Killara HIGH SCHOOL

MATHEMATICS DEPARTMENT

2021

Year 12

HSC Mathematics Extension 1

TASK 2

Date issued: 26/02/20 **Date due:16/03/2020**

LEARNING OUTCOMES:

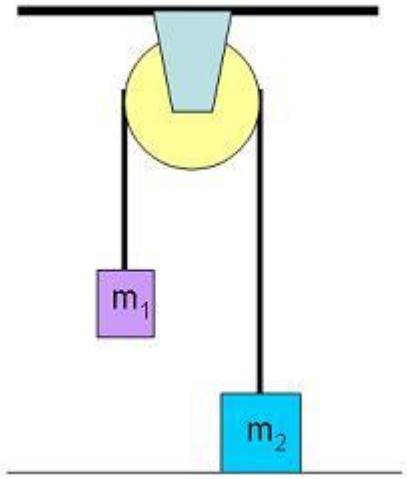
- Applies concepts and techniques involving vectors and projectiles to solve problems (ME12-2)
- Chooses and uses appropriate technology to solve problems in a range of contexts (ME12-6)
- Evaluates and justifies conclusions, communicating in a range of contexts (ME12-7)

Section I 40%	Part A – Modelling	/42
	Part B – Geometry	/23
	Part C – Modelling Orbits	/10
Section II 60%	45-minute examination	

1 Three boys are pulling a heavy trolley by means of three ropes. The boy in the middle is exerting a pull of 100N. The other boys, whose ropes both make an angle of 30^o with the central rope, are pulling with forces of $50\sqrt{3}$N and $100\sqrt{3}$N. Draw a diagram representing these forces and find the magnitude of the resultant pull on the trolley.

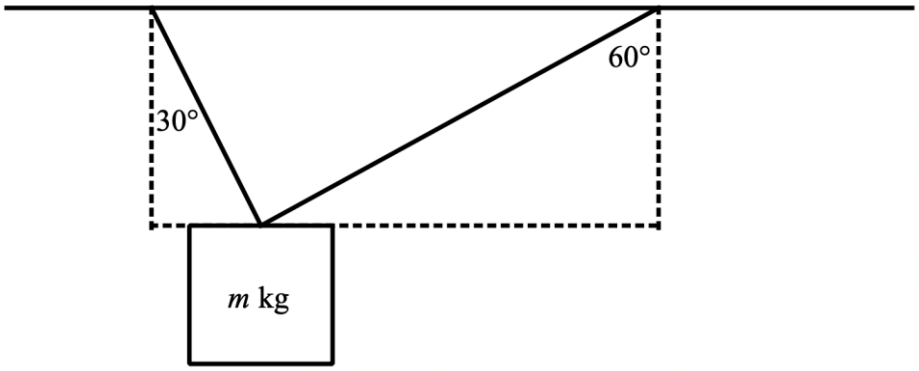
4

2 Two blocks with masses m_1 and m_2 are connected by a cord that passes over a massless and frictionless pulley.



Find the mass of m_2 if m_1=3kg and is accelerating upwards at 2.5ms^{-2} 3

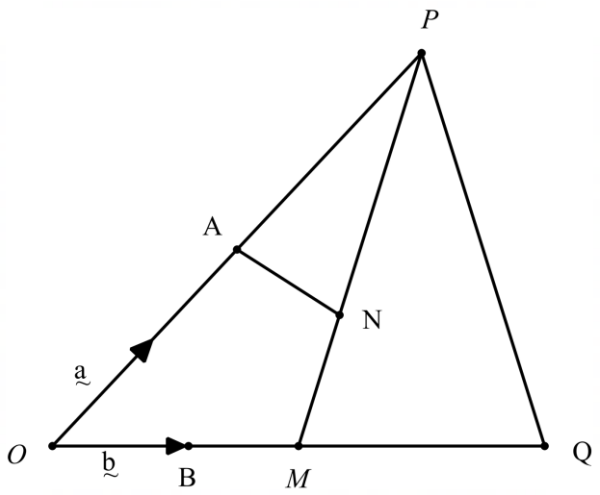
3 A flowerpot of mass m kilograms is held in equilibrium by two light ropes, both of which are connected to a ceiling. The first rope makes an angle of 30^o to the vertical and has tension T_1 newtons. The second rope makes an angle of 60^o to the vertical and has a tension T_2 newtons.



(a) Show that $T_2 = \frac{T_1}{\sqrt{3}}$. 1

(b) The first rope is strong, but the second rope will break if the tension in it exceeds 98N. Find the maximum value of m for which the flowerpot will remain in equilibrium. 3

4 In the diagram, $OA = \frac{1}{2}AP$ and $OB = \frac{1}{4}OQ$. M is the midpoint of OQ and $MN = \frac{1}{4}NP$. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.



