

Student Name

2022 YEAR 12

Mathematics Extension 1

Task 2

Q	Marks
1	/12
2	/10
3	/14
Total	/36

8th March, 2022

General

Instructions:

- Reading time – 5 minutes
 - Working time – 55 minutes
 - Write using blue or black pen
 - NESA approved calculators may be used
 - Show relevant mathematical reasoning and/or calculations
 - No white-out may be used
-

This question paper must not be removed from the examination room.

This validation test constitutes 60% of the assessment task.

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Question 1 (12 marks) Answer each question in a NEW booklet.

- (a) Find vector $\vec{b} = \begin{pmatrix} p \\ q \end{pmatrix}$, such that

2

$$\text{proj}_{\vec{b}} \vec{a} = \frac{87}{117} \vec{b}$$

$$\text{proj}_{\vec{b}} \vec{c} = \frac{60}{117} \vec{b}$$

given $\vec{a} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$, and $\vec{c} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$.

- (b) Consider the vector $\vec{a} = \vec{i} + \sqrt{3}\vec{j}$.

- (i) Find the unit vector in the direction of $\vec{a}$.

1

- (ii) Find the acute angle that $\vec{a}$ makes with the positive direction of the x -axis.

1

- (iii) Given that vector $\vec{b} = m\vec{i} - 2\vec{j}$ is perpendicular to $\vec{a}$, find the value of m .

1

- (c) A ship needs to travel on a bearing of 240° . The ship is sailing at a constant 60 km/h. However, there is a current from the north with a constant speed of 9 km/h.

2

On what constant bearing, to the nearest degree, is the ship sailing?

Examination continues on next page

- (d) A crate of mass 25.0 kg is pulled across the floor with a force of 168 N. The force makes an angle of 15.0° with the horizontal. Assume there is no friction between the crate and the floor and that the crate does not lift. You may assume that $g = 10\text{ms}^{-2}$.

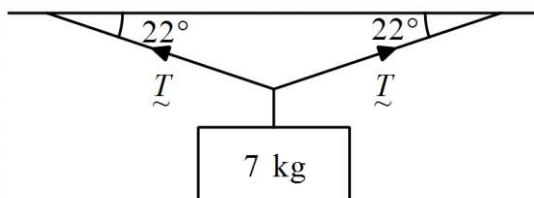
Find the:

- | | | |
|-------|---|---|
| (i) | Resultant force acting on the crate, correct to the nearest Newton. | 2 |
| (ii) | The acceleration of the crate, correct one decimal place. | 1 |
| (iii) | The magnitude of the normal force, correct to the nearest Newton. | 2 |

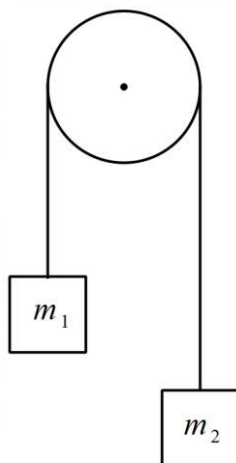
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Question 2 (10 marks) Answer each question in a NEW booklet.

- (a) A mass of 7 kg is suspended from a horizontal ceiling by two inextensible light strings of equal length. Each string is at 22° to the ceiling. Find the tension T in each string, correct to the nearest Newton. You may assume $g = 10 \text{ ms}^{-2}$. 2



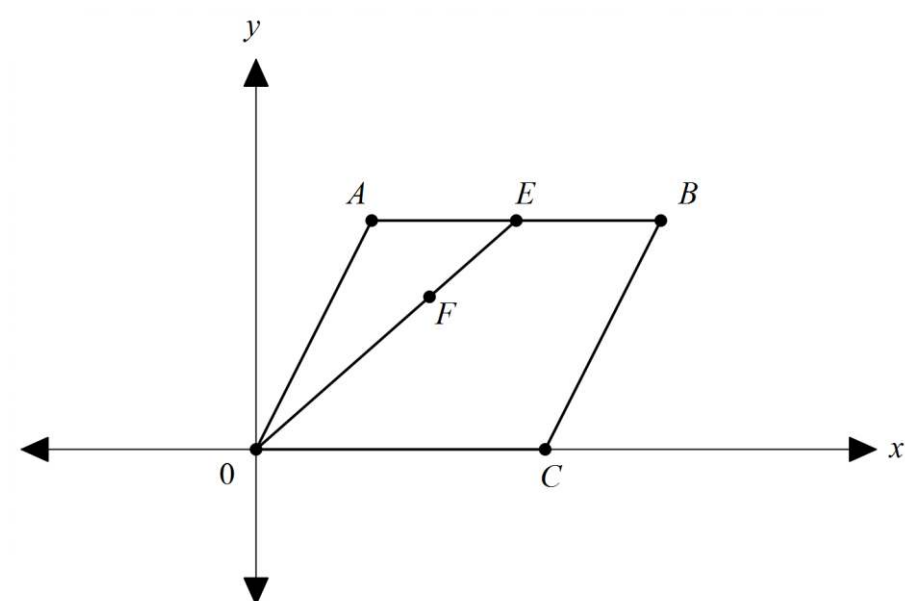
- (b) Two masses are connected by a light inextensible string over a smooth pulley.



