Year 11 – Extension 1 Mathematics – 2019
Preliminary HSC Assessment Task 1
Term 2 – Week 4 – 2019 **MES**

Name: .....

Teacher: .....

Tuesday 21st May, 2019

General Instructions

- Reading time – 5 minutes.
- Start each question on a new page.
- Write using black or blue pen.
- NESA approved calculators may be used.
- Show all relevant mathematical reasoning and/or calculations.
- Total possible mark: **50**
- Time allowed: **80 minutes**

Marks

Question 1 (8 marks)

(a) (i) Expand the expression $\left(x-\frac{1}{x}\right)^3$. **1**

(ii) Given that $x-\frac{1}{x}=5$, use your answer in part (i) to find the value of $x^3-\frac{1}{x^3}$ without finding the value of x. **1**

(b) Classify the relation $x=\sqrt{9-y^2}$ as one-to-one, one-to-many, many-to-one or many-to-many. **1**

(c) Make y the subject of the equation $x=y^2-6y$. **1**

(d) Consider the functions $f(x)=x^2-3x+2$ and $g(x)=\sqrt{x}$.

(i) Find $f(g(x))$. **1**

(ii) Solve $f(g(x))\geq 0$. **3**

Question 2 (8 marks)

(a) Suppose $x^3-2x^2+a\equiv(x+2)Q(x)+3$ where $Q(x)$ is a polynomial. Find the value of a. **1**

(b) Solve the inequality $\frac{5}{x-4}\geq x$. **3**

(c) If $g(x)=x^2+1$, sketch $y=\frac{1}{g(x)}$ and hence state its range. **2**

(d) Find the Cartesian equation of the curve defined by the parametric equations $x=2\tan\theta$ and $y=3\sec\theta$. **2**

Marks

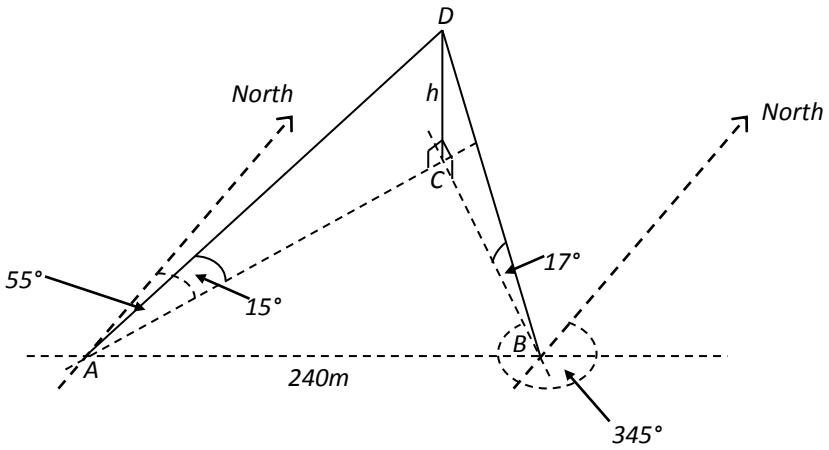
Question 3 (8 marks)

(a) Solve $2\sin x\cos x = \sin x$ for $0° \le x \le 360°$. **3**

(b) Ellie walks along a straight road *AB*. At point *A* she notices a tower. From this point, the tower is on a bearing of 055°. The top of the tower has an angle of elevation of 15°.

After walking 240 metres, she arrives at point *B*. From here, the tower is on a bearing of 345° with an angle of elevation to the top of the tower of 17°.

Let the height of the tower be *h*.



