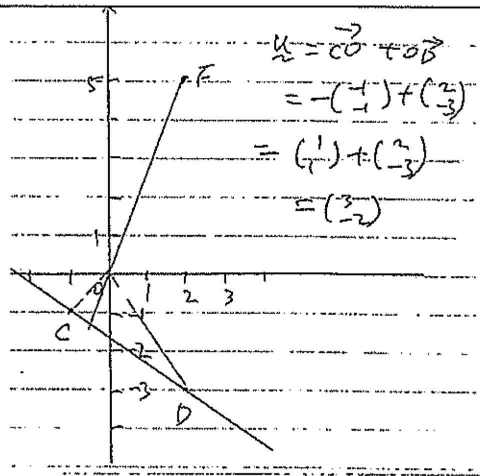
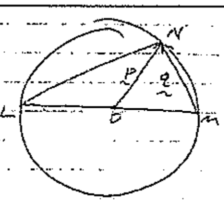
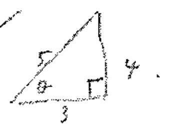
12 Ext 1 Task 2 Marking Scheme Question 1

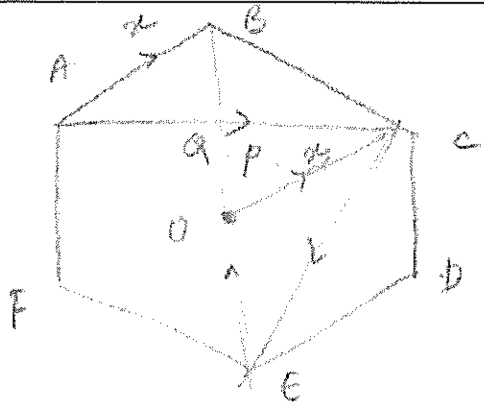
1a)	Q1 a) $\begin{cases} 2x+3=-3 & ① \\ 6-3y=-6 & ② \end{cases}$ from ①: $2x=-6$ $\therefore x=-3$ from ②: $12=3y$ $\therefore y=4$	1 mark: correctly calculates x 1 mark: Correctly calculates y	Quite a few students wrote 6 instead of −6 as vertical component of the resultant vector.
1b) (i)	 $\underset{\sim}{u} = \overrightarrow{CO} + \overrightarrow{OB}$ $= -\binom{-1}{-1} + \binom{2}{-3}$ $= \binom{1}{1} + \binom{2}{-3}$ $= \binom{3}{-2}$ b) i) $\hat{\underset{\sim}{u}} = \frac{\underset{\sim}{u}}{\lvert \underset{\sim}{u} \rvert}$ $= \frac{3\underset{\sim}{i} - 2\underset{\sim}{j}}{\sqrt{(2+1)^2+(3+1)^2}}$ $= \frac{3\underset{\sim}{i} - 2\underset{\sim}{j}}{\sqrt{9+4}}$ $= \frac{3\underset{\sim}{i} - 2\underset{\sim}{j}}{\sqrt{13}}$	2 marks : Correct answer from full working. 1 mark: correct calculation of $\lvert \underset{\sim}{u} \rvert$.	In general, don well. A few tried to calculate the unit vector of $\underset{\sim}{F}$ instead of the unit vector of $\underset{\sim}{u}$.
1b (ii)	$Proj_{\underset{\sim}{l}}\underset{\sim}{F} = \frac{\underset{\sim}{F}.\underset{\sim}{l}}{\lvert \underset{\sim}{l} \rvert^2}$ $= \frac{(2\underset{\sim}{i}+5\underset{\sim}{j}).(3\underset{\sim}{i}-2\underset{\sim}{j})}{\lvert \underset{\sim}{l} \rvert^2} \times (3\underset{\sim}{i} - 2\underset{\sim}{j})$ $= \frac{6-10}{13} \times (3\underset{\sim}{i} - 2\underset{\sim}{j})$ $= \frac{-4}{13} \times (3\underset{\sim}{i} - 2\underset{\sim}{j})$	2 marks: full correct working with correct answer. 1 mark off: incorrect working.	Quite a few students did not understand or misinterpreted what the question was really asking for.
1c)	Q1 c) $\overrightarrow{OM} = \overrightarrow{ON} + \overrightarrow{NM}$ $= \underset{\sim}{p} - \underset{\sim}{q}$ $\overrightarrow{LM} = 2(\underset{\sim}{p} - \underset{\sim}{q})$ $\overrightarrow{LN} = \overrightarrow{LM} + \overrightarrow{MN}$ $= 2(\underset{\sim}{p} - \underset{\sim}{q}) + \underset{\sim}{q}$ $= 2\underset{\sim}{p} - \underset{\sim}{q}$ 	2 marks: full correct working with correct answer.	A few students were treating $\underset{\sim}{p}$ and $\underset{\sim}{q}$ as magnitudes instead of vectors.