- (i) Write equations for the tension in the string, assuming $m_1 < m_2$. 2
- (ii) The second mass has an acceleration of $0.25g$. Find the ratio $m_2 : m_1$. 2

Examination continues on next page

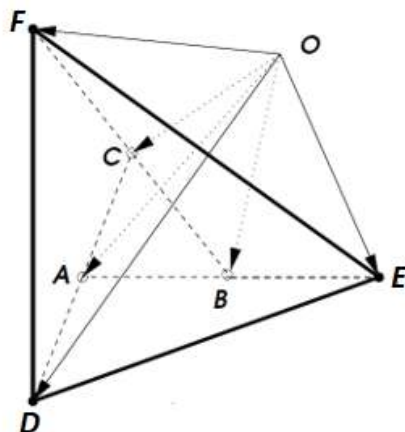
- (c) $OABC$ is a parallelogram. $\vec{OA} = \vec{a}$, $\vec{AB} = \vec{b}$, E is the midpoint of AB . F lies on OE such that $OE = 3FE$. 4



Prove that A, F and C are collinear.

Question 3 (14 marks) Answer each question in a NEW booklet.

- (a) Given a $\triangle ABC$, line CA is extended to D , so that $CA = AD$, line AB is extended to E so that $AB = BE$ and line BC is extended to F so that $BC = CF$. The new $\triangle DEF$ is formed. Let the position vectors $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, $\vec{OC} = \vec{c}$, $\vec{OD} = \vec{d}$, $\vec{OE} = \vec{e}$ and $\vec{OF} = \vec{f}$.

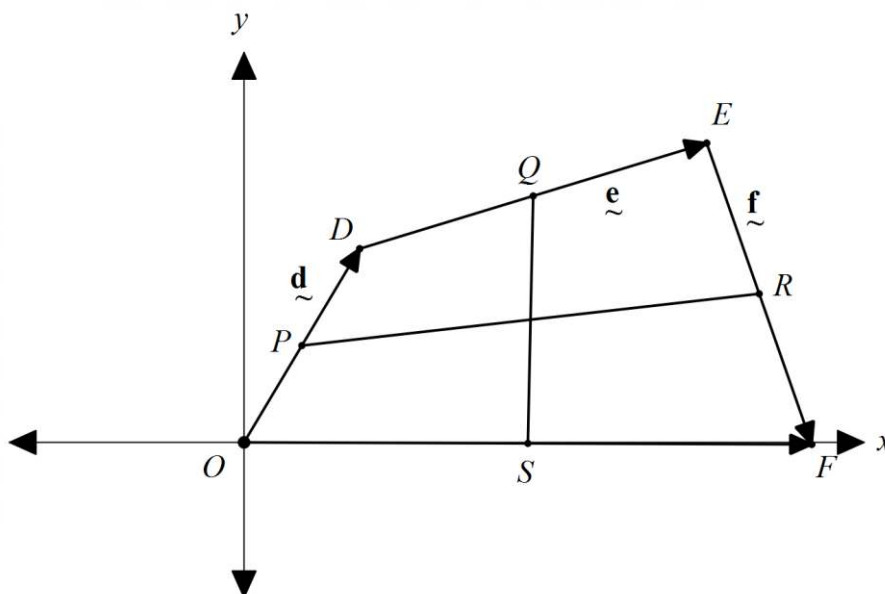


- | | | |
|-------|---|---|
| (i) | Find $\vec{a}$ in terms of $\vec{c}$ and $\vec{d}$. | 2 |
| (ii) | Find $\vec{b}$ in terms of $\vec{a}$ and $\vec{e}$. | 1 |
| (iii) | Find $\vec{c}$ in terms of $\vec{b}$ and $\vec{f}$. | 1 |
| (iv) | Then find $\vec{b}$ in terms of $\vec{d}$, $\vec{e}$ and $\vec{f}$. | 2 |

