Year 11 Mathematics Extension 1
Preliminary Assessment Task 2
Term 3, Week 4, 2020

Name: ____________________

Wednesday, 12th August 2020

- Start each question on a new page.
- NESA approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- Total possible mark: **32**
- Time allowed**: 45 minutes + 2 minutes reading time**
- For any multiple choice question, **write the letter** that corresponds to your answer on your writing paper.

Question 1 (8 marks) **Marks**

(a) What is the value of $f'(2)$ if $f(x)=\dfrac{1}{3x}$? **1**

(A) $-\dfrac{1}{36}$

(B) $-\dfrac{1}{12}$

(C) $\dfrac{1}{6}$

(D) $\dfrac{1}{3}$

(b) Find the value of c such that $y=-10x+c$ is a tangent to the parabola $y=3x^2+2x-4$. **3**

(c) Given that $y=(3x+1)^2$. Show that $y-y'+4=x(y'-12)-y$. **2**

(d) (i) On a Cartesian plane, draw a neat sketch of the function of the function $y=f(x)$ where: **1**

$$f(x)=\begin{cases} x^2-4, & for\ x<2 \\ 4x-8, & for\ x\geq 2 \end{cases}$$

(ii) Is the function above differentiable at $x=2$? Justify your answer. **1**

End of Question 1

Question 2 starts on the next page

Question 2 (8 marks) *Start a new page* **Marks**

(a) For $x > 0$, which expression is NOT equivalent to $e^{\ln x}$? **1**

(A) $\ln(e^x)$

(B) $x^{\ln e}$

(C) $(\ln e)^x$

(D) $\dfrac{1}{e^{\ln\frac{1}{x}}}$

(b) Solve for x: **2**

$$\log_3 x + 2 = \log_3(x+2)$$

(c) By using a suitable substitution, solve $4^x - 5\times 2^{x+1} + 16 = 0$. **2**

(d) Find the equation of the normal to the curve $y = e^{2x+2}$ at the point where $x = -1$. **3**

End of Question 2

Question 3 starts on the next page

Question 3 (7 marks) *Start a new page* **Marks**

(a) How many solutions does the equation $\sin^2 x + 3\sin x\cos x + 2\cos^2 x = 0$ have in the domain $0 \le x \le 2\pi$? **1**

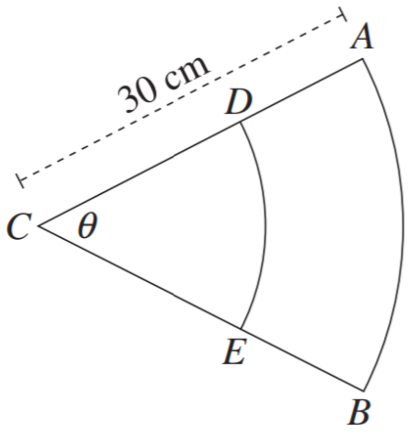
(A) 1

(B) 2

(C) 3

(D) 4

(b) The region *ABC* is a sector of a circle with radius 30 cm, centred at *C*. The angle of the sector is θ. The arc *DE* lies on a circle also centred at *C*, as shown in the diagram. **2**



The arc *DE* divides the sector *ABC* into two regions of equal area.

Find the exact length of the interval *CD*.

Question 3 continues on the next page.

(c) Two hundred rabbits in a region with an estimated population of 200 000 rabbits have a highly contagious disease.

The disease is known to spread at the weekly rate of 1% of the remaining healthy rabbits such that:

$$\frac{dP}{dt} = 0.01(200\,000 - P)$$

where P is the number of infected rabbits after t weeks.

(i) Show that $P = 200\,000 - 199\,800e^{-0.01t}$ satisfies the differential equation. **1**

(ii) How many days does it take for half of the rabbit population to become infected? **3**

End of Question 3

Question 4 starts on the next page

Question 4 (9 marks) *Start a new page* **Marks**

(a) The radius of a circle is decreasing at a constant rate of 0.1 cm/s. **1**

In terms of the circumference, C, what is the rate of change of the area of the circle in cm^2/s?

