Killara HIGH SCHOOL

MATHEMATICS DEPARTMENT

2020

Year 12

HSC Mathematics Extension 1

TASK 2

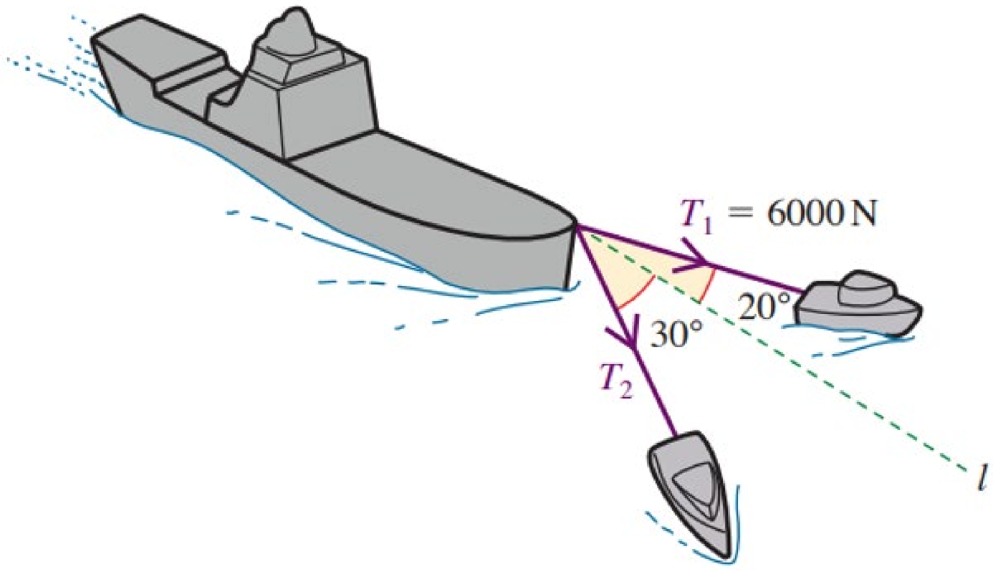
Date issued: 26/02/20 **Date due:16/03/2020**

LEARNING OUTCOMES:

- Applies concepts and techniques involving vectors and projectiles to solve problems (ME12-2)
- Chooses and uses appropriate technology to solve problems in a range of contexts (ME12-6)
- Evaluates and justifies conclusions, communicating in a range of contexts (ME12-7)

Section I 40%	Part A – Modelling	/42
	Part B – Geometry	/23
	Part C – Modelling Orbits	/10
Section II 60%	35-minute examination	

1 A ship is being towed by two tugs. Each tug exerts forces on the ship as indicated. There is also a drag force on the ship.

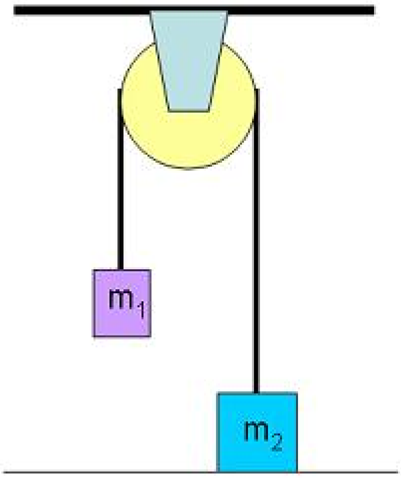


(a) Write down the components of the tensions in the towing cables along and perpendicular to the line of motion, $\boldsymbol{l}$, of the ship. 2

(b) There is no resultant force perpendicular to the line $\boldsymbol{l}$. Find $\boldsymbol{T}_2$ 2

(c) The ship is traveling at a constant speed. Find the magnitude of the drag force. 2

2 Two blocks with masses m_1 and m_2 are connected by a cord that passes over a massless and frictionless pulley.



Find the mass of m_2 if m_1=2kg and is accelerating upwards at 1.3ms^{-2} 3

....

....

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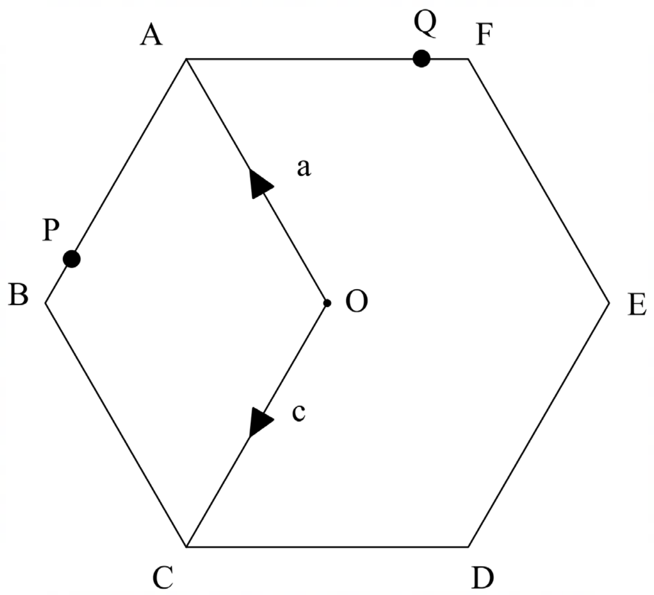
....