Examination continues on next page

- (b) $ODEF$ is a quadrilateral. $\overrightarrow{OD} = \vec{d}$, $\overrightarrow{DE} = \vec{e}$, $\overrightarrow{EF} = \vec{f}$.

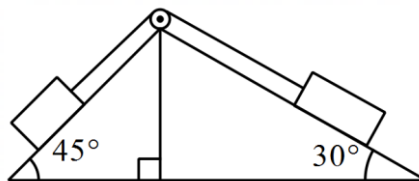
4



P, Q, R, S are the midpoints of OD, DE, EF and FO , respectively.

Show that the intervals joining the midpoints of opposite sides of a quadrilateral bisect one another.

- (c) Masses of 30 kg and 50 kg on adjacent smooth inclined planes are connected by a light inextensible string over a frictionless pulley, as shown in the diagram below. The 50 kg mass is on a plane inclined at 30° to the horizontal and the 30 kg mass is on a plane at 45° to the horizontal.



- (i) Write an expression in terms of g involving the tension in the string for each mass. 2
- (ii) Find the magnitude of the acceleration of the masses in terms of g and state which is descending. 2

End of Examination

2022 Ext Validation task Solutions

Friday, 10 March 2023 12:07 AM



Student Name

2022 YEAR 12

Mathematics Extension 1

Task 2

8th March, 2022

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Question 1 (12 marks) Answer each question in a NEW booklet.

- (a) Find vector $\underline{b} = \begin{pmatrix} p \\ q \end{pmatrix}$, such that

2

$$\text{proj}_{\underline{b}} \underline{a} = \frac{87}{117} \underline{b}$$

$$\text{proj}_{\underline{b}} \underline{c} = \frac{60}{117} \underline{b}$$

given $\underline{a} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$, and $\underline{c} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$.

$$a) \text{proj}_{\underline{b}} \underline{a} = \frac{\underline{a} \cdot \underline{b}}{\underline{b} \cdot \underline{b}} \cdot \underline{b} = \frac{87}{117} \underline{b}. \quad \text{proj}_{\underline{b}} \underline{c} = \frac{\underline{c} \cdot \underline{b}}{\underline{b} \cdot \underline{b}} \cdot \underline{b} = \frac{60}{117} \underline{b}.$$

$$\frac{\begin{pmatrix} 4 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} p \\ q \end{pmatrix}}{\begin{pmatrix} p \\ q \end{pmatrix} \cdot \begin{pmatrix} p \\ q \end{pmatrix}} = \frac{87}{117}.$$

$$\frac{\begin{pmatrix} -2 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} p \\ q \end{pmatrix}}{\begin{pmatrix} p \\ q \end{pmatrix} \cdot \begin{pmatrix} p \\ q \end{pmatrix}} = \frac{60}{117}.$$

$$\frac{4p+7q}{p^2+q^2} = \frac{87}{117} \quad (1)$$

$$\frac{-2p+8q}{p^2+q^2} = \frac{60}{117} \quad (2)$$

$$(1) \div (2)$$

$$\frac{4p+7q}{-2p+8q} = \frac{87}{60}.$$

$$p^2+q^2 = 117.$$

$$p^2 + \frac{9}{4}p^2 = 117.$$

$$240p + 420q = -174p + 696q.$$

$$\frac{13}{4}p^2 = 117$$

$$p^2 = 36.$$

$$p = \pm 6$$

$$q = 9 \text{ or } -9.$$

$$414p = 276q.$$

$$\frac{3}{2}p = q.$$

$$\therefore 4p+7q = 87.$$

$$\therefore p, q \text{ positive}$$

$$\therefore \underline{b} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}.$$

$$\begin{cases} p=6 \\ q=9 \end{cases} \quad \begin{cases} p=-6 \\ q=-9 \end{cases}$$

(b) Consider the vector $\underline{a} = \underline{i} + \sqrt{3}\underline{j}$.

