Year 11 Mathematics Extension 1 Assessment Task 1 Term 1 Week 4 2020

Name: ....

Teacher: ....

Tuesday 19th May 2020

Setter: WIL

General Instructions

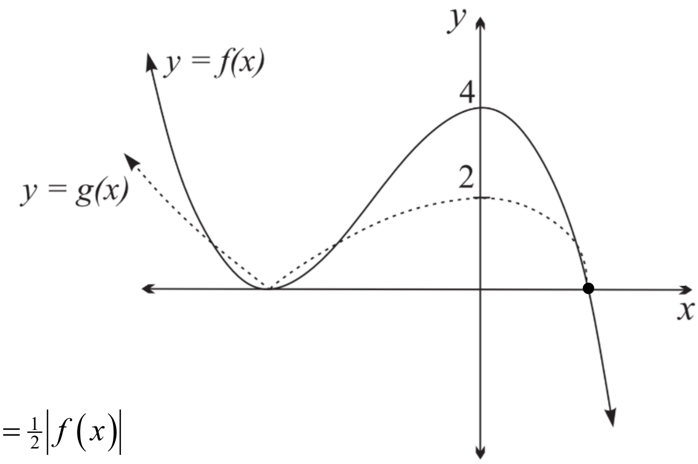
- Working time – 55 minutes, inclusive of printing or downloading, writing, scanning, naming and uploading the task to Google Classroom.
- Answer all questions on your own lined paper using black or blue pen.
- It is your responsibility to ensure that your marker can read your repsonses in the PDF file that you create.
- NESA approved calculators may be used and you can reference any notes and/or your textbook during the task.
- Show all relevant mathematical reasoning and/or calculations.
- **Total marks – 32.**
- Attempt Questions 1 – 4.
- All questions are of equal value.

Question 1 (8 marks)

For **Multiple Choice** questions, write down the correct option on your writing paper.

(a) The diagram below shows the graphs of the functions $y = f(x)$ and $y = g(x)$.

Which of the following could represent the relationship between $f(x)$ and $g(x)$? **1**



(A) $g(x) = \frac{1}{2}\left|f(x)\right|$

(B) $g(x) = \sqrt{f(x)}$

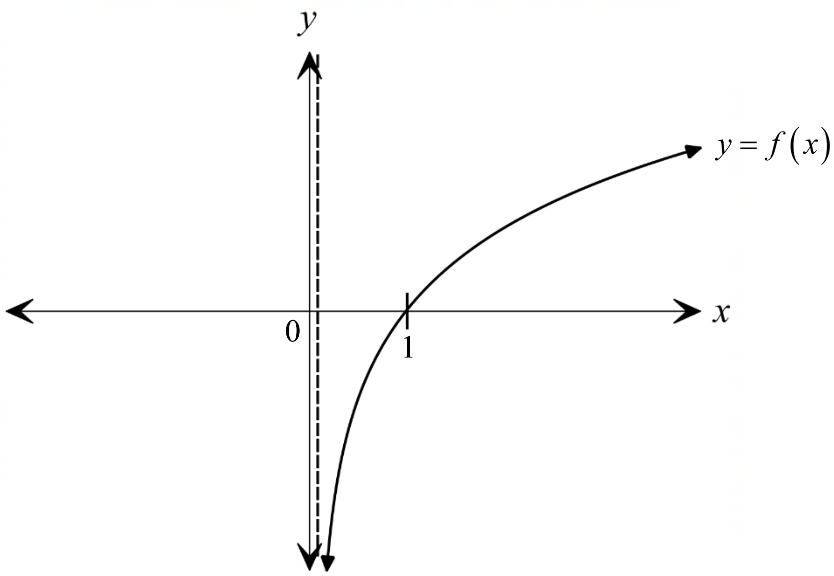
(C) $g(x) = \dfrac{1}{f(x)}$

(D) $\left(g(x)\right)^2 = f(x)$.