(i) With the aid of a diagram, show that $\angle ACB = 70°$. **1**

(ii) Find expressions for *AC* and *BC* in terms of *h*. **2**

(iii) Calculate the height, *h*, of the tower to the nearest metre. **2**

Marks

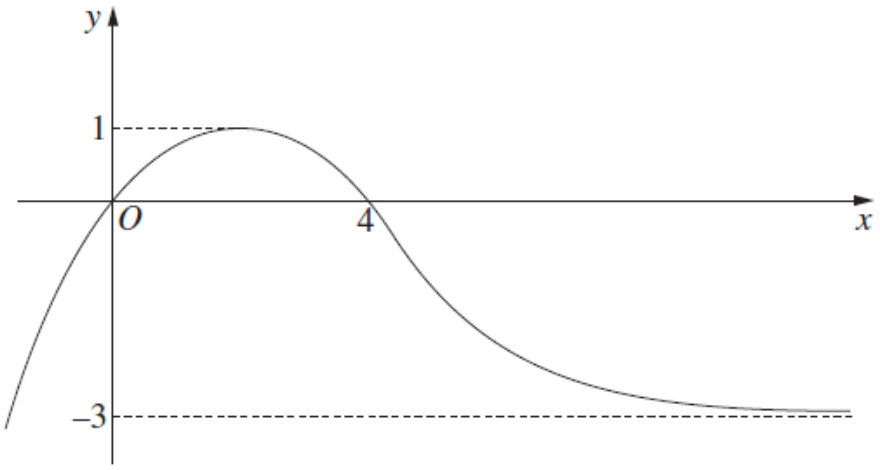
Question 4 (8 marks)

(a) The sign of the function $y = h(x)$ changes either side of $x = a$. **2**

Give two possible reasons as to what may happen at $x = a$ for this to occur.

(b) Use the fact that $P(1) = 0$ to factorise $P(x) = x^4 + x^3 - 9x^2 + 11x - 4$ completely. **3**

(c) The diagram shows the graph of $y = f(x)$.



Draw separate one-third page sketches of the graphs of the following:

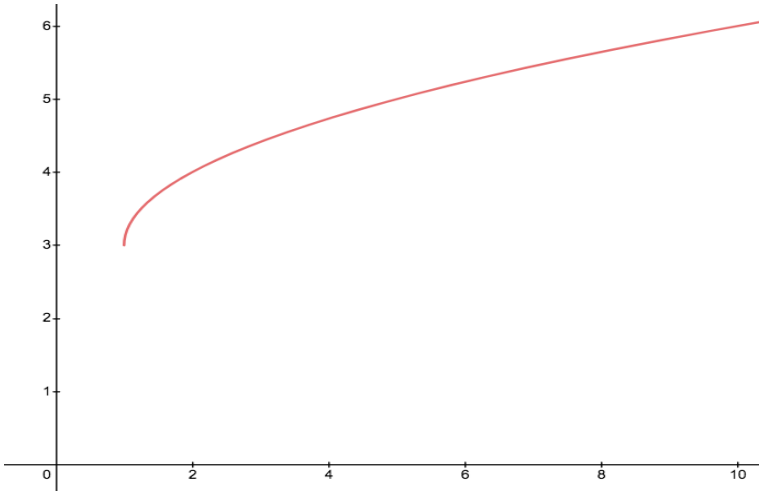
(i) $y = [f(x)]^2$ **2**

(ii) $|y| = f(x)$ **1**

Marks

Question 5 (8 marks)

(a) The function $f(x) = 3 + \sqrt{x-1}$ is sketched below.



(i) Explain why the inverse function $f^{-1}(x)$ exists. **1**

(ii) Find a formula for $f^{-1}(x)$ noting any restrictions on the domain of $f^{-1}(x)$. **2**

(iii) Hence or otherwise find the point(s) of intersection of $f(x)$ and $f^{-1}(x)$. **2**

(b) The cubic polynomial $P(x) = x^3 + rx^2 + sx + t$, where r, s and t are real numbers, has three real zeroes, 1, α and $-\alpha$.

(i) Find the value of r. **1**

(ii) Find the value of $s + t$. **2**

Marks

Question 6 (10 marks)

(a) Consider the function $f(x) = |5-x| + |x-2|$.

(i) Find $f(0)$. **1**

(ii) For what values of x is $f(x)$ constant? **1**

(iii) What is the value of $f(x)$ for these values of x? **1**

(iv) Draw a neat sketch of $y = f(x)$ showing any intercepts with the axes. **2**

(v) Hence or otherwise solve $|5-x| + |x-2| > 7$. **1**

(b) Consider the polynomial $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d and e are all integers.

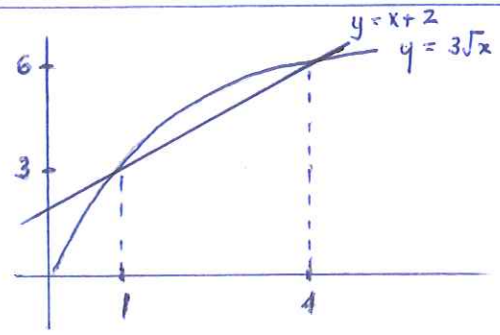
(i) Suppose $\beta \neq 0$ is an integer such that $P(\beta) = 0$. **2**

Prove that $\frac{e}{\beta}$ is also an integer.