....

3 $ABCDEF$ is a regular hexagon, centre O. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$. P is a point on BA such that $BP:BA = 1:3$ and Q is a point on AF such that $AQ:QF = 2:1$.



(a) Express, as simply as possible, in terms of **a** and/or **c**.

(i) $\overrightarrow{AF}$ 1

....

....

(ii) $\overrightarrow{OQ}$ 2

....

....

....

....

(iii) $\overrightarrow{PA}$ 1

....

....

(iv) $\overrightarrow{BF}$ 1

....

....

(v) $\overrightarrow{PQ}$ 1

....

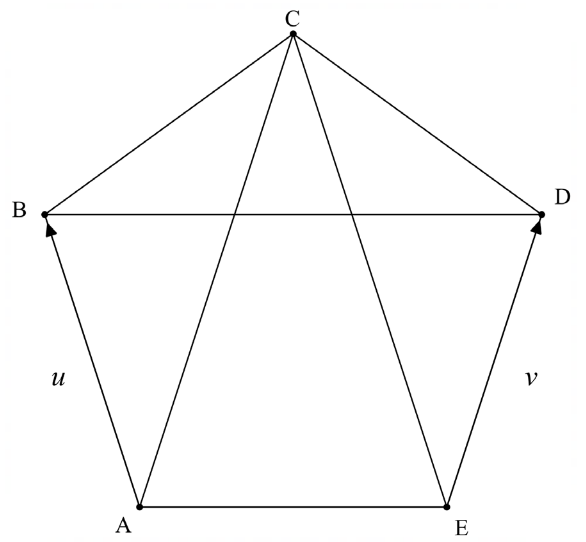
....

2

(b) Show that ΔAPQ is similar to ΔAPQ.

(c) PQ intersects OA at the point R. Find $\overrightarrow{PR}$. 2

4 Consider the following pentagon $ABCDE$ with 5 equal lengths.



Let $\boldsymbol{u} = \overrightarrow{AB}$ and $\boldsymbol{v} = \overrightarrow{ED}$.

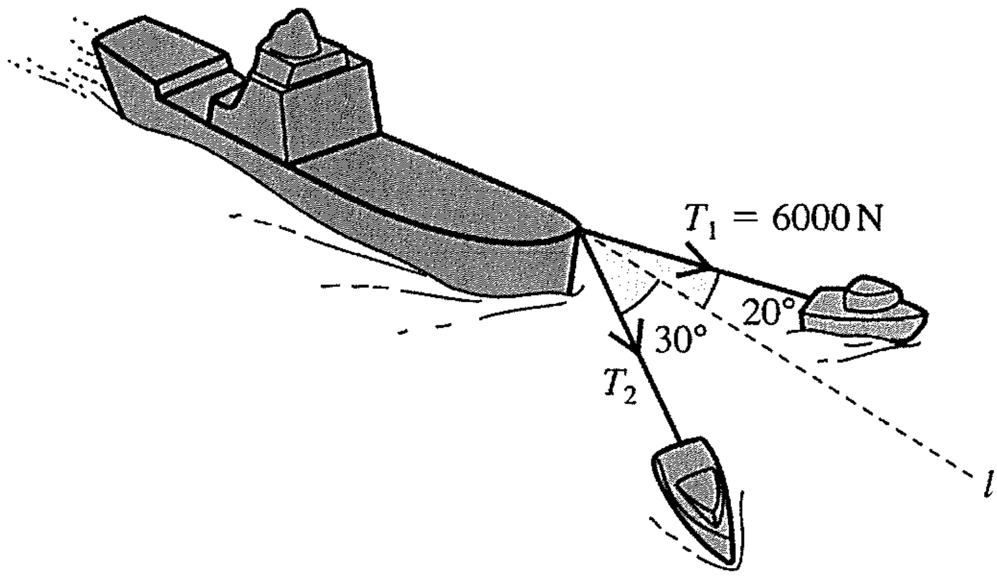
Observe that $\overrightarrow{BD}$ is parallel to $\overrightarrow{AE}$ and hence $\overrightarrow{BD} = \lambda\left(\overrightarrow{AE}\right)$ for some $\lambda > 1$.

Similarly $\overrightarrow{AC} = \lambda\left(\overrightarrow{ED}\right) = \lambda\boldsymbol{v}$ and $\overrightarrow{EC} = \lambda\left(\overrightarrow{AB}\right) = \lambda\boldsymbol{u}$.