Question 1 continued on page 2

Question 1 continued

(b)



Use the sheet on page 7 to sketch the following graphs, clearly showing any intercepts with axes, asymptotes and changes to the curves' shape. Please go over $f(x)$ on the template with your pen where necessary.

(i) $y = f^{-1}(x)$ **1**

(ii) $y = |f(x)|$ **1**

(iii) $y = f(|x|)$ **1**

(iv) $y^2 = f(x)$ **1**

(c) (i) Show that $\sqrt{\dfrac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = |\tan x|$. **2**

(ii) Find the value(s) of x if $\sqrt{\dfrac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = 1$ for $0° \le x \le 180°$. **1**

End of Question 1

Question 2 (8 marks) (Start a new page)

For **Multiple Choice** questions, write down the correct option on your writing paper.

(a) Which of the following is a correct simplification of $\dfrac{\cos(180°-\theta)}{\cos(90°-\theta)}$? **1**

(A) $\cos 90°$

(B) $-\tan\theta$

(C) $\tan\theta$

(D) $-\cot\theta$.

(b) Solve $\cos(x-60°)=\dfrac{1}{\sqrt{2}}$ for $0°\le x\le 360°$. **2**

(c) Use the expansion of $\left(\sqrt{15-6\sqrt{6}}-\sqrt{15+6\sqrt{6}}\right)^2$ to simplify $\sqrt{15-6\sqrt{6}}-\sqrt{15+6\sqrt{6}}$

leaving your answer in simplest surd form. **2**

(d) Let $g(x)=\dfrac{1}{x-2}$.

(i) Draw $g(x)$ and $\dfrac{1}{g(x)}$ on one diagram on your lined paper. Make your diagram at least one quarter of a page, clearly showing any intercepts with axes and asymptotes. **2**

(ii) Let $h(x)=g(x)+\dfrac{1}{g(x)}$. Sketch $h(x)$ on the same diagram, clearly showing any intercepts with axes and both asymptotes. **1**

End of Question 2

Question 3 (8 marks) (Start a new page)

For **Multiple Choice** questions, write down the correct option on your writing paper.

(a) The polynomial $P(x)$ has degree 4 and the polynomial $Q(x)$ has degree 2. If you divide $P(x)$ by $Q(x)$, the remainder $R(x) \neq 0$. The remainder has degree: **1**

(A) 1

(B) 2

(C) 0 or 1

(D) 0, 1 or 2

(b) The polynomial $Q(x) = x^3 + bx^2 + cx + d$ has zeros 0, 3 and –3.

(i) What are the values of b, c and d? **2**

(ii) Without using calculus, sketch the graph of $y = Q(x)$. **1**

(iii) Hence or otherwise, solve the inequality $\dfrac{x^2 - 9}{x} > 0$. **1**

(c) Find the roots of the equation $x^3 + 6x^2 - x - 30 = 0$ given one of the roots of the equation is the sum of the other roots. **3**

End of Question 3

Question 4 (8 marks) (Start a new page)

For **Multiple Choice** questions, write down the correct option on your writing paper.

(a) It is known that $x+2$ is a factor of the polynomial $P(x)$ and that $P(x)=(x^2+x+1)\times Q(x)+(2x+3)$ for some polynomial $Q(x)$. From this information alone, it can be deduced that: **1**