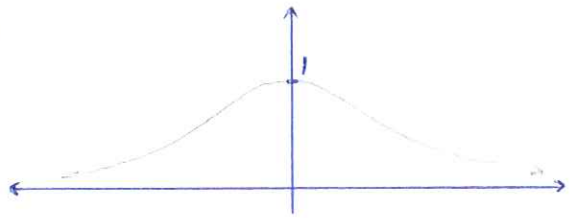
(ii) Deduce that $Q(x) = 4x^4 - x^3 + 3x^2 + 2x - 3$ does not have an integer zero. **2**

End of Assessment Task

2019 - Year 11 Mathematics Extension 1 - Task 1 - SOLUTIONS

Suggested Solution (s)	Comments
Question 1:	
(a) (i) $x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}$	R/W
(ii) $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$ $= 5^3 + 3(5)$ $= 140$	Aw 1 for $5^3 + 3 \times 5$
(b) one to many	R/W
(c) $x = y^2 - 6y$ $x + 9 = y^2 - 6y + 9$ $x + 9 = (y-3)^2$ $y - 3 = \pm\sqrt{x+9}$ $y = 3 \pm \sqrt{x+9}$ Note: Accept $y = \frac{6 \pm \sqrt{4x+36}}{2}$ by use of the quadratic formula	Either expression for y: Aw 1.
(d) (i) $f(g(x)) = (\sqrt{x})^2 - 3(\sqrt{x}) + 2$ $= x - 3\sqrt{x} + 2$	← Aw 1
(ii) $(\sqrt{x})^2 - 3(\sqrt{x}) + 2 \geq 0$ $(\sqrt{x} - 2)(\sqrt{x} - 1) \geq 0$ $\sqrt{x} \leq 1$, $\sqrt{x} \geq 2$ But $x \geq 0$ $\therefore x \geq 4$ $\therefore 0 \leq \sqrt{x} \leq 1$ $0 \leq x \leq 1$	1 for factorisation 1 for $\sqrt{x} \leq 1$, $\sqrt{x} \geq 2$ 1 for answer #
 Graphically: $x - 3\sqrt{x} + 2 \geq 0$ $\therefore x + 2 \geq 3\sqrt{x}$ $\therefore 0 \leq x \leq 1$, $x \geq 4$	

2019 - Year 11 Mathematics Extension 1 - Task 1 - SOLUTIONS

Suggested Solution (s)	Comments
Question 2:	
(a) $x^3 - 2x^2 + a \equiv (x+2)Q(x) + 3$ If $x = -2$ $(-2)^3 - 2(-2)^2 + a \equiv (-2+2)Q(-2) + 3$ $-16 + a = 3$ $a = 19$	R/W
(b) $\frac{5}{x-4} \geq x$ $x \neq 4$ $5(x-4) \geq x(x-4)^2$ $5(x-4) - x(x-4)^2 \geq 0$ $(x-4)[5 - x(x-4)] \geq 0$ $(x-4)(5 - x^2 + 4x) \geq 0$ $(x-4)(x^2 - 4x - 5) \leq 0$ $(x-4)(x-5)(x+1) \leq 0$  $x \leq -1, \quad 4 < x \leq 5$	1 for excluding $x=4$ from solution 1 for mult. by $(x-4)^2$ OR 1 for setting $\frac{5}{x-4} = x$ 1 for final solution
Alternatively: $\frac{5}{x-4} \geq x$ $x \neq 4$ Solve $\frac{5}{x-4} = x$ $5 = x^2 - 4x$ $0 = x^2 - 4x - 5$ $0 = (x-5)(x+1)$ $\therefore x = -1, 5$  $x \leq -1, \quad 4 < x \leq 5$	
(c)  Range: $0 < y \leq 1$	1 for graph 1 for range
(d) $x = 2\tan\theta$ $\quad y = 3\sec\theta$ $\tan\theta = \frac{x}{2}$ $\quad \sec\theta = \frac{y}{3}$ Since $1 + \tan^2\theta = \sec^2\theta$ $1 + \frac{x^2}{4} = \frac{y^2}{9}$ $\frac{y^2}{9} - \frac{x^2}{4} = 1$	1 for $\tan\theta$ & $\sec\theta$ 1 for answer

