{4}} \sin x\, cos^3 x\ dx$ $= -\int_0^{\frac{\pi}{4}} -\sin x\, cos^3 x\ dx$ $= -\left[\frac{\cos^4 x}{4}\right]_0^{\frac{\pi}{4}} = \frac{-1}{4}[\left(\frac{1}{\sqrt{2}}\right)^4 - 1]$ $= \frac{-1}{4}\left[\frac{1}{4} - 1\right] = \frac{3}{16}$	3 marks correct answer from correct working 2 marks: correct integration, minor error in evaluation 1 mark: correct integral	
(d)	$\lim\limits_{x\to 0}\dfrac{\tan 3x}{x} = \lim\limits_{x\to 0}\dfrac{\tan 3x}{3x} \times \dfrac{3x}{x}$ $= 3\lim\limits_{x\to 0}\dfrac{\tan 3x}{3x}$ $= 3$	2 marks: correct answer from correct working 1 mark: shows that $\lim\limits_{x\to 0}\frac{tanx}{x} = 1$	

	QUESTION 2		
(a)	$\int_0^{\frac{\pi}{3}} \sin^2 2x dx = \int_0^{\frac{\pi}{3}} \frac{1-\cos 4x}{2} \; dx$ $= \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{3}}$ $= \frac{1}{2}\left[\frac{\pi}{3} - \frac{\sin\frac{4\pi}{3}}{4} - (0 - \frac{\sin 0}{4})\right]$ $= \frac{\pi}{6} + \frac{\sqrt{3}}{16}$	3 marks: correct answer from correct working 2 marks: correct integral minor error in substitution 1 mark: correct expression for $\sin^2 2x$	
(b)	$\int x\sqrt{4-x}\,dx$ $u = 4 - x \quad \Rightarrow \quad x = 4 - u$ $dx = -du$ $\int x\sqrt{4-x}\,dx = -\int (4-u)\sqrt{u}\,du$ $= \int u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\,du$ $= \frac{2}{5}\,u^{\frac{5}{2}} - 4 \times \frac{2}{3}u^{\frac{3}{2}} + C$ $= u^{\frac{3}{2}}\left(\frac{2}{5}\,u - \frac{8}{3}\right) + C$ $= (4-x)\sqrt{4-x}\left(\frac{2}{5}(4-x) - \frac{8}{3}\right) + C$ 1 mark $= -\frac{2}{15}(4-x)\sqrt{4-x}\big(20 - 3(4-x)\big) + C$ $= \frac{2}{15}(4-x)\sqrt{4-x}(8+3x) + C$	1 mark: correctly converts the integrand in terms of *u* 1 mark: correct integration 1 mark: Expresses the answer in terms of *x*	
(c)	$\int \frac{dx}{(x^2+16)^{\frac{3}{2}}}$ Let $x = 4tan\theta$ Then $dx = 4\sec^2\theta\; d\theta$ 1 mark $\int \frac{dx}{(x^2+16)^{\frac{3}{2}}} = \int \frac{4\sec^2\theta\; d\theta}{(16tan^2\theta+16)^{\frac{3}{2}}}$ $=. \int \frac{4\sec^2\theta\; d\theta}{((16sec^2\theta)^{\frac{3}{2}}}$ 1 mark $= \int \frac{4\sec^2\theta\; d\theta}{64\sec^3\theta}$ $= \int \frac{d\theta}{16sec\theta}$ $= \int \frac{1}{16}\; cos\theta\; d\theta.$ 1 mark $= \frac{1}{16}\; sin\theta + C.$ 1 mark	1 mark: converts all components of the integrand in terms of θ 1 mark: expresses the integrand in the simplest form 1 mark: correct integration 1 mark: Converts the result back to *x*.	

	$sin\theta = \frac{x}{\sqrt{16-x^2}}$ $\int \frac{dx}{(x^2+16)^{\frac{3}{2}}} = \frac{x}{16\sqrt{16-x^2}} + C$		
(d)(i)	$\frac{d}{dx}(xsinx) = xcosx + sinx$	1 mark: for each part	
(ii)	Hence $\int xcosx + \sin x\, dx = xsinx + C$ $\therefore \int xcosx\, dx - cosx = xsinx + C$ $\therefore \int xcosx\, dx = xsinx + cosx + C$	1 mark: shows the antiderivative statement from their answer in (i) 1 mark: integrates the integrable part ($sinx$) and hence, finds $\int xcosx\, dx$	