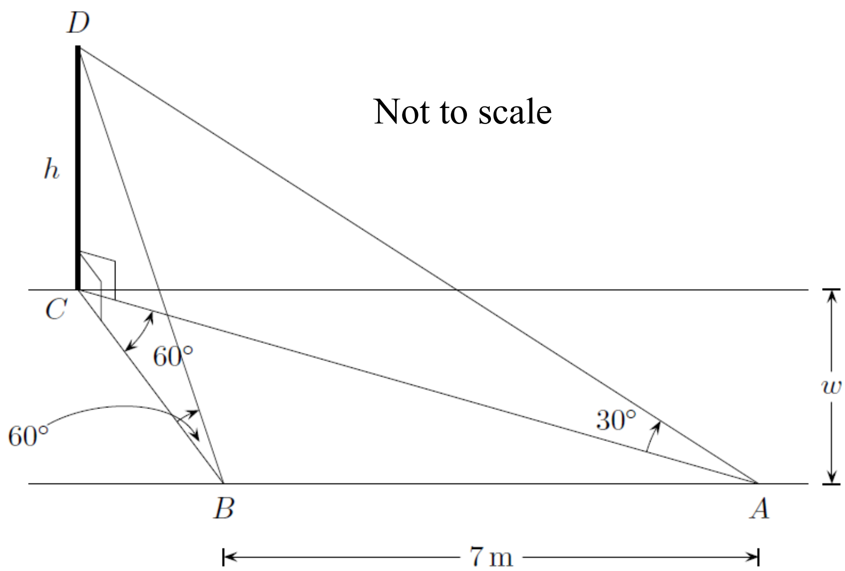
(A) $Q(-2)=-\frac{1}{3}$

(B) $Q(-2)=\frac{1}{3}$

(C) $Q(-2)=-1$

(D) $Q(-2)=1$.

(b) A footpath on horizontal ground has two parallel edges. CD is a vertical street sign pole of height h metres which stands with its base C on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB=7\text{ m}$ and $\angle ACB=60°$. From A and B, the angles of elevation to the top of the pole at D are $30°$ and $60°$ respectively.



(i) Show that $CA=h\sqrt{3}$ and $CB=\dfrac{h}{\sqrt{3}}$. **1**

(ii) Show that the exact height of the flagpole is $h=\sqrt{21}$ metres. **1**

(iii) Find the area of ΔABC. **2**

(iv) Hence or otherwise, find the exact width w of the footpath. **1**

Question 4 continued on page 6

Question 4 continued

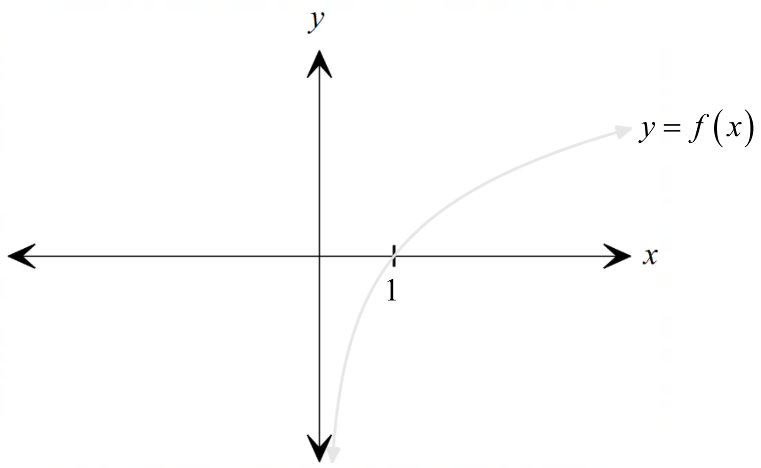
(c) (i) Sketch the function $g(x)$ which is defined by $g(x)=\begin{cases}1+\dfrac{|x|}{x}, & \textit{for } x \neq 0 \\ 2, & \textit{for } x=0\end{cases}$. **1**