(a) Express, as simply as possible, in terms of **a** and/or **b**.

(i) $\overrightarrow{OP}$ 1

(ii) $\overrightarrow{OQ}$ 1

(iii) $\overrightarrow{AQ}$ 1

(iv) $\overrightarrow{MN}$ 1

(v) $\overrightarrow{AN}$ 1

(b) Prove that AN, when produced, will pass through Q. 1

(c) Find the ratio $AN:NQ$ 1

(d) Calculate the value of $\frac{Area\ of\ \Delta OPQ}{Area\ of\ \Delta PMQ}$ 2

This is the end of the examination.

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Answers

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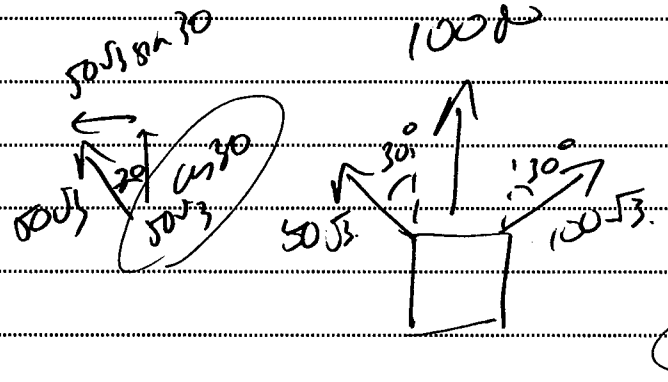
Section I 40%	Part A – Modelling	/42
	Part B – Geometry	/23
	Part C – Modelling Orbits	/10
Section II 60%	45-minute examination	

↳ 35 – 40 mins
(Last question!).

1 Three boys are pulling a heavy trolley by means of three ropes. The boy in the middle is exerting a pull of 100N. The other boys, whose ropes both make an angle of 30° with the central rope, are pulling with forces of $50\sqrt{3}$N and $100\sqrt{3}$N. Draw a diagram representing these forces and find the magnitude of the resultant pull on the trolley. 4

① diagram



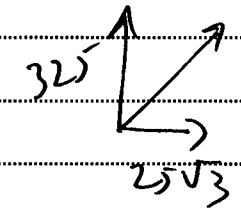
½
½
½

$\uparrow \Rightarrow 100 + 50\sqrt{3}\,\frac{\sqrt{3}}{2} + 100\sqrt{3}\,\frac{\sqrt{3}}{2} = 325$

① for vertical (cos)

$\rightarrow \Rightarrow 100\sqrt{3}\times\frac{1}{2} - 50\sqrt{3}\times\frac{1}{2} = 25\sqrt{3}$

① for horizontal (sin)



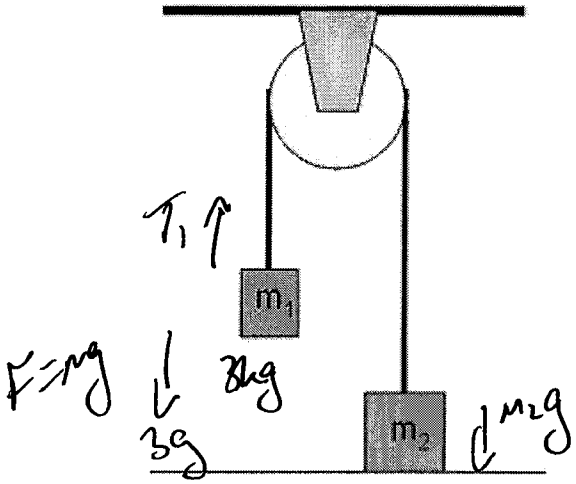
$\sqrt{325^2 + (25\sqrt{3})^2}$

$= 50\sqrt{43}$ N.

$= 327{\cdot}87$ N.

① for resultant

2 Two blocks with masses m_1 and m_2 are connected by a cord that passes over a massless and frictionless pulley.



① for $F=mg$
$=mg$.

& some work on diagram

Find the mass of m_2 if m_1=3kg and is accelerating upwards at $2.5ms^{-2}$ 3

$F-T-3g$ $\quad F=Mg$

$F=M_2g-3g$

$F=mg$

2.5×3.

$F_R=7.5$.

$F_2=W_2-T=m_2g$.

$F_2=M_2$

$F_1=T-W_1=m_1g$

$W_2-T=m_2g$.

$T-3g=2.5\times3$.

$m_2g-36.9=m_2\times2.5$.

$T=7.5+3g$

$m_2(g-2.5)=36.9$

$=36.9$.

$m_2=5.05$ kg ?