	Question 3		
(i)	$\sin\frac{\pi}{6} = \frac{1}{2}$ $\cos\left(2 \times \frac{\pi}{6}\right) = \frac{1}{2}$ $x = \frac{\pi}{6}$ is a root.	1 mark: $x = \frac{\pi}{6}$ is satisfied by both equations	
(ii)	 Area = $\int_0^{\frac{\pi}{6}} cos2x - sinx\, dx +$ $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} sinx - \ cos2xdx$ $= \left[\frac{sin2x}{2} + cosx\right]_0^{\frac{\pi}{6}} +$ $\left[-cosx - \frac{sin2x}{2} +\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} - 0 - 1\right) + \left(0 - 0 + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right)$ $= \frac{3\sqrt{3}}{2} - 1\ unit^2$	1 mark: correct graphs (rough sketch is sufficient) 1 mark: correct expressions for the area formula 1 mark: correct evaluation	
(b)(i)	$2x^2 \ - 4x \quad - 2$ $x^2 - 3x + 1 \overline{)\, 2x^4 \ - 10x^3 \ + 12x^2 \ + 2x \ - 3}$ $2x^4 \ - \ 6x^3 \ + \ 2x^2$ $- \ 4x^3 \ + 10x^2 \ + 2x$ $-4x^3 \ + 12x^2 \ - 4x$ $- \ 2x^2 \ + 6x \ - \ 3$ $- \ 2x^2 \ + 6x \ - \ 2$ -1 $\begin{array}{r\|rrrrr} & 2 & -10 & 12 & 2 & -3 \\ 3 & & 6 & -12 & -6 & 3 \\ -1 & & & -2 & 4 & 2 \\ \hline & 2 & -4 & -2 & 0 & -1 \end{array}$	2 marks: correct division. (Students can choose to do synthetic division) 1 mark: Minor error	

<table>
<tr>
<td>(ii)</td>
<td>$2x^4 - 10x^3 + 12x^2 + 2x - 3 =$
$(x^2 - 3x + 1)(2x^2 \ - 4x \quad - 2\ ) - 1,$
$Deg\ r(x) < 2$</td>
<td>1 mark: Correct form of division transformation given</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>If $f(x)$ and $g(x)$ have a common zero, say α.
Hence, $f(\alpha) = g(\alpha) = 0$
But, $f(\alpha) = g(\alpha).q(\alpha) + r(\alpha)$
$0 = 0 \times q(\alpha) + r(\alpha)$
$\Longrightarrow r(\alpha) = 0$
But $r(\alpha) = 1 \neq 0.$
Hence, $f(x)$ and $g(x)$ have no zeros in common.</td>
<td>2 marks: must show all reasoning and working to get 2 marks. (must prove conclusively using an analytic method)

1 mark: states $g(\alpha) = 0$ and attempts to prove $f(\alpha) = 0$</td>
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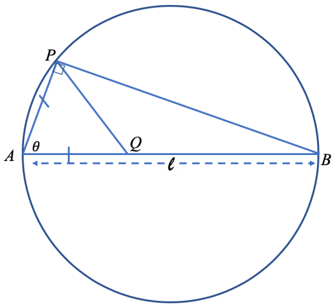
<tr>
<td>(d)(i)</td>
<td>

$cos\theta = \frac{AP}{l} \therefore AP = l\cos\theta$
$S = \frac{1}{2} \times l\ cos\theta \times lcos\theta sin\theta$
$= \frac{l^2}{2}\cos^2\theta sin\theta$ as required.
$= \frac{l^2}{2}(1 - \sin^2\theta)\ sin\theta$
$= \frac{l^2}{2}(sin\theta - \sin^3\theta)$</td>
<td>2 marks correct expression with working

1 mark: correct expression for S
$S = \frac{1}{2} \times l\ cos\theta \times lcos\theta sin\theta$</td>
<td></td>
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<tr>
<td>(ii)</td>
<td>$\frac{ds}{d\theta} = \frac{l^2}{2}(cos\theta - 3\sin^2\theta cos\theta)$
$= \frac{l^2}{2}cos\theta(1 - 3\sin^2\theta)$
$\frac{ds}{d\theta} = 0$ when $cos\theta = 0$ or $sin\theta = \pm\frac{1}{\sqrt{3}}$
$Since\ 0 < \theta < 90°$
$sin\theta = \frac{1}{\sqrt{3}}$ is the only possible solution.
<table>
<tr><td>$sin\theta$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{\sqrt{3}}$</td><td>$\frac{3}{4}$</td></tr>
<tr><td>$\frac{ds}{d\theta}$</td><td>$+ve$</td><td>0</td><td>$-ve$</td></tr>
</table>
$\therefore$ when $sin\theta = \frac{1}{\sqrt{3}}$, S is a maximum.
$S = \frac{l^2}{2}\left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right) = \frac{l^2}{2}\left(\frac{3-1}{3\sqrt{3}}\right) = \frac{l^2}{3\sqrt{3}}$
$=. \frac{\sqrt{3}l^2}{9}$ is the maximum area</td>
<td>3 marks: correct answer with correct working

2 marks: Correct solution $sin\theta = \frac{1}{\sqrt{3}}$

And verification of max by a table. No calculation of S

1 mark correct derivative and identifying $\frac{ds}{d\theta} = 0$ for $cos\theta = 0$ or $sin\theta = \pm\frac{1}{\sqrt{3}}$</td>
<td></td>
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</table>