(ii) Given $f(x)=-2x$. Sketch $f(g(x))$ on a new number plane. **1**

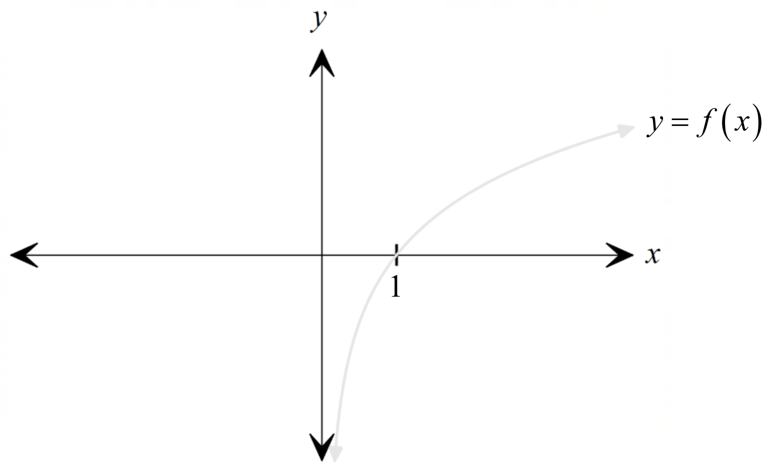
End of Assessment Task

Answer Sheet for Question 1 (b)