(A) $-0.1C$

(B) $\dfrac{-0.1C}{2\pi}$

(C) $0.1^2 C$

(D) $0.1^2 \pi C$

(b) Solve for $-\pi \le x \le \pi$: **1**

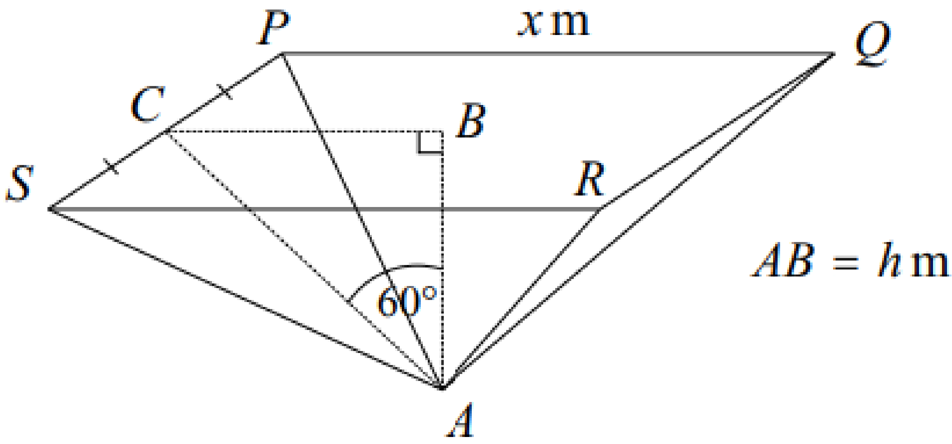
$$\tan\left(x - \frac{\pi}{6}\right) = \sqrt{3}$$

(c) Solve the equation for $0 \le \theta \le 2\pi$, leaving your answer in terms of π. **3**

$$4\sin^2\theta + 2\cos\theta = 3$$

Question 4 continues on the next page.

(d)



The diagram above shows a container in the shape of a square pyramid with ‘base’ *PQRS*.

The centre of the base is the point *B*, and the perpendicular height *AB* of the pyramid is *h* m. The side length of the square base is *x* m, and the slant height *AC* of the pyramid makes an angle of 60° to the vertical, as shown in the diagram.

(i) Show that $x = 2h\sqrt{3}$. **1**

(ii) Hence show that $h = \left(\frac{V}{4}\right)^{\frac{1}{3}}$, where *V* is the volume of the pyramid in cubic metres. (Volume of pyramid: $V = \frac{1}{3} \times \textit{area of base} \times \textit{perpendicular height}$) **1**

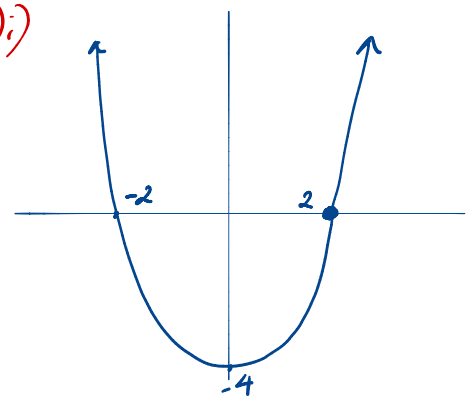
(iii) The pyramid is initially empty and is filled with water at a constant rate of 0.5 m^3/min. Find that rate at which the height of the water is increasing after it has been filled for 8 minutes. **2**