(i) Find the unit vector in the direction of $\underline{a}$.

1

(ii) Find the acute angle that $\underline{a}$ makes with the positive direction of the x -axis.

1

(iii) Given that vector $\underline{b} = m\underline{i} - 2\underline{j}$ is perpendicular to $\underline{a}$, find the value of m .

1

$$b) \quad \sqrt{1 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2.$$

$$(i) \quad \frac{\mathbf{a}}{|\mathbf{a}|} = \left(\frac{\frac{1}{2}}{\frac{2}{2}} \right) = \frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j}.$$

$$(ii) \quad \begin{array}{c} \mathbf{a} \\ \nearrow \\ \text{ } \end{array} \quad \begin{array}{c} \sqrt{3} \\ \text{ } \end{array} \quad \begin{array}{c} \tan \theta = \frac{\sqrt{3}}{1} \\ \theta = 60^\circ \end{array}$$

$$(iii) \quad \begin{array}{c} \mathbf{b} \\ \nwarrow \\ \text{ } \end{array} \quad \begin{array}{c} 1 \\ \text{ } \end{array} \quad \perp \quad \begin{array}{c} \mathbf{a} \\ \nearrow \\ \text{ } \end{array}$$

$$\therefore \mathbf{b} \cdot \mathbf{a} = 0$$

$$\begin{pmatrix} m \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} = 0$$

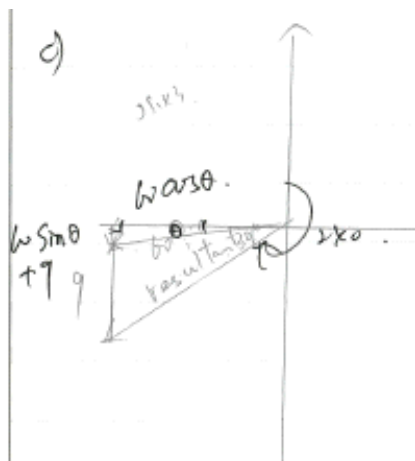
$$m - 2\sqrt{3} = 0$$

$$m = 2\sqrt{3}$$

- (c) A ship needs to travel on a bearing of 240° . The ship is sailing at a constant 60 km/h. However, there is a current from the north with a constant speed of 9 km/h.

2

On what constant bearing, to the nearest degree, is the ship sailing?



$$\frac{2}{\sqrt{3}} \cdot V_{\text{ship}} = V_{\text{resultant}} - V_{\text{current}}$$

=

$$V_{\text{ship}} + V_{\text{current}} = V_{\text{resultant}}$$

$$\tan 30^\circ = \frac{60 \cos \theta}{60 \sin \theta + 9}$$

$$60\sqrt{3}\sin\theta + 9\sqrt{3} = 60\cos\theta.$$

$$9\sqrt{3} = 60\cos\theta - 60\sqrt{3}\sin\theta$$

$$\frac{3 \times 9\sqrt{3}}{40\sqrt{3}} = \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$$

$$\frac{3}{40}\sqrt{3} = \cos(60^\circ + \theta)$$

$$60^\circ + \theta = \cos^{-1}\left(\frac{3}{40}\sqrt{3}\right)$$

$$\theta = \cos^{-1}\left(\frac{3}{40}\sqrt{3}\right) - 60^\circ.$$

$$= 22.536^\circ.$$

$$\text{Bearing} = 270^\circ - \theta$$

$$= 247.46^\circ.$$

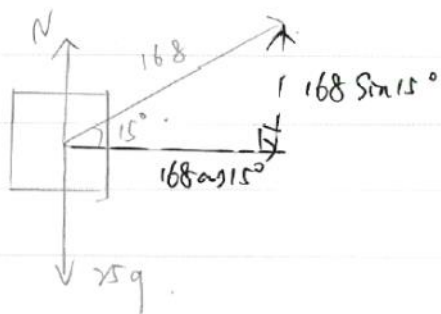
$$= 247^\circ \text{ T}^\Delta.$$

- (d) A crate of mass 25.0 kg is pulled across the floor with a force of 168 N. The force makes an angle of 15.0° with the horizontal. Assume there is no friction between the crate and the floor and that the crate does not lift. You may assume that $g = 10\text{ms}^{-2}$.