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Suggested Solution (s)	Comments
Question 3:	
(a) $2\sin x \cos x = \sin x$ $2\sin x \cos x - \sin x = 0$ $\sin x (2\cos x - 1) = 0$ $\therefore \sin x = 0$ or $\cos x = \frac{1}{2}$ $x = 0^\circ, 180^\circ 360^\circ$ $\quad$ $x = 60^\circ, 300^\circ$	1 for factorised equation 1 for each set of solutions (0, 180 360) (60, 300)
(b) (i) $\angle ACB = 55^\circ + 15^\circ$ $= 70^\circ$ or $\angle ACB = 180^\circ - 35^\circ - 75$ $= 70^\circ$	
(ii) In $\triangle ACD$, $\tan 15^\circ = \frac{h}{AC}$ $\therefore AC = \frac{h}{\tan 15^\circ}$ or $h\cot 15$ or $h\tan 75^\circ$ Similarly, in $\triangle BCD$, $BC = \frac{h}{\tan 17}$ or $h\cot 17$ or $h\tan 73^\circ$	1 for each expression
(iii) By cosine rule, $240^2 = h^2\tan^2 75 + h^2\tan^2 73 - 2(h\tan 75)(h\tan 73)\cos 70^\circ$ $= h^2\tan^2 75 + h^2\tan^2 73 - 2h^2\tan 75\tan 73\cos 70$ $\therefore h^2 = \frac{240^2}{\tan^2 75 + \tan^2 73 - 2\tan 75\tan 73\cos 70}$ $\therefore h \approx 59m$	1 for correct use of cosine rule 1 for 59m

Suggested Solution (s)	Comments

Question 4:

(a) • A vertical asymptote exists at $x = a$ with $h(a-\varepsilon)$ and $h(a+\varepsilon)$ being opposite in sign

• $h(x)$ is monotonic increasing or decreasing with $h(a) = 0$

(b) $P(x) = x^4 + x^3 - 9x^2 + 11x - 4$

$P(1) = 0 \quad \therefore x-1$ is a factor

$\therefore x^4 + x^3 - 9x^2 - 11x - 4 = (x-1)\,Q(x)$ ← Award 1

By trial and error $P(-4) = 0$

$= (x-1)(x+4)\,R(x)$ ← Award 2

$= (x-1)(x+4)(x^2+kx+1)$

$\therefore 4x - x - 4kx = 11x$

$-4kx = 8x$

$k = -2$

$\therefore P(x) = (x-1)(x+4)(x^2-2x+1)$

$= (x-1)^3(x+4)$ ← Award 3

Alternatively:

$$
\begin{array}{r}
x^3 + 2x^2 - 7x + 4 \\
x-1 \;\overline{)\; x^4 + x^3 - 9x^2 + 11x - 4} \\
\underline{x^4 - x^3} \\
2x^3 - 9x^2 \\
\underline{2x^3 - 2x^2} \\
-7x^2 + 11x \\
\underline{-7x^2 + 7x} \\
4x - 4 \\
\underline{4x - 4} \\
0
\end{array}
$$

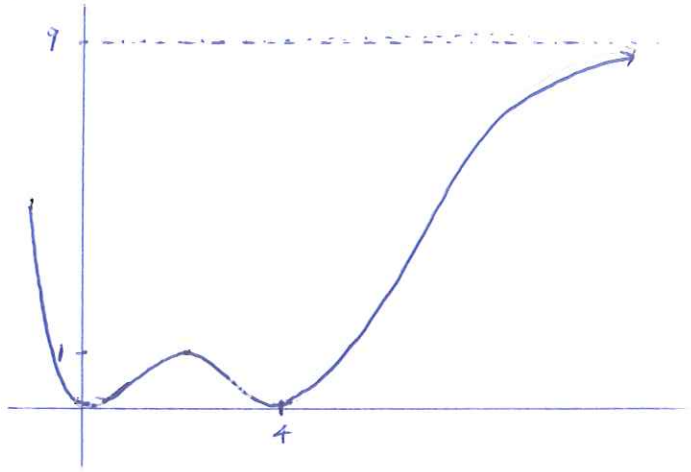
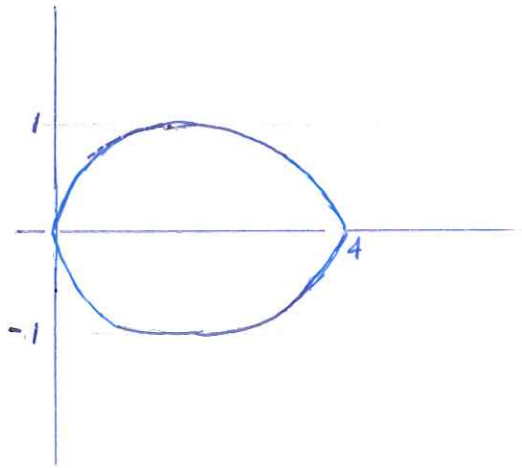
$\therefore P(x) = (x-1)(x^3 + 2x^2 - 7x + 4)$ ← 1 for cubic factor