End of Question 4

End of Assessment

2020 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments
Question 1	
a) $f(x)=\frac{1}{3}(x)^{-1}$	
$f'(x)=-\frac{1}{3}\times\frac{1}{x^2}$	
$f'(2)=\frac{-1}{3}\times\frac{1}{2^2}=\frac{-1}{12}$	
Answer: B	
b) $y=3x^2+2x-4$	
$y'=6x+4$	
from $y=-10x+c$, $m=-10$, $y'=-10$	
$-10=6x+2$	
$-12=6x$	
$x=-2$	1 for x
sub $x=-2$ into y	
$y=3(-2)^2+2(-2)-4$	
$y=4$	1 for y
sub x & y into tangent	
$4=-10(-2)+c$	
$4=20+c$	
$c=-16$	1 correct sub of x & y
c) $y=(3x+1)^2$	
$y'=2(3x+1)\times 3$	
$=6(3x+1)$	
LHS/ $(3x-1)^2-6(3x+1)+4$	
$9x^2+6x+1-18x-6+4$	
$9x^2-12x-1$	1 correct working for LHS

Suggested Solution (s)	Comments
RHS	1 correct working for RHS.
$x(18x+6-12)-3(x+1)^2$	
$18x^2-6x-9x^2-6x-1$	
$9x^2-12x-1$	
$\therefore$ LHS = RHS	
d) i) 	1 must be smooth @ $x=2$ 1 linear function must be smooth
ii) it is smooth @ $x=2$ hence differentiable	1 stating smooth
note: error carry for part (i) incorrect graph	

2020 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 2 a) Answer: C $(\ln e^x) = 1^x$ b) $2 = \log_3(x+2) - \log_3 x$ $\log_3\left(\frac{x+2}{x}\right) = 2$ ✓ $\frac{x+2}{x} = 9$ $x+2 = 9x$ $2 = 8x$ $x = \frac{1}{4}$ ✓ c) $4^x - 5\times 2^{x+1} = 16$ let $u = 2^x$ $(2^x)^2 - 5\times(2^x)\times 2 + 16 = 0$ $(2^x)^2 - 10(2^x) + 16 = 0$ ✓ $u^2 - 10u + 16 = 0$ $(u-8)(u-2) = 0$ $u = 8$ or $u = 2$ $2^x = 8$ $2^x = 2$ $x = 3$ $x = 1$ ✓	① use of log laws ① answer ① correct sub ① answer note: no marks awarded for guess & check.	d) $y = e^{2x+2}$ $y' = 2e^{2x+2}$ at $x = -1$, $y' = 2e^{-2+2}$ $y' = 2e^0$ $y' = 2$ $m_1 = 2$, normal : $m_2 = -\frac{1}{2}$ ✓ @ $x = -1$, $y = e^0$ $y = 1$ $(-1, 1)$ with normal $= -\frac{1}{2}$ ✓ Equation of normal $y - 1 = -\frac{1}{2}(x+1)$ $2y - 2 = -x - 1$ $x + 2y - 1 = 0$ ✓	① finding normal ① x, y coordinate ① eqn of normal.

2020 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

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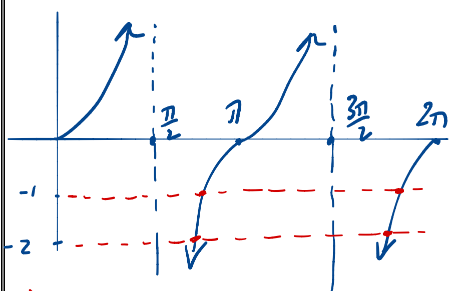
<td>
Question 3
a) Answer: D
$(\sin x+2\cos x)(\sin x+\cos x)=0$
$\sin x=-2\cos x$ $\sin x=-\cos x$
$\tan x=-2$ $\tan x=-1$
2 solutions   2 solutions


b) Area of sector ACB
$\frac{1}{2}r^2\theta$
$\frac{1}{2}\times(30)^2\times\theta$
$=450\theta$ ✓
Area of sector DCE
$\frac{1}{2}\times450\theta$
$=225\theta$
$\therefore \frac{1}{2}r^2\theta=225\theta$
$r^2=450$
$r=15\sqrt{2}$, $r>0$ ✓
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1 for either sector ACB or DCE