Find the:

- | | | |
|-------|---|---|
| (i) | Resultant force acting on the crate, correct to the nearest Newton. | 2 |
| (ii) | The acceleration of the crate, correct one decimal place. | 1 |
| (iii) | The magnitude of the normal force, correct to the nearest Newton. | 2 |

d).



i) $f_{\text{resultant}} = 168 \cos 15^\circ$

$= 162.28 \text{ N}$

$= 162 \text{ N}$

Towards right.

Along horizontal.

ii) $F = ma$

$a = \frac{F}{m} = \frac{168 \cos 15^\circ}{25} = 6.47$
 $= 6.5 \text{ m/s}^2 \text{ (1 d.p.)}$

iii). As there is no vertical acceleration,

$\Sigma F_{\text{vertical}} = 0$

$\therefore N + 168 \sin 15^\circ = 25g$

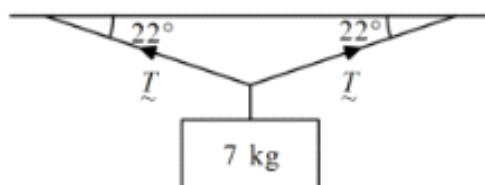
$N = 25g - 168 \sin 15^\circ$

$= 206.52 \text{ N}$

$= 207 \text{ N}$

Question 2 (10 marks) Answer each question in a NEW booklet.

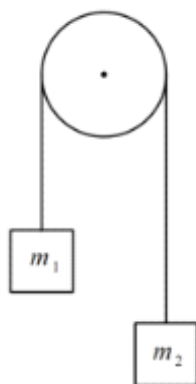
- (a) A mass of 7 kg is suspended from a horizontal ceiling by two inextensible light strings of equal length. Each string is at 22° to the ceiling. Find the tension T in each string, correct to the nearest Newton. You may assume $g = 10 \text{ ms}^{-2}$. 2



a) ② $2 \times 9.8 \sin 22^\circ = T$. $\because$ equilibrium
 $\therefore \Sigma f = 0$
 $|T| = \frac{7 \times 10}{2 \times \sin 22^\circ}$
 $= 93.43 \text{ N}$
 $= 93 \text{ N}$ ✓

③ $\cos 22^\circ T_1 = \cos 22^\circ T_2$
 $\therefore T_1 = T_2$

(b) Two masses are connected by a light inextensible string over a smooth pulley.



(i) Write equations for the tension in the string, assuming $m_1 < m_2$. 2

(ii) The second mass has an acceleration of $0.25g$. Find the ratio $m_2 : m_1$. 2

b) .

i) for m_2 .

$$m_2 a = m_2 g - T$$

$$T = m_2 (g - a)$$
 ✓

for m_1

~~$$T + m_1 a = m_1 g$$~~

$$T = m_1 (g - a)$$

$$m_1 a = T - m_1 g$$

$$T = m_1 (a + g)$$
 ✓

ii) . $a = 0.25g$ ↓ .