(a) By considering the triangle ACE prove that $\overrightarrow{AE} = \lambda(\boldsymbol{v} - \boldsymbol{u})$. 1

(b) Find a different expression for $\overrightarrow{AE}$ in terms of $\lambda, \boldsymbol{v}$ and $\boldsymbol{u}$ and hence find the exact value of λ, expressing your answer in surd form. 3

A ship is being towed by two tugs. Each tug exerts forces on the ship as indicated. There is also a drag force on the ship.



(i) Write down the components of the tensions in the towing cables along and perpendicular to the line of motion, l, of the ship.

① Missing a component or small error

$$T_1 = \begin{pmatrix} 6000\cos 20° \\ 6000\sin 20° \end{pmatrix} \qquad T_2 = \begin{pmatrix} T_2\cos 30° \\ -T_2\sin 30° \end{pmatrix}$$

② All components in some form (does not have to be column vectors)

(Defining the vector as $\begin{pmatrix} \text{Along} \\ \text{perpendicular} \end{pmatrix}$)

(ii) There is no resultant force perpendicular to the line l. Find T_2

$F_{\text{Perpendicular}} = 6000\sin 20° - T_2\sin 30° = 0$ — ① mark

$6000\sin 20° = T_2 \sin 30°$

$T_2 = \dfrac{6000\sin 20°}{\sin 30°}$

$= 4104\,\text{N}$ ← ① mark

(ii) The ship is traveling at a constant speed. Find the magnitude of the drag force.

$F_{\text{Along}} = 6000\cos 20° + 4104\cos 30° - D = 0$

1 mark

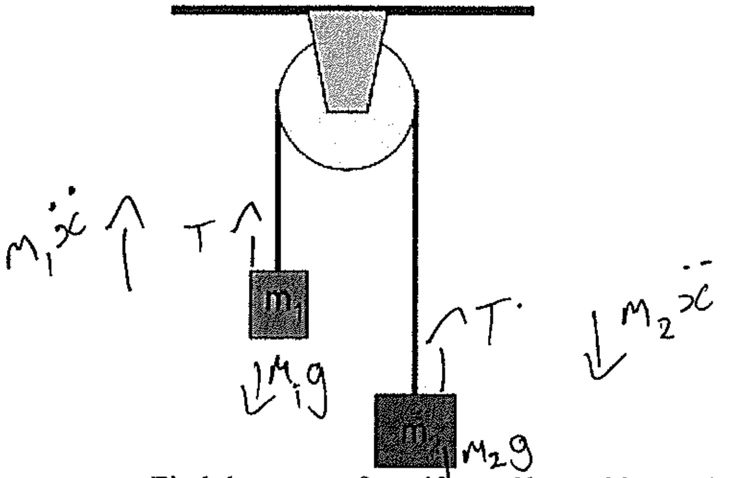
$D = 6000\cos 20° + 4104\cos 30°$

$= 9193\,\text{N}$ (9192N)

1 mark

If they use 4104

Two blocks with masses m_1 and m_2 are connected by a cord that passes over a massless and frictionless pulley.



① mark setting up vectors / vector diagrams

Find the mass of m_2 if m_1=2kg and is accelerating upwards at $1.3ms^{-2}$

$2\times 1.3 = T - m_1 g$ $\qquad$ $m_2 \times 1.3 = m_2\times 9.8 - T$ — ②

$= T - 2\times 9.8$ — ①

① + ②

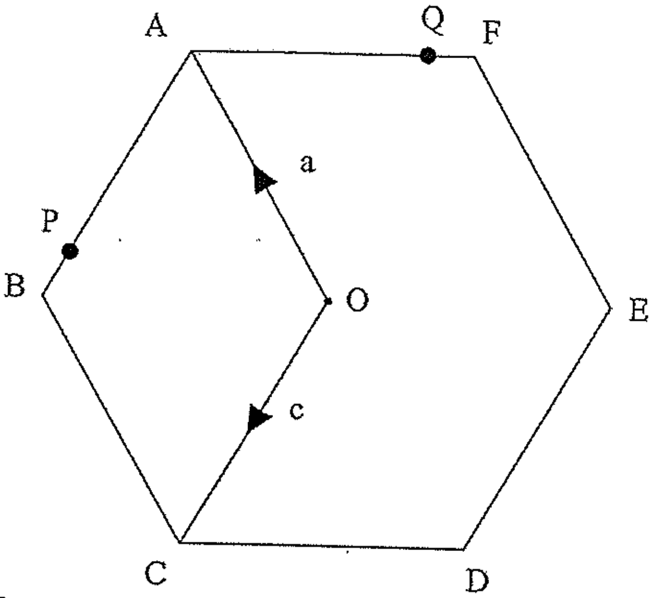
$2\times 1.3 + m_2\times 1.3 = m_2\times 9.8 - 2\times 9.8$ ← 1 mark sum of forces in some form

$2\times 11.1 = 8.5 m_2$

$m_2 = \dfrac{22.2}{8.5}$ ← 1 mark answer.