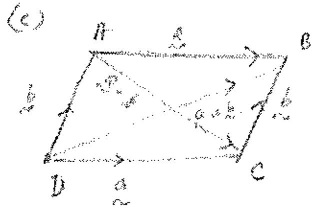
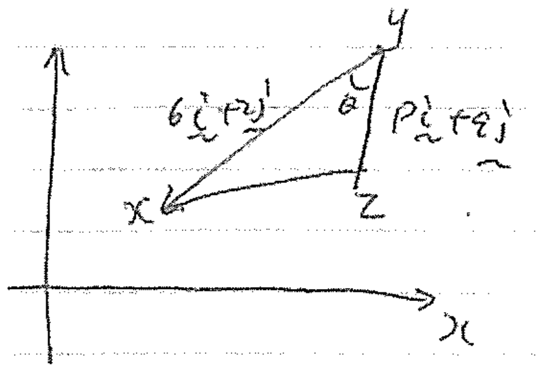
		1 mark for correct answer of $\overrightarrow{LM}$ or $\overrightarrow{OM}$ depending on which vector is used for the part of the calculation of $\overrightarrow{LN}$.	Quite a few got the vector directions mixed up.

1d)	**Solution 1:** Since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel, $\underset{\sim}{a} = \lambda \underset{\sim}{b}$, where λ is a real number. $\underset{\sim}{a} = p\underset{\sim}{i} + 8\underset{\sim}{j}$ and $\underset{\sim}{b} = q\underset{\sim}{i} + 4\underset{\sim}{j}$ $= 2(q\underset{\sim}{i} + 4\underset{\sim}{j})$ $= 2q\underset{\sim}{i} + 8\underset{\sim}{j}$ $\therefore p = 2q$ **Solution 2:** Since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel, The gradient of $\underset{\sim}{a} = \frac{8}{p}$. The gradient of $\underset{\sim}{b} = \frac{4}{q}$. $\frac{8}{p} = \frac{4}{q}$ $4p = 8q$ $\therefore p = 2q$	2 marks: full correct working with correct answer by using any of the solutions. One mark off with incorrect answer.	In general, done well by using either of the methods.

12 Ext 1 Task 2 Marking Scheme Question 2

2a)	$\underset{\sim}{a}\begin{pmatrix}1\\3\end{pmatrix}$ $\underset{\sim}{b}\begin{pmatrix}3\\1\end{pmatrix}$ $\cos\theta = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert} = \frac{3+3}{\sqrt{10}\sqrt{10}}$ $= \frac{6}{10} = \frac{3}{5}$ ✓  	1 mark: correctly calculates $cos\theta$ 1 mark: Correctly calculates $\cos 2\theta$	Well done

	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$ ✓		
2b (i)	 $\vec{AQ} = \frac{1}{2}\underset{\sim}{p} = \frac{1}{2}\vec{AC}$ $\vec{BQ} = \vec{AQ} - \vec{AB}$ $= \frac{1}{2}\underset{\sim}{p} - \underset{\sim}{x}$ ✓ $\vec{BE} = 4\left(\frac{1}{2}\underset{\sim}{p} - \underset{\sim}{x}\right)$ $= 2\underset{\sim}{p} - 4\underset{\sim}{x}$ ✓	2 marks : Correct answer from full working, including showing the vectors involved such as $\overrightarrow{AB}$ 1 mark: All the component vectors are correctly used and minor mistakes in the direction, such as $\overrightarrow{CO} = \underset{\sim}{x}$ instead of $-\underset{\sim}{x}$	
2b (ii)	(ii) $\vec{EO} + \vec{OC} = \vec{EC}$ $-\frac{1}{2}\vec{BE} + \underset{\sim}{x} = \vec{EC}$ $= 2\underset{\sim}{x} - \underset{\sim}{p} + \underset{\sim}{x}$ $= 3\underset{\sim}{x} - \underset{\sim}{p}$ ✓ $\therefore \vec{CE} = \underset{\sim}{p} - 3\underset{\sim}{x}$	1 mark: Must show all the component vectors in terms of A, B, C and correct answer	Note: For Qb, students who did the working out only using $\underset{\sim}{p}$ and $\underset{\sim}{x}$ received only 1 mark out of 3. Most students showed the full working and received full marks for Qb.