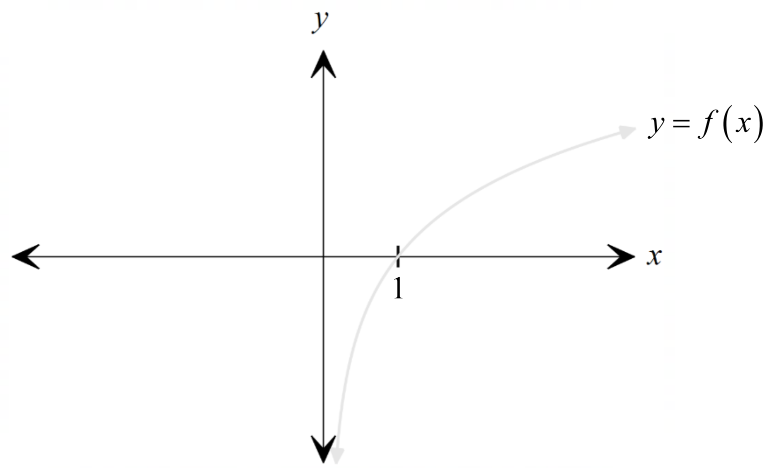
(b) (i) $y = f^{-1}(x)$



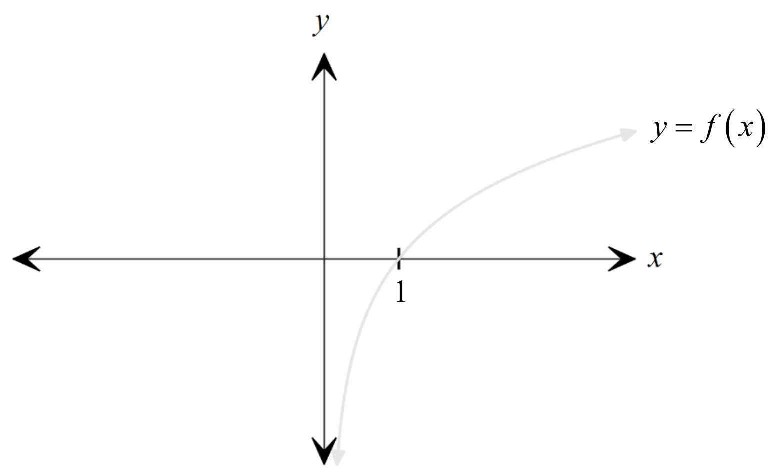
(b) (ii) $y = |f(x)|$



(b) (iii) $y = f(|x|)$



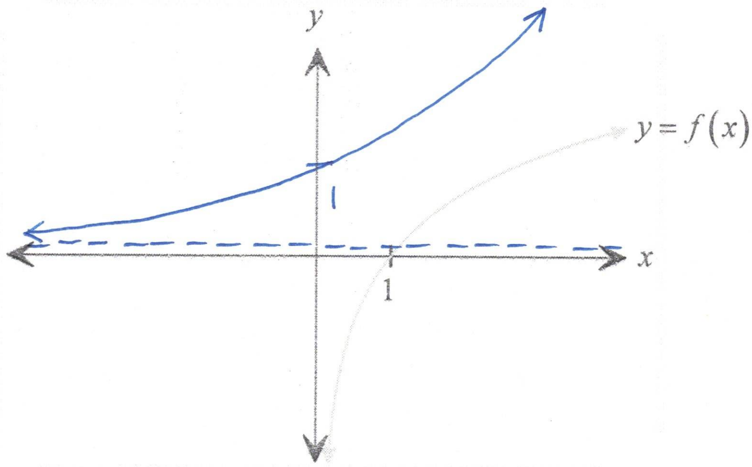
(b) (iv) $y^2 = f(x)$



Suggested Solution (s)	Comments
Question 1 (a) $\frac{1}{2}\lvert f(x)\rvert$ would have reflected curve below x-axis $\frac{1}{f(x)}$ would have a y-intercept of $\frac{1}{4}$ $(g(x))^2$ would have reflected $\sqrt{f(x)}$ about x-axis. $\therefore$ B. (b) see scheme on the next page for requirements each graph. (c) (i) $\text{LHS} = \sqrt{\frac{(1+\cot^2 x) - \cot^2 x - \cos^2 x}{\cos^2 x}}$ $= \sqrt{\frac{1-\cos^2 x}{\cos^2 x}}$ ✓ $= \sqrt{\frac{\sin^2 x}{\cos^2 x}}$ $= \sqrt{\tan^2 x}$ ✓ or $\sqrt{\left(\frac{\sin x}{\cos x}\right)^2}$ $= \lvert\tan x\rvert$ (ii) $\lvert\tan x\rvert = 1$ $\therefore \tan x = \pm 1$ $x = 45°, 135°$ ✓ must have both **Question 2** (a) $\frac{\cos(180-\theta)}{\cos(90-\theta)} = \frac{-\cos\theta}{\sin\theta}$ $= -\cot\theta \ \therefore$ D. (b) $\cos(x-60) = \frac{1}{\sqrt{2}}$ ref $\angle = 45°$ S | A ✓, T | C ✓ $-60° \le x - 60° \le 300°$ ✓ $x - 60° = -45°, 45°$ $x = 15°, 105°$ ✓ both	

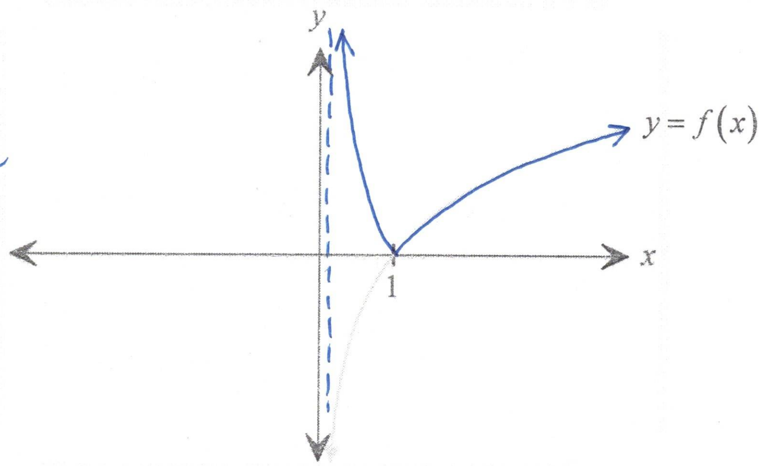
Answer Sheet for Question 1 (b)