Then by trial and error & division again,

$P(x) = (x-1)^2(x^2+3x-4)$ or $P(x) = (x-1)(x+4)(x^2-2x+1)$ ← 2 for 2 linear factors

$P(x) = (x-1)^3(x+4)$ $\qquad$ $P(x) = (x-1)^3(x+4)$

3 for answer

Suggested Solution (s)	Comments
(c) (i) 	1 for correct hor. asym. 1 for correct turning pts
(ii) 	1 for graph (must be "pointy" at (0,0) & (4,0))

Suggested Solution (s)	Comments

Question 5 :

(a) (i) Because $f(x)$ is a one to one function or a monotonic increasing function — R/W

(ii) $x = 3 + \sqrt{y-1}$ — 1 for interchanging x & y

$x - 3 = \sqrt{y-1}$

$(x-3)^2 = y - 1$

$y = (x-3)^2 + 1 \quad (x \geq 3)$ — 1 for answer

(iii) $y = (x-3)^2 + 1$

$y = x$

$\therefore (x-3)^2 + 1 = x$

$x^2 - 6x + 10 = x$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$\therefore x = 2, 5$ — ← 1 for x values

But $x \geq 3$ $\therefore$ discard $x = 2$

Point of intersection is $(5, 5)$ — 1 for $(5, 5)$

(b) $P(x) = x^3 + rx^2 + sx + t$

(i) Sum of roots $= -r$ Sum of roots $= 1 + \alpha - \alpha$ $= 1$ — R/W

$\therefore r = -1$

(ii) $P(1) = 0$ — 1 for $P(1) = 0$

$\therefore 1^3 - 1 + s.1 + t = 0$

$\therefore s + t = 0$ — 1 for answer

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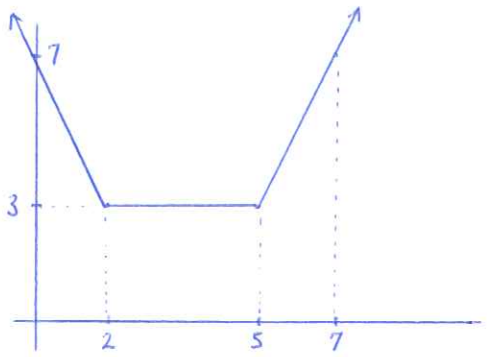
Suggested Solution (s)	Comments
Question 6 :	
(a) (i) $f(0) = 7$	R/W
(ii) $2 \leq x \leq 5$ (Accept $2 < x \leq 5$, $2 \leq x < 5$, $2 < x < 5$)	R/W
(iii) 3	R/W
(iv) 	1 for correct graph 1 for (0,7)
(v) $x < 0$, $x > 7$	R/W
(b) (i) $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ $P(\beta) = 0$ $\therefore 0 = a\beta^4 + b\beta^3 + c\beta^2 + d\beta + e$ $-e = \beta(a\beta^3 + b\beta^2 + c\beta + d)$ $\frac{e}{\beta} = -(a\beta^3 + b\beta^2 + c\beta + d)$ $\therefore$ since a, b, c, d and β are all integers, the RHS is an integer and, as such, $\frac{e}{\beta}$ is an integer.	1 for $\frac{e}{\beta}$ 1 for conclusion
(ii) We require the integer $\beta \neq 0$ to divide -3. ie, $\beta = \pm 1$ or $\beta = \pm 3$ only Since, $Q(1) = 5 \neq 0$ $Q(-1) = 3 \neq 0$ $Q(3) = 327 \neq 0$ $Q(-3) = 369 \neq 0$ $\therefore Q(x)$ has no integer zero.	← 1 for possible β's 1 for establishing $\beta \notin \mathbb{Z}$