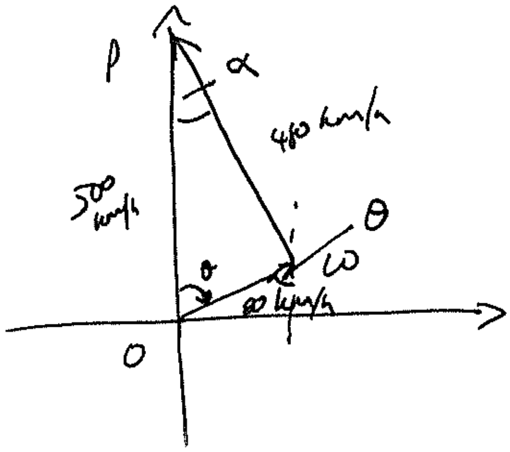
2C	(c)  $\vec{AC} = \underset{\sim}{a} - \underset{\sim}{b}$ $\vec{DB} = \underset{\sim}{a} + \underset{\sim}{b}$ Consider $\lvert \underset{\sim}{a} - \underset{\sim}{b} \rvert^2 + \lvert \underset{\sim}{a} + \underset{\sim}{b} \rvert^2$ ✓ $= (\underset{\sim}{a} - \underset{\sim}{b})\cdot(\underset{\sim}{a} - \underset{\sim}{b}) + (\underset{\sim}{a} + \underset{\sim}{b})\cdot(\underset{\sim}{a} + \underset{\sim}{b})$ $\underset{\sim}{a}\cdot\underset{\sim}{a} - \underset{\sim}{a}\cdot\underset{\sim}{b} - \underset{\sim}{b}\cdot\underset{\sim}{a} + \underset{\sim}{b}\cdot\underset{\sim}{b} +$ $\underset{\sim}{a}\cdot\underset{\sim}{a} + \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{a} + \underset{\sim}{b}\cdot\underset{\sim}{b}$. $= \lvert a\rvert^2 + \lvert b\rvert^2 + \lvert a\rvert^2 + \lvert b\rvert^2$ ✓ Hence $\lvert\vec{AC}\rvert^2 + \lvert\vec{DB}\rvert^2 = \lvert\vec{AB}\rvert^2 + \lvert\vec{BC}\rvert^2 + \lvert\vec{DC}\rvert^2 + \lvert\vec{DA}\rvert^2$. Sum of squares of the lengths of the diagonals equals sum of squares of the lengths of the sides	1 mark: correctly interprets the statement as lengths of vectors and represents the lengths diagonals using vectors. 1 mark: demonstrates that dot product gives the scalar which is the length of the side in this case. (Demonstrates $\underset{\sim}{a}.\underset{\sim}{a} = \lvert a\rvert^2$. Needs to clearly state this. 1 mark: Simplifies the expansion and converts into corresponding side lengths to complete the proof.	Many students think that expressing $\overrightarrow{AC}$ and $\overrightarrow{BC}$ is sufficient to meet the requirement of using vector methods. Note: There are only two types of products in vectors and in our syllabus we simply have the dot product. So, expanding like an algebraic expression is not accepted. You need to have the dot in between. Also, many students cannot recognise the difference between a vector and a scalar. You cannot say, $(\underset{\sim}{a} - \underset{\sim}{b})^2 + (\underset{\sim}{a} + \underset{\sim}{b})^2 = 2(\lvert\underset{\sim}{a}\rvert^2 + \lvert\underset{\sim}{b}\rvert^2)$
2d	 Since $\lvert\vec{YZ}\rvert = \sqrt{p^2+q^2} = 8\sqrt{5}$ $\therefore p^2+q^2 = 320$ ✓① $\lvert\vec{YX}\rvert = \sqrt{36+4} = \sqrt{40}$ Area $= \frac{1}{2} \times \lvert\vec{YX}\rvert\lvert\vec{YZ}\rvert \sin\theta$ $\frac{1}{2} \times \sqrt{40} \times \sqrt{p^2+q^2}\ \sin\theta = 40$	1 mark: Uses the $\lvert\overline{YZ}\rvert = 8\sqrt{5}$ and writes $p^2+q^2 = 320$ 1 mark: Correctly applies the area formula and calculates the angle 1 mark: Uses the dot product and gets the result $6p + 2q = 80$	This was the method chosen by most students. Many students did not remove the common factor 10, and it became a very difficult numerical expression to handle.

