Year 11 – Extension 1 Mathematics – 2019
Preliminary HSC Assessment Task
Term 3 – Week 4 – 2019

Name: ....

Teacher: ....

Thursday 15th August, 2019

Period 2

General Instructions

- Reading time – 5 minutes.
- Write your solutions on the lined paper provided.
- Start each question on a new page.
- Write using black or blue pen.
- NESA approved calculators may be used.
- Show all relevant mathematical reasoning and/or calculations.
- Total possible mark: **20**
- Time allowed: **30 minutes**

Setter: **VUL**

Marks

Question 1 (10 marks)

(a) If the degrees of $P(x)$ and $Q(x)$ are p and q respectively where $p \neq q$, comment on the degree of $P(x) + Q(x)$. **1**

(b) If $P(x)$ is divided by $x - a$, prove that the remainder is given by $P(a)$. **2**

(c) Factorise $x^3 - 2x^2 + x - 12$ completely. **2**

(d) A polynomial $P(x)$ is divided by $(x+4)(x-3)$. If $P(-4) = 11$ and $P(3) = -3$, what is the remainder. **3**

(e) If $ax^2 + bx + c = 0$ has roots α and β, prove $\alpha\beta = \dfrac{c}{a}$. **2**

Question 2 is on the next page.

Question 2 (10 marks) **START A NEW PAGE**

(a) If two polynomials are of degrees *m* and *n* respectively, where $m > n$, what is the maximum number of intersection points their graphs can have? **1**

(b) If $x^3 + 2x^2 - 11x - 12 = 0$ has roots α, β and γ , evaluate:

(i) $\alpha + \beta + \gamma$. **1**

(ii) $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. **1**

(c) Two of the roots of the polynomial equation $x^3 + 3x^2 - 4x + d = 0$ are opposite in sign. Find the value of *d* and the three roots of the equation. **2**

(d) Suppose the polynomial equation $2x^3 - 3x^2 + 5x - 8 = 0$ has roots α, β and γ . **2**
By considering the value of $\alpha^2 + \beta^2 + \gamma^2$, explain why the equation does not have three real roots.

(e) If the three zeros of $P(x) = x^3 + x^2 + 1$ are r_1, r_2, r_3 and $Q(x) = x^2 - 3$, **3**
evaluate $Q(r_1) \times Q(r_2) \times Q(r_3)$.

End of Assessment Task

2019 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments
Question 1	
(a) The degree of $P(x)+Q(x)$ will be the larger of either p or q.	✓
(b) By the division transformation $P(x) = (x-a)Q(x) + r$, where r is the remainder constant.	✓
$P(a) = (a-a).Q(a) + r$ $= 0 \cdot Q(a) + r$ $= 0 + r$ $= r$ which is the remainder	✓
Thus the remainder is given by $P(a)$.	
(c) $P(3) = 0$ $\therefore x-3$ is a factor	✓
$$\begin{array}{r} x^2 + x + 4 \\ x-3\,\overline{)\,x^3 - 2x^2 + x - 12} \\ \underline{x^3 - 3x^2} \\ x^2 + x \\ \underline{x^2 - 3x} \\ 4x - 12 \\ \underline{4x - 12} \\ 0 \end{array}$$	✓
$\therefore P(x) = (x-3)(x^2+x+4)$ over $\mathbb{R}$	
(d) If $A(x)$ is quadratic let $R(x) = mx+n$	
$\therefore P(x) = (x+4)(x-3)Q(x) + mx + n$	
$P(-4) = -4m + n = 11 \ldots$ (1) $P(3) = 3m + n = -3 \ldots$ (2)	✓
(1)−(2): $-7m = 14$ $m = -2$ $\therefore n = 3$	✓
$\therefore R(x) = -2x + 3$ is remainder.	
(e) $ax^2 + bx + c = 0$ $\therefore x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ $\therefore x^2 + \frac{b}{a}x + \frac{c}{a} \equiv (x-\alpha)(x-\beta)$ $\equiv x^2 - (\alpha+\beta)x + \alpha\beta$.	✓
Equating co-efficients $\alpha\beta = \frac{c}{a}$	✓

2019 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 2 (a) no points of intersection ✓ (b)(i) $\alpha+\beta+\gamma = -\frac{b}{a}$ $= -2$ ✓ (ii) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ $= \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ $= \frac{c}{a} / -\frac{d}{a}$ $= \frac{-11}{12}$ ✓ (c) Let roots be $\alpha, -\alpha, \beta$ $\therefore \alpha+(-\alpha)+\beta = -\frac{3}{1}$ $\therefore \beta = -3$ $\therefore P(-3) = -27+18+12+d = 0$ $\therefore d = -12$ ✓ $\therefore -\alpha^2\beta = \frac{-d}{a} = 12$ (Product of Roots) $\alpha^2 = 4$ $\alpha = \pm 2$ $\therefore$ Roots are $-3, \pm 2$ ✓		(d) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $= \left(\frac{-3}{2}\right)^2 - 2\left(\frac{5}{2}\right)$ $= \frac{9}{4} - 5$ $= -\frac{11}{4}$ since $\alpha^2+\beta^2+\gamma^2 < 0$ and $\alpha^2+\beta^2+\gamma^2 \geq 0$ for real values of α, β, γ then the equation cannot have 3 real roots. (e) $Q(r_1)\times Q(r_2)\times Q(r_3)$ $= (r_1^2-3)(r_2^2-3)(r_3^2-3)$ $= (r_1+\sqrt{3})(r_1-\sqrt{3})(r_2+\sqrt{3})(r_2-\sqrt{3})(r_3+\sqrt{3}) \times (r_3-\sqrt{3})$ ✓ $= (-1)^3(\sqrt{3}-r_1)(\sqrt{3}-r_2)(\sqrt{3}-r_3) \times (-1)^3(-\sqrt{3}-r_1)(-\sqrt{3}-r_2)(-\sqrt{3}-r_3)$ $= (-1)^3 \times P(\sqrt{3}) \times (-1)^3 \times P(-\sqrt{3})$ ✓ as $P(x) = (x-r_1)(x-r_2) \times (x-r_3)$ $= -1 \times (3\sqrt{3}+4) \times -1 \times (-3\sqrt{3}+4)$ $= 16 - 27 = -11$ ✓	✓ ✓