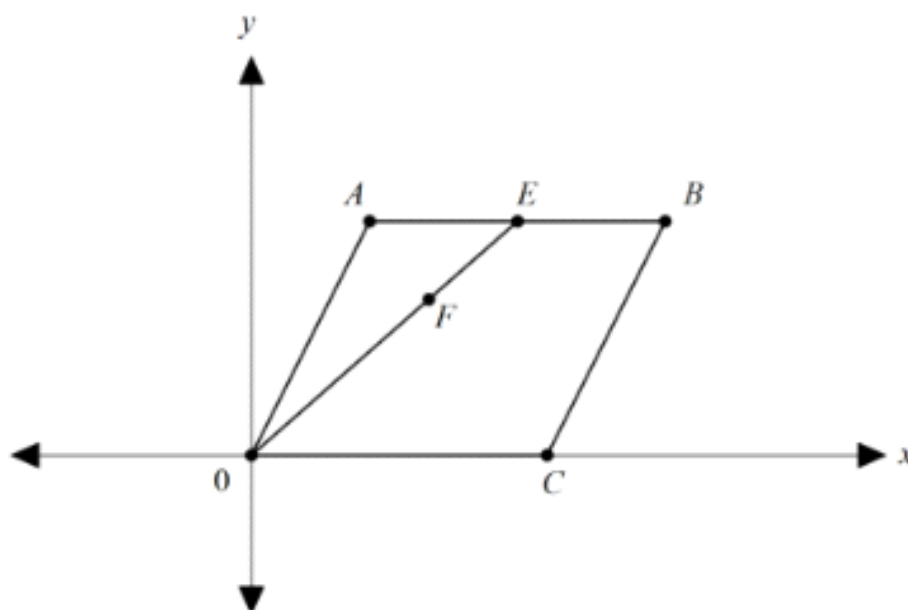
$$m_2 (g - 0.25g) = m_1 (0.25g + g)$$

$$0.75 m_2 = 1.25 m_1$$

$$\frac{3}{4} m_2 = \frac{5}{4} m_1$$

$$m_2 : m_1 = 5 : 3$$
 ✓

- (c) $OABC$ is a parallelogram. $\vec{OA} = \underline{a}$, $\vec{AB} = \underline{b}$, E is the midpoint of AB . F lies on OE such that $OE = 3FE$. 4



Prove that A, F and C are collinear.

c). As $OABC$ is parallelogram

$$\therefore \vec{AB} = \vec{OC} = \underline{b}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = \underline{b} - \underline{a}$$

$$\vec{OF} = \frac{2}{3} \vec{OE} = \frac{2}{3} (\vec{OA} + \frac{1}{2} \vec{AB})$$

$$= \frac{2}{3} \times (\underline{a} + \frac{1}{2} \underline{b})$$

$$= \frac{2}{3} \underline{a} + \frac{1}{3} \underline{b}$$

$$\vec{AF} = \vec{OF} - \vec{OA} = \frac{2}{3} \underline{a} + \frac{1}{3} \underline{b} - \underline{a}$$

$$= \frac{1}{3} \underline{b} - \frac{1}{3} \underline{a}$$

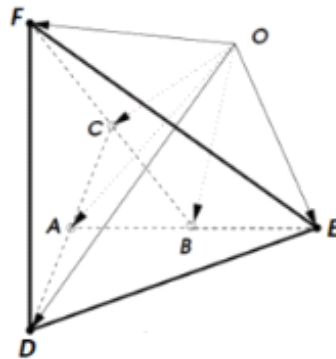
$$= \frac{1}{3} (\underline{b} - \underline{a})$$

$$\therefore \vec{AF} = \frac{1}{3} \vec{AC}$$

$\therefore A, F, C$ are collinear.

Question 3 (14 marks) Answer each question in a NEW booklet.

- (a) Given a $\triangle ABC$, line CA is extended to D , so that $CA = AD$, line AB is extended to E so that $AB = BE$ and line BC is extended to F so that $BC = CF$. The new $\triangle DEF$ is formed. Let the position vectors $\vec{OA} = \underline{a}$, $\vec{OB} = \underline{b}$, $\vec{OC} = \underline{c}$, $\vec{OD} = \underline{d}$, $\vec{OE} = \underline{e}$ and $\vec{OF} = \underline{f}$.



- | | | |
|-------|---|---|
| (i) | Find $\underline{a}$ in terms of $\underline{c}$ and $\underline{d}$. | 2 |
| (ii) | Find $\underline{b}$ in terms of $\underline{a}$ and $\underline{e}$. | 1 |
| (iii) | Find $\underline{c}$ in terms of $\underline{b}$ and $\underline{f}$. | 1 |
| (iv) | Then find $\underline{b}$ in terms of $\underline{d}$, $\underline{e}$ and $\underline{f}$. | 2 |

a) i) $\vec{CD} = \vec{OD} - \vec{OC}$
 $= \underline{d} - \underline{c}$

$\vec{CA} = \frac{1}{2} \vec{CD} = \frac{1}{2} (\underline{d} - \underline{c})$

$\underline{a} = \vec{OA} = \vec{OC} + \vec{CA}$
 $= \underline{c} + \frac{1}{2} \underline{d} - \frac{1}{2} \underline{c}$
 $= \frac{1}{2} (\underline{d} + \underline{c})$