	$\sin\theta = \frac{80}{\sqrt{40}\times 8\sqrt{5}}$ $= \frac{10}{\sqrt{200}} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\therefore \theta = 45^\circ$ ✓ $\because \overrightarrow{YX}\cdot\overrightarrow{YZ} = 6p+2q$ $6p+2q = \lvert\overrightarrow{YX}\rvert\lvert\overrightarrow{YZ}\rvert\cos 45^\circ$ $= \sqrt{40}\times 8\sqrt{5}\times\frac{1}{\sqrt{2}}$ $= 10\sqrt{2}\times 8\times\frac{1}{\sqrt{2}}$ $6p+2q = 80$ $\rightarrow 3p+q = 40$ (2) $q = 40-3p$ (3) ✓ $p^2+(40-3p)^2=320$ $p^2+1600+9p^2-240p=320$ $10p^2-240p+1600=320$ $p^2-24p+160-32=0$ $p^2-24p+128=0$ $(p-16)(p-8)=0$ $p=16$ or $p=8$ from : (3) : $q = 40-3\times16$ or $q=40-3\times8$ $\therefore q=-8$ or $q=16$. Method 2: Area of $\triangle xyz = \frac{1}{2}\lvert xy\rvert\times\lvert yz\rvert\sin\theta$ $40 = \frac{1}{2}\times\sqrt{40}\times8\sqrt{5}\sin\theta$ $\therefore\sin\theta = \frac{1}{\sqrt{2}}$ $\theta = 45^\circ$ $\text{Arg}(\overrightarrow{YX}) = \tan^{-1}\left(\frac{2}{6}\right)$ $= 18.43^\circ$	and correctly solves to get one of the solutions. 1 mark: Correctly solves to get both solutions.	

	when X is above Z, $\text{Arg } \vec{YZ} = 18.43^\circ - 45^\circ$ $= -26.57^\circ$ $\vec{YZ} = \begin{pmatrix} p \\ q \end{pmatrix} =$ $= 8\sqrt{2}\begin{pmatrix} \cos(-26.57^\circ) \\ \sin(-26.57^\circ) \end{pmatrix}$ $p = 16 \quad q = -8$ when x is below Z: $\text{Arg}(\vec{YZ}) = 18.43^\circ + 45^\circ$ $= 63.43^\circ)$ $\vec{YZ} = \begin{pmatrix} p \\ q \end{pmatrix} = 8\sqrt{5}\begin{pmatrix} \cos 63.43^\circ \\ \sin 63.43^\circ \end{pmatrix}$ $= \begin{pmatrix} 8 \\ 16 \end{pmatrix}$		

Question 3

3ai	Since the system at in equilibrium: $\lvert T_1 \rvert = \lvert T_2 \rvert = t$ Hence: $T_2 = -t\cos 30^o\, \boldsymbol{i} + t\sin 30^o\, \boldsymbol{j}$ $= -\frac{\sqrt{3}}{2}t\boldsymbol{i} + \frac{t}{2}\boldsymbol{j}$	Must identify equivalence of tensions Must reference trigonometry	Many students did not identify equivalence of tension and therefore t came out of nowhere in their working.
3aii	Gravity of $5g = \begin{bmatrix} 0 \\ 5g \end{bmatrix}$ From (i): $\begin{bmatrix} \frac{-\sqrt{3}}{2}t \\ \frac{t}{2} \end{bmatrix} + \begin{bmatrix} \frac{n}{2} \\ \frac{\sqrt{3}}{2}n \end{bmatrix} = \begin{bmatrix} 0 \\ 5g \end{bmatrix}$ therefore, equating horizontal and vertical components: $-\frac{\sqrt{3}}{2}t + \frac{n}{2} = 0$ And $\frac{t}{2} + \frac{\sqrt{3}}{2}n - 5g = 0$ Solving to get: $\frac{\sqrt{3}}{2}t = -\frac{n}{2}, \quad n = \sqrt{3}t\ N$ And	1 mark for progress towards the solution, equating horizontal and vertical components. 2 marks final answers correct.	Students rarely attempted this question and when they did so, rarely set up the equations necessary to complete the question.