(b) (i) $y = f^{-1}(x)$



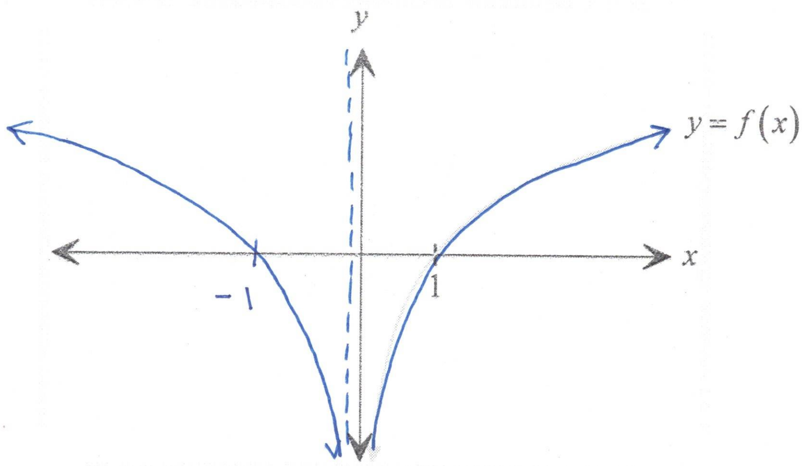
1
- correct shape
- asymptote
- y-intercept

(b) (ii) $y = |f(x)|$



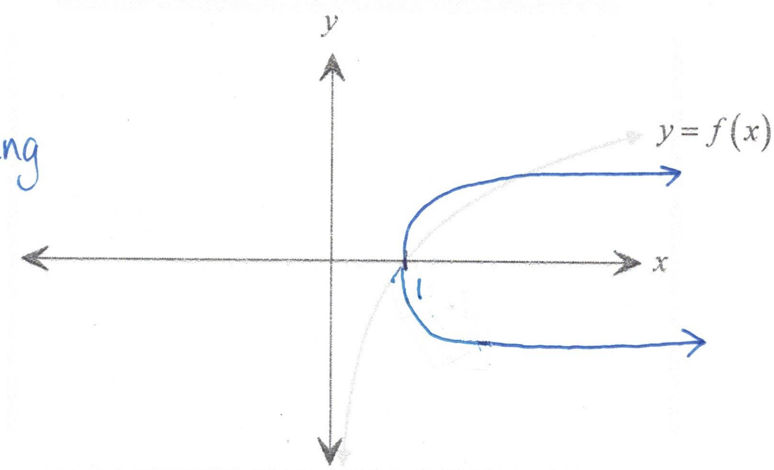
1
- correct shape
- asymptote

(b) (iii) $y = f(|x|)$



1
- shape, both sides
- asymptote
- x-intercept

(b) (iv) $y^2 = f(x)$



1
- shape, including cross over
- reflection in x-axis

Suggested Solution (s)	Comments
Question 2 (continued)	

(c) $\left(\sqrt{15-6\sqrt{6}}\right)^2 - 2\left(\sqrt{15-6\sqrt{6}}\right)\left(\sqrt{15+6\sqrt{6}}\right) + \left(\sqrt{15+6\sqrt{6}}\right)^2$