1 working
</td>
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c) i) $P=200\,000-199\,800e^{-0.01t}$
$\frac{dP}{dt}=-1998e^{-0.01t}$
given $\frac{dP}{dt}=0.01(200\,000-P)$
sub in P
$0.01(200\,000-(200\,000-199\,800e^{-0.01t}))$
$0.01(199\,800e^{-0.01t})$
$1998e^{-0.01t}$ QED
ii) when $P>100\,000$
$200\,000-199\,800e^{-0.01t}>100\,000$
$199\,800e^{-0.01t}<100\,000$
$e^{-0.01t}<\frac{100000}{199800}$ ✓
$-0.01t<\ln\left(\frac{500}{999}\right)$
$t>100\ln\left(\frac{500}{999}\right)$ ✓
$t>69.214..$ weeks
$t>484.502...$ days ✓
$\therefore$ Half rabbit population is infected after 485 days.
note: if answer written as 69.24 days but rounded to 70 days → mark still awarded.
</td>
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1 working
* allow for alternate method.

1 in terms of e

1 for t

1 answer
no mark awarded for 484 days
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2020 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 4 a) $\frac{dr}{dt} = -0.1$ cm/s $A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $= 2\pi r \times -0.1$ $= -0.1C$ Answer: A b) $\tan\left(x - \frac{\pi}{6}\right) = \sqrt{3}$ $x - \frac{\pi}{6} = \frac{\pi}{3}$ or $-\frac{2\pi}{3}$ $x = \frac{\pi}{2}$ or $\frac{-\pi}{2}$ ✓ c) $4\sin^2\theta + 2\cos\theta = 3$ $4(1-\cos^2\theta) + 2\cos\theta = 3$ $4 - 4\cos^2\theta + 2\cos\theta = 3$ $4\cos^2\theta - 2\cos\theta - 1 = 0$ ✓ $\cos\theta = \frac{2 \pm \sqrt{4+16}}{8}$ $\cos\theta = \frac{2 \pm 2\sqrt{5}}{8} \Rightarrow \frac{1 \pm \sqrt{5}}{4}$ ✓ $\theta = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$ ✓	① answer ① in terms of $\cos^2\theta$ ① using quadratic formula ① solution	d) i) $\tan 60° = \frac{\frac{1}{2}x}{h}$ $\frac{(BC)}{(BA)}$ $\sqrt{3} = \frac{\frac{1}{2}x}{h}$ $\sqrt{3}h = \frac{1}{2}x$ $x = 2\sqrt{3}h$ ✓ ii) $V = \frac{1}{3}x^2h$ $V = \frac{1}{3}(2\sqrt{3}h)^2 \times h$ $V = \frac{12h^2}{3} \times h$ $V = 4h^3 \Rightarrow \frac{V}{4} = h^3 \Rightarrow h = \left(\frac{V}{4}\right)^{\frac{1}{3}}$ iii) $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $\frac{dV}{dt} = 0.5$ $\frac{dh}{dV} = \frac{1}{3}\left(\frac{1}{4}V\right)^{-2/3} \times \frac{1}{4}$ $= \frac{1}{12\left(\frac{1}{4}V\right)^{2/3}}$ ✓ after 8 mins: $V = 0.5 \times 8$ $V = 4$ $\frac{dh}{dt} = \frac{1}{12\left(\frac{1}{4} \times 4\right)^{2/3}} \times 0.5$ $= \frac{1}{24}$ m/min ✓	① working ① correct working to h. ① correct differentiation for $\frac{dh}{dV}$ ① answer

2020 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Alternate Solution $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $0.5 = 12h^2 \times \frac{dh}{dt}$ Also when $t=8$, $V=4$ as $\frac{dV}{dt} = 0.5\,m^3/min$ & h is $\left(\frac{4}{4}\right)^{1/3} = 1m$ So $\frac{dh}{dt} = \frac{1}{24h^2} = \frac{1}{24}\ m/min$			