	$\frac{t}{2}+\frac{3}{2}t-50=0$ $2t=50$ $t=25\ N$		
3aiii	$N_1+T_1=\begin{bmatrix}0\\80\end{bmatrix}$ $N_1+\begin{bmatrix}0\\t\end{bmatrix}=\begin{bmatrix}0\\80\end{bmatrix}$ $N_1+\begin{bmatrix}0\\25\end{bmatrix}=\begin{bmatrix}0\\80\end{bmatrix}$ $N_1=\begin{bmatrix}0\\55\end{bmatrix}$	1 mark for correct answer with working	Very few students successfully completed the previous questions and so could not approach this question. Time also appeared to be a factor
3bi	 $480^2=500^2+80^2-2\times80\times500\times\cos\theta$ $-26000=-2\times80\times500\times\cos\theta$ $\cos\theta=0.325$ $\theta=71^o$	1 mark for an accurate diagram, not to scale 1 mark for substitution into cosine rule 1 mark for correct anser from correct working	Many students did not read the question correctly and found it challenging to draw a correct diagram, which hindered their progress in this question.
3bii	$\frac{\sin\alpha}{80}+\frac{\sin\theta}{480}$ $\alpha=\sin^{-1}\left[\frac{80\sin\theta}{480}\right]$ $\alpha=9.07^o$ So bearing is $360-\alpha=351^oT$	1 mark for sine rule correct use or equivalent 1 mark for expressing the correct answer as a bearing	If part (i) was poorly done, this was often also poorly done. Many overcomplicated this question.
3biii	$\binom{0}{500}$km/h	1 mark for correct groundspeed vector	Those who attempted this question mistook the plane's propulsive vector for the resultant vector, and answered incorrectly. The plane is so trimmed and angled to result in a directly northern path.

MATHEMATICS DEPARTMENT

Student Number: ____________________

$\boxed{2023}$ **Task 2 Validation Component**

Mathematics Extension 1

General Instructions

- Working time - 50 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For all questions, show mathematical reasoning and/or calculations

Total Marks: 32

Long Response – 32 marks (Pages 1–4)

- **Attempt Questions 1–3**

Question:	1	2	3	Total
Marks:	10	12	10	32
Awarded:				

Attempt Questions 1–3
Answer each question in the appropriate writing booklet.
All necessary working should be shown in every question.

Question 1 (10 Marks) Use a SEPARATE writing booklet.

(a) The resultant vector of **2**

$$2\begin{pmatrix} x \\ 3 \end{pmatrix} - 3\begin{pmatrix} -1 \\ y \end{pmatrix}$$

is $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$. Evaluate x and y.

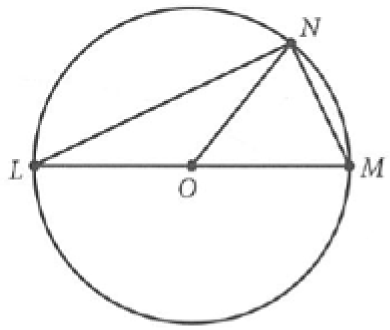
(b) A force described by the vector $\underset{\sim}{F} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, moves a particle along the line l from $C(-1,-1)$ to $D(2,-3)$.

(i) Find the unit vector $\hat{\underset{\sim}{u}}$ in the direction of $\overrightarrow{CD}$. **2**

(ii) Find the component of $\underset{\sim}{F}$ in the direction of the line l. **2**

(c) LOM is the diameter of a circle with centre O and N is a point on the circumference. **2**

If $\mathbf{p} = \overrightarrow{ON}$ and $\mathbf{q} = \overrightarrow{MN}$, express $\overrightarrow{LN}$ in terms of $\mathbf{p}$ and $\mathbf{q}$



(d) Given $\mathbf{a} = p\mathbf{i} + 8\mathbf{j}$ and $\mathbf{b} = q\mathbf{i} + 4\mathbf{j}$, find the relation of p and q when $\mathbf{a}$ and $\mathbf{b}$ are parallel. **2**

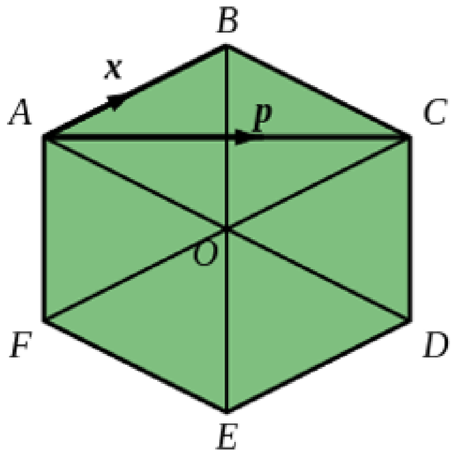
End of Question 1

Question 2 (12 Marks) Use a SEPARATE writing booklet.