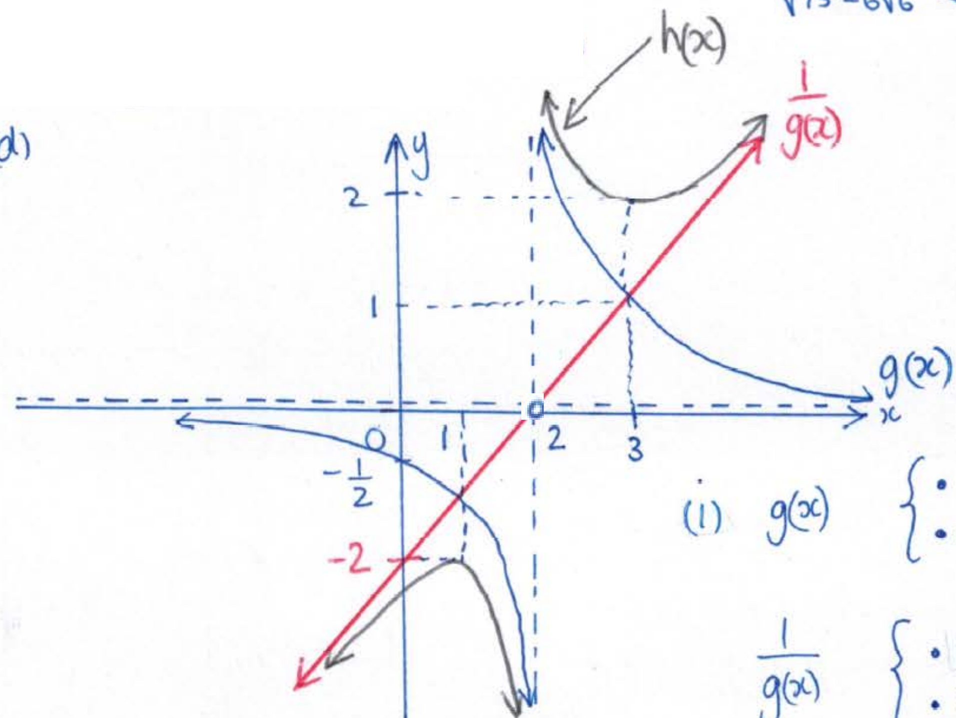
$= 15 - 6\sqrt{6} - 2\sqrt{\left(15^2 - (6\sqrt{6})^2\right)} + 15 + 6\sqrt{6}$ ✓

$= 30 - 2(3)$

$= 24$ $\quad \therefore \sqrt{15-6\sqrt{6}} - \sqrt{15+6\sqrt{6}} = -\sqrt{24} = -2\sqrt{6}$ ✓

$\sqrt{15-6\sqrt{6}} < \sqrt{15+6\sqrt{6}}$

(d)



(i) $g(x)$ { • shape • asymptotes

$\dfrac{1}{g(x)}$ { • line • x-intercept

(ii) $h(x)$ { • shape - both parts • approaching other curves.

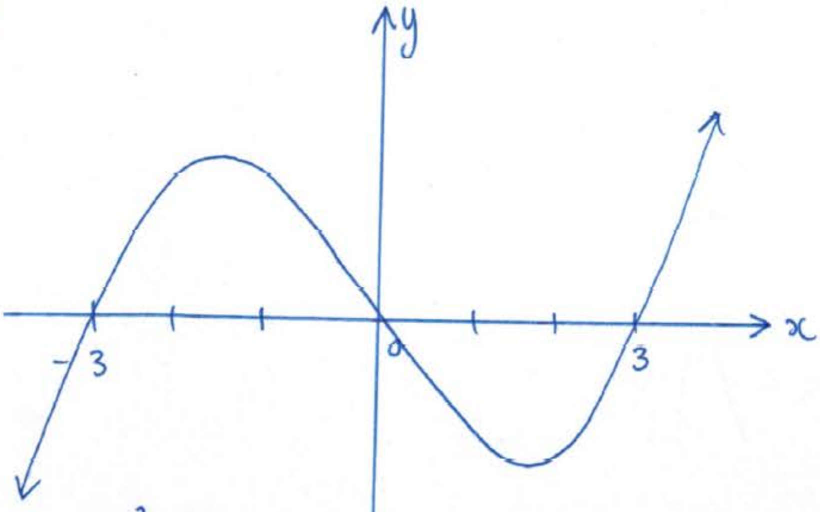
Question 3

(a) $R(x) \neq 0 \therefore 0$ or $1 \Rightarrow C$

(b)(i) $x^3 + bx^2 + cx + d = x(x-3)(x+3)$

$= x^3 - 9x$ ✓

$\therefore \left.\begin{array}{l} b = 0 \\ d = 0 \\ c = -9 \end{array}\right\}$ ✓