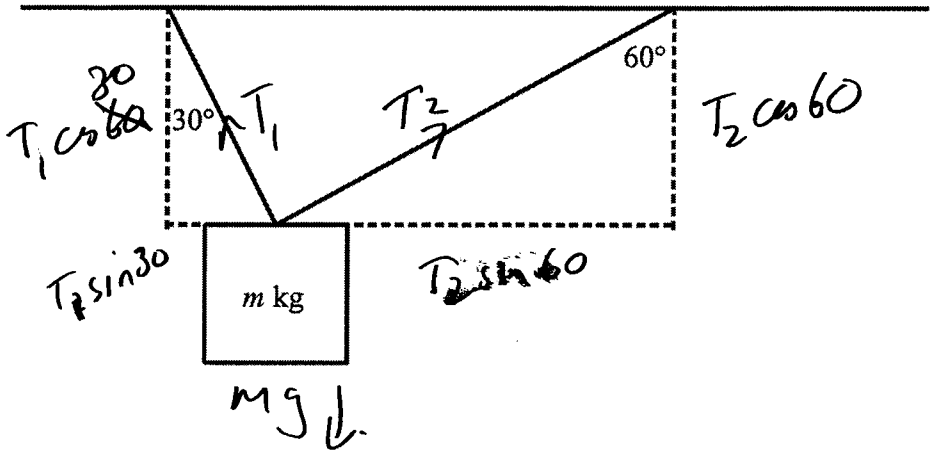
① for T.

① for m_2

37.5

w/10

3 A flowerpot of mass m kilograms is held in equilibrium by two light ropes, both of which are connected to a ceiling. The first rope makes an angle of 30^o to the vertical and has tension T_1 newtons. The second rope makes an angle of 60^o to the vertical and has a tension T_2 newtons.



(a) Show that $T_2 = \frac{T_1}{\sqrt{3}}$. 1

$T_1 \sin 30 = T_2 \sin 60$

$T_1/2 = T_2 \times \frac{\sqrt{3}}{2}$

$\sqrt{3} T_2 = T_1$

$T_2 = \frac{T_1}{\sqrt{3}}$ (1)

(b) The first rope is strong, but the second rope will break if the tension in it exceeds 98N. Find the maximum value of m for which the flowerpot will remain in equilibrium. 3

$T_1 \cos 30 + T_2 \cos 60 = mg$ (1) for statement

$\frac{\sqrt{3} T_1}{2} + \frac{T_2}{2} = mg$

$\sqrt{3} T_1 + T_2 = 2mg$

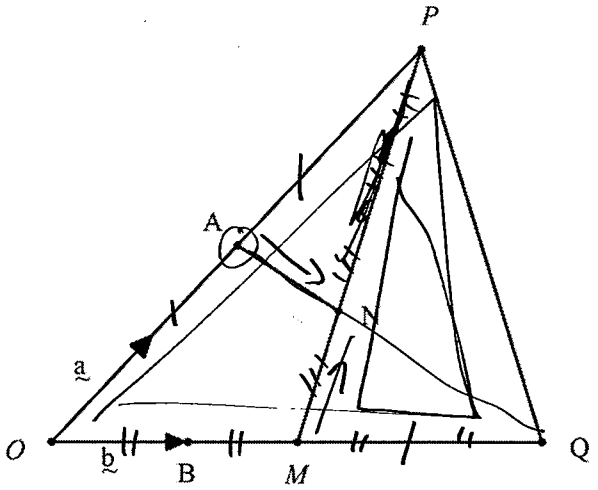
$3T_2 + T_2 = 2mg$ (1) for substitution

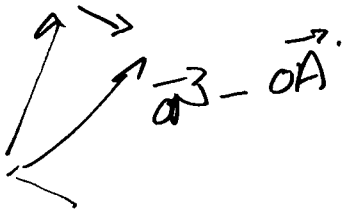
$4T_2 = 2mg$

$2 \times 98 = mg$

$m = 20$ kg (1) for result

4 In the diagram, $OA = \frac{1}{2}AP$ and $OB = \frac{1}{4}OQ$. M is the midpoint of OQ and $MN = \frac{1}{4}NP$. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.





(a) Express, as simply as possible, in terms of **a** and/or **b**.

(i) $\overrightarrow{OP}$ 1

~~2a~~ 3a ✓1

(ii) $\overrightarrow{OQ}$ 1

4b ✓1

(iii) $\overrightarrow{AQ}$ 4b − a 1

✓1

(iv) $\overrightarrow{MN}$ $\overrightarrow{MP} = ~~2b~~\ 3a - 2b$ $\overrightarrow{MN} = \frac{3a - 2b}{5}$ 1

✓1

(v) $\overrightarrow{AN}$ $\frac{2b + 3a - 2b}{5} - a$ 1

$\frac{10b + 3a - 2b - 5a}{5} = \frac{8}{5}b - \frac{2}{5}a$ ✓1