ii) $\vec{AE} = \vec{OE} - \vec{OA}$
 $= \underline{e} - \underline{a}$

$\vec{AB} = \frac{1}{2} \vec{AE} = \frac{1}{2} (\underline{e} - \underline{a})$

$\underline{b} = \vec{OB} = \vec{OA} + \vec{AB}$
 $= \frac{1}{2} \underline{a} - \frac{1}{2} \underline{a} + \underline{a}$
 $= \frac{1}{2} (\underline{a} + \underline{e})$

$$\text{iii) } \vec{BP} = \vec{f} - \vec{b}.$$

$$\vec{BC} = \frac{1}{2}\vec{BF} = \frac{1}{2}\vec{f} - \frac{1}{2}\vec{b}$$

$$\vec{c} = \vec{OC} = \vec{OB} + \vec{BC} = \frac{1}{2}(\vec{b} + \vec{f}).$$

$$\text{iv) } \vec{b} = \frac{1}{2}(\vec{a} + \vec{c}) \quad (\text{from ii})$$

$$= \frac{1}{2} \times \frac{1}{2}(\vec{d} + \vec{e}) + \frac{1}{2}\vec{c} \quad (\text{from i})$$

$$= \frac{1}{4}\vec{d} + \frac{1}{2}\vec{e} + \frac{1}{4}\vec{c}.$$

$$= \frac{1}{4}\vec{d} + \frac{1}{2}\vec{e} + \frac{1}{4} \times \frac{1}{2}(\vec{b} + \vec{f}) \quad (\text{from iii})$$

$$= \frac{1}{4}\vec{d} + \frac{1}{2}\vec{e} + \frac{1}{8}\vec{b} + \frac{1}{8}\vec{f}.$$

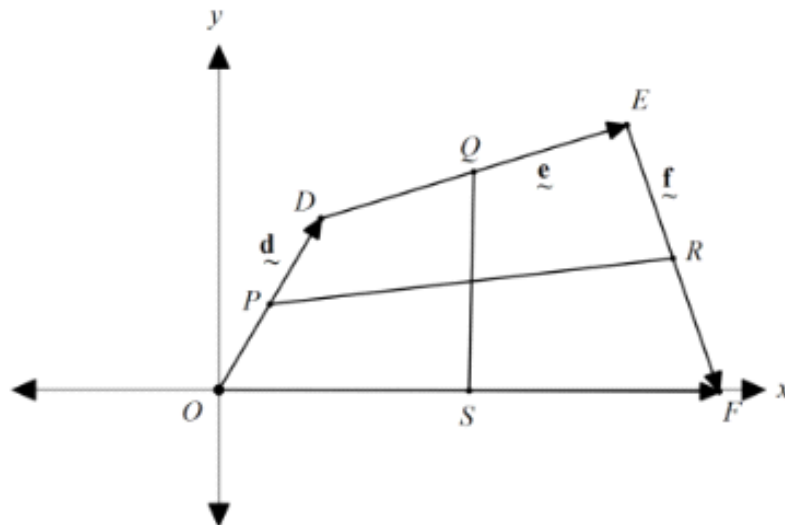
$$\frac{7}{8}\vec{b} = \frac{1}{4}\vec{d} + \frac{1}{2}\vec{e} + \frac{1}{8}\vec{f}.$$

$$\vec{b} = \frac{2}{7}\vec{d} + \frac{4}{7}\vec{e} + \frac{1}{7}\vec{f}.$$

$$= \frac{1}{7}(2\vec{d} + 4\vec{e} + \vec{f}).$$

(b) $ODEF$ is a quadrilateral. $\vec{OD} = \vec{d}$, $\vec{DE} = \vec{e}$, $\vec{EF} = \vec{f}$.

4



P, Q, R, S are the midpoints of OD, DE, EF and FO , respectively.

Show that the intervals joining the midpoints of opposite sides of a quadrilateral bisect one another.

$$b). \vec{OP} = \frac{1}{2}\vec{d}.$$

$$\vec{OR} = \vec{d} + \vec{e} + \frac{1}{2}\vec{f}.$$

$$\begin{aligned}\vec{PR} &= \vec{OR} - \vec{OP} \\ &= \vec{d} + \vec{e} + \frac{1}{2}\vec{f} - \frac{1}{2}\vec{d} \\ &= \frac{1}{2}\vec{d} + \frac{1}{2}\vec{f} + \vec{e}.\end{aligned}$$