Suggested Solution (s)	Comments
Question 3 (continued)	
(b) (ii) 	
(iii) $x^2 \times \frac{x^2-9}{x} > 0$ $x(x+3)(x-3) > 0$ $-3 < x < 0$ or $x > 3$ or $(-3,0)$, $(3,\infty)$	
(c) $x^3 - 6x^2 - x - 30$ roots are $\alpha, \beta, \alpha+\beta$ sum: $\alpha + \beta + \alpha + \beta = -\frac{-6}{1}$ $2(\alpha+\beta) = -6$ $\alpha + \beta = -3$ product: $\alpha\beta(\alpha+\beta) = -\frac{-30}{1}$ $-3\alpha\beta = 30$ $\alpha\beta = -10$	1 mark sum and product
form a quadratic whose roots have a sum -3 and a product -10 $x^2 - (\alpha+\beta)x + \alpha\beta = 0$ $x^2 + 3x - 10 = 0$	1 mark for utilising line!
$(x+5)(x-2) = 0$ roots are $2, -5, -3$	1 mark final solution.

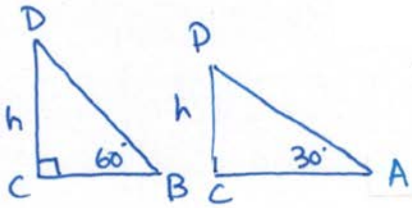
2020 Year 11 AT1 Mathematics Extension 1 Examination SOLUTIONS

Suggested Solution (s)	Comments

Question 4

(a) B

(b)(i)



$\tan 60^\circ = \frac{h}{CB}$ $\quad \tan 30^\circ = \frac{h}{CA}$ ✓ both equations

$\sqrt{3} = \frac{h}{CB}$ $\quad \frac{1}{\sqrt{3}} = \frac{h}{CA}$

$\therefore CB = \frac{h}{\sqrt{3}}, \; CA = h\sqrt{3}$

(ii) $AB^2 = BC^2 + AC^2 - 2(BC)(AC)\cos 60^\circ$

$AB^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + \left(h\sqrt{3}\right)^2 - \frac{h}{\sqrt{3}} \times h\sqrt{3}$ $\quad \left(\cos 60^\circ = \frac{1}{2}\right)$

one correctly using cosine rule

$7^2 = \frac{h^2}{3} + 3h^2 - h^2$

$\frac{7}{3}h^2 = 49$

$h^2 = 21$

$\therefore h = \sqrt{21}$ as required.

(iii) $CB = \frac{\sqrt{21}}{\sqrt{3}} = \sqrt{7}$, $CA = \sqrt{21} \times \sqrt{3}$ ✓ both.

$= 3\sqrt{7}$

$A = \frac{1}{2}ab\sin C$

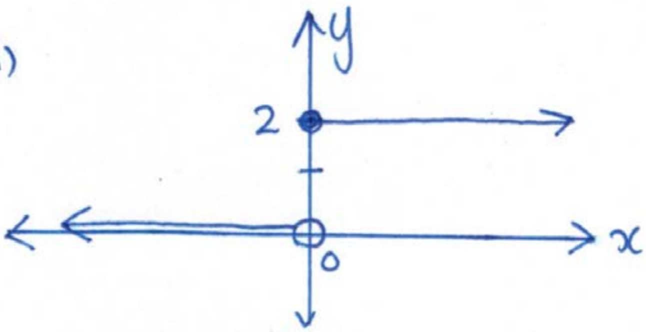
$= \frac{1}{2}\left(\sqrt{7}\right)\left(3\sqrt{7}\right)\sin 60^\circ$

$= \frac{21\sqrt{3}}{4}$ ✓

(iv) $\frac{7}{2}w = \frac{21\sqrt{3}}{4}$

$w = \frac{21\sqrt{3}}{4} \times \frac{2}{7}$

$w = \frac{3\sqrt{3}}{2}$ m ✓

Suggested Solution (s)	Comments
Question 4 (continued) (c)(i)  $x > 0 \quad g(x) = 2$ $x = 0 \quad g(x) = 2$ $x < 0 \quad g(x) = 0$ (ii)