(a) The angle between $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + \mathbf{j}$ is θ. **2**

Find the exact value of $\cos 2\theta$ by use of the dot product.

(b) $ABCDEF$ is a regular hexagon where $\overrightarrow{AB} = \underset{\sim}{x}$ and $\overrightarrow{AC} = \underset{\sim}{p}$.

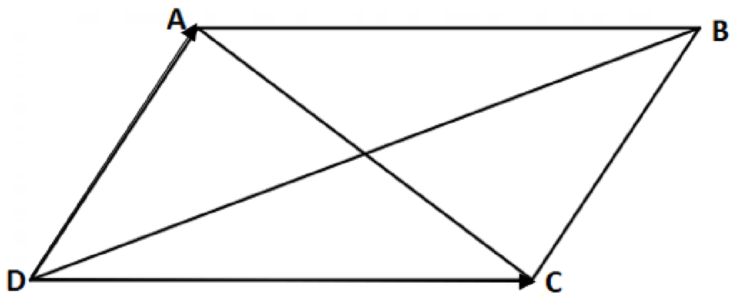


Express the following vectors in terms of $\underset{\sim}{x}$ and $\underset{\sim}{p}$.

(i) $\overrightarrow{BE}$ **2**

(ii) $\overrightarrow{CE}$ **1**

(c) Use vectors to prove that the sum of squares of the lengths of the two diagonals of a parallelogram is equal to the sum of the squares of the lengths of the four sides. **3**



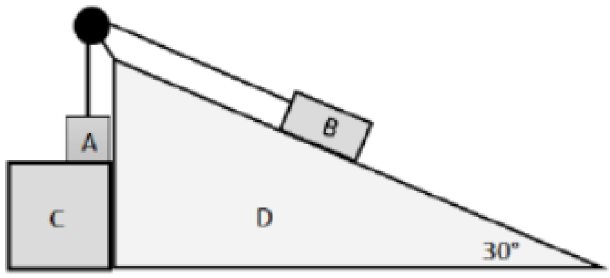
(d) An acute-angled triangle XYZ has an area of 40 square units. **4**

The vector $\overrightarrow{YX} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and $\overrightarrow{YZ} = \begin{pmatrix} p \\ q \end{pmatrix}$. Given $|\overrightarrow{YZ}| = 8\sqrt{5}$, find the possible values of p and q.

End of Question 2

Question 3 (10 Marks) Use a SEPARATE writing booklet.

(a) Both object A of mass 8 kg and object B of mass 5 kg are attached to the ends of a light inelastic string that passes over a smooth pulley. Object C is fixed to the ground and object D is a fixed plane at an inclination of 30^o as shown.



Gravity is given by $g = 10ms^{-2}$.

Let N_1 be the normal force of object C acting on object A and N_2 be the normal force of the inclined plane acting on the object B. Let T_1 and T_2 be the tension forces in the strings applied to object A and object B respectively.

Given $|T_1| = t$ Newtons and $N_2 = \begin{pmatrix} \frac{n}{2} \\ \frac{\sqrt{3}}{2}n \end{pmatrix}$

(i) Show $T_2 = \begin{pmatrix} \frac{\sqrt{3}}{2}t \\ \frac{t}{2} \end{pmatrix}$ **1**

(ii) By setting up vector equations, find the values of t and n. **2**

(iii) Find N_1 in column vector form. **1**

This question continues on the next page.

(b) Aberdeen is 750 km north of London. There is a wind of 80km/h blowing **from** a direction whose bearing is $(\theta + 180)$, where $0 < \theta < 90$. A plane whose speed in still air is 480 km/h flies directly from London to Aberdeen, with this wind blowing, in a time of 1.5 hours.

(i) Find the value of θ. **3**

(ii) Find the direction in which the pilot heads the plane. **2**

(iii) Hence, find the vector representing the plane's relative groundspeed. **1**

End of Paper