$$\vec{OP} = \vec{d} + \vec{e} + \frac{1}{2}\vec{f}.$$

$$\vec{OS} = \frac{1}{2}\vec{OP} = \frac{1}{2}\vec{d} + \frac{1}{2}\vec{e} + \frac{1}{4}\vec{f}.$$

$$\vec{OA} = \vec{d} + \frac{1}{2}\vec{e}.$$

$$\begin{aligned}\vec{AS} &= \vec{OS} - \vec{OA} \\ &= \frac{1}{2}\vec{d} + \frac{1}{2}\vec{e} + \frac{1}{4}\vec{f} - \vec{d} - \frac{1}{2}\vec{e} \\ &= \frac{1}{2}(\frac{1}{2}\vec{f} - \vec{d}).\end{aligned}$$

Let the intersect point of PR, AS be A.

$$\vec{PA} = \lambda \vec{PR}.$$

$$\vec{OA} = \mu \vec{AS}.$$

$$\vec{PA} = \vec{OA} - \vec{OP}$$

$$= \vec{d} + \frac{1}{2}\vec{e} - \frac{1}{2}\vec{d}$$

$$= \frac{1}{2}\vec{e} + \frac{1}{2}\vec{d}.$$

$$\lambda(\frac{1}{2}\vec{d} + \frac{1}{2}\vec{f} + \vec{e})$$

$$\vec{PA} + \vec{AO} = \vec{PO}.$$

$$\lambda(\frac{1}{2}\vec{d} + \frac{1}{2}\vec{f} + \vec{e}) + \mu(\frac{1}{2}\vec{d} - \frac{1}{2}\vec{f}) = \frac{1}{2}\vec{e} + \frac{1}{2}\vec{d}.$$

$$\frac{1}{2}\lambda + \frac{1}{2}\mu = \frac{1}{2} \quad \frac{1}{2}\lambda - \frac{1}{2}\mu = 0.$$

$$\lambda + \mu = 1$$

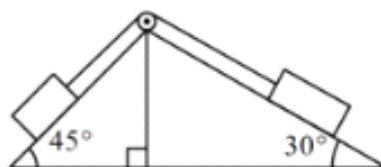
$$\lambda = \mu.$$

$$\therefore \lambda = \mu = \frac{1}{2}.$$

$$\therefore \vec{PA} = \frac{1}{2}\vec{PR} \quad \vec{OA} = \frac{1}{2}\vec{AS}.$$

$\therefore$ PR bisect, AS bisect each other

- (c) Masses of 30 kg and 50 kg on adjacent smooth inclined planes are connected by a light inextensible string over a frictionless pulley, as shown in the diagram below. The 50 kg mass is on a plane inclined at 30° to the horizontal and the 30 kg mass is on a plane at 45° to the horizontal.



- (i) Write an expression in terms of g involving the tension in the string for each mass. 2
- (ii) Find the magnitude of the acceleration of the masses in terms of g and state which is descending. 2

c) i) Assume $\boxed{50\text{kg}}$ is sliding down. $\boxed{30\text{kg}}$ is sliding upward.

$$30a = T - 30g \sin 45^\circ \quad (1) \quad 50a = 50g \sin 30^\circ - T \quad (2)$$

$$(1) \times 5: 150a = 5T - 150g \sin 45^\circ \quad (2) \times 3: 150a = 150g \sin 30^\circ - 3T$$

$$5T - 150g \sin 45^\circ = 150g \sin 30^\circ - 3T$$

$$8T = 150g (\sin 30^\circ + \sin 45^\circ)$$

$$T = \frac{150}{8} g (\sin 30^\circ + \sin 45^\circ)$$

$$= \frac{12+1}{8} \times \frac{150}{2} g$$

$$= \frac{75(12+1)}{8} g$$

ii) $\downarrow$ $\frac{1}{a}$ $(1) \quad 30a = T - 30g \sin 45^\circ$

$$30a = \frac{75\sqrt{2}+75}{8} g - 30 \sin 45^\circ g$$

$$= \left(\frac{75\sqrt{2}+75}{8} - 15\sqrt{2} \right) g$$

$$= \frac{75 - 45\sqrt{2}}{8} g$$

$$a = \frac{5-3\sqrt{2}}{16} g$$

As this a is positive. $5-3\sqrt{2} > 0$.

Therefore Assumption is true.

~~the~~ $\boxed{50\text{kg}}$ is descending (sliding down).