≈ 2.6 (1.D.P.)

3 $ABCDEF$ is a regular hexagon, centre O. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is a point on BA such that $BP : BA = 1 : 3$ and Q is a point on AF such that $AQ : QF = 2 : 1$.



(a) Express, as simply as possible, in terms of **a** and/or **c**.

(i) $\overrightarrow{AF} = \overrightarrow{AO} + \overrightarrow{OF} = -\underset{\sim}{a} - \underset{\sim}{c}$ 1

(ii) $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ} = \underset{\sim}{a} - \frac{2}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{c}$ 2

$= \frac{1}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{c}$

(iii) $\overrightarrow{PA} = \frac{2}{3}\overrightarrow{BA} = -\frac{2}{3}\underset{\sim}{c}$ 1

(iv) $\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF}$ 1

$= -\underset{\sim}{c} - \underset{\sim}{a} - \underset{\sim}{c}$

$= -2\underset{\sim}{c} - \underset{\sim}{a}$

(v) $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = -\frac{2}{3}\underset{\sim}{c} - \frac{2}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{c}$ 1

$= -\frac{2}{3}\underset{\sim}{a} - \frac{4}{3}\underset{\sim}{c}$

2

(b) Show that ΔAPQ is similar to ΔAPQ.

$\overrightarrow{PQ} = \frac{2}{3}(-2\underset{\sim}{c} - \underset{\sim}{a}) = \frac{2}{3}\overrightarrow{BF}$

$\therefore PQ \parallel BF$

$\angle PAQ = \angle BAF$ (common $\angle$)

$\angle APQ = \angle ABF$ (corresponding angles in parallel lines)

$\therefore \Delta APQ \parallel\!\mid \Delta ABF$ (equiangular)

(c) PQ intersects OA at the point R. Find $\overrightarrow{PR}$. 2

$\overrightarrow{PR} = \overrightarrow{PA} + \overrightarrow{AR} = -\frac{2}{3}\underset{\sim}{c} + \frac{2}{3}\left(-\frac{1}{2}\underset{\sim}{a}\right)$

$= -\frac{1}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{c}$

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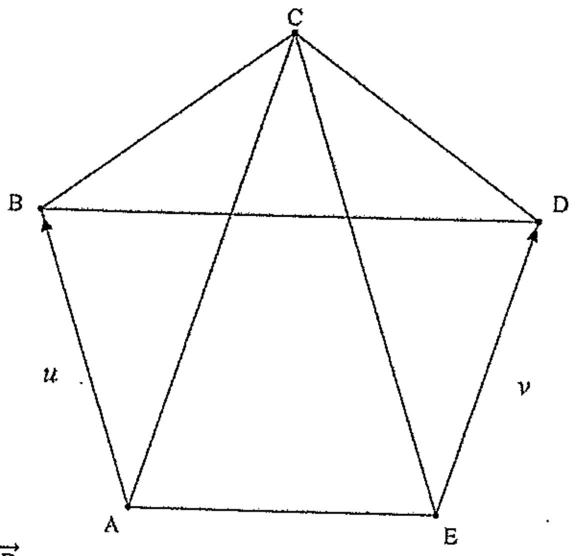

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(a) By considering the triangle ACE prove that $\overrightarrow{AE} = \lambda(\boldsymbol{v} - \boldsymbol{u})$. 1

$\overrightarrow{AE} = \lambda \underset{\sim}{v} - \lambda \underset{\sim}{u} = \lambda(\underset{\sim}{v} - \underset{\sim}{u})$ 1

(b) Find a different expression for $\overrightarrow{AE}$ in terms of λ, $\boldsymbol{v}$ and $\boldsymbol{u}$ and hence find the exact value of λ, expressing your answer in surd form. 3

$\overrightarrow{AE} = \underset{\sim}{u} + \lambda \overrightarrow{AE} \cancel{+} - \underset{\sim}{v}$ — 1 for expression

$\overrightarrow{AE}\, \cancel{4}(1-\lambda) = \underset{\sim}{u} - \underset{\sim}{v}$

$\overrightarrow{AE} = \dfrac{1}{1-\lambda}(\underset{\sim}{u} - \underset{\sim}{v})$

$\therefore \lambda = \dfrac{1}{\lambda - 1}$ — 1 for λ quadratic

$\lambda^2 - \lambda - 1 = 0$

$(\lambda > 0)$

$\lambda = \dfrac{\sqrt{5}+1}{2}$ (Golden Ratio ϕ) — 1 for positive solution