Year 11 Mathematics Extension 1
Assessment Task 1
Term 2, Week 5, 2021

Name: ....

Teacher: ....

Friday 21st May, 2021

Setter: WIL

General Instructions

- **Working time – 45 minutes.**
- Answer all questions on the lined paper provided using black or blue pen.
- NESA approved calculators may be used.
- Show all relevant mathematical reasoning and/or calculations.
- **Total marks – 32.**
- Attempt Questions 1 – 5.

Question 1 (6 marks) **Marks**

For **Multiple Choice** questions, write down the correct option on your writing paper.

(a) Which one of the following transformations is **not** required to obtain $g(x) = -1 + \frac{1}{2} f(3-2x)$ from $f(x)$? **1**

A. A reflection in the *y*-axis

B. A reflection in the *x*-axis

C. A translation parallel to the *x*-axis

D. A compression in the *y* direction.

(b) Suppose that the equation $x^2 + 5x - 1 = 0$ has roots α and β. Without solving the equation, find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha - \beta)^2$ **2**

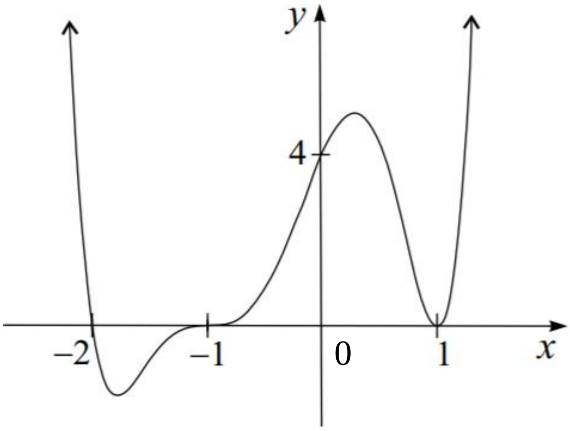
(iv) $|\alpha - \beta|$ **1**

End of Question 1

Question 2 (6 marks) (Start a new page) **Marks**

For **Multiple Choice** questions, write down the correct option on your writing paper.

(a) The diagram below shows the graph of $y = a(x+b)(x+c)^2(x+d)^3$.
What are the possible values of *a*, *b*, *c* and *d*? **1**



A. $a = 1,\ b = 1,\ c = -2,\ d = -1$

B. $a = 1,\ b = 2,\ c = -1,\ d = 1$

C. $a = 2,\ b = 2,\ c = -1,\ d = 1$

D. $a = 2,\ b = 2,\ c = 1,\ d = -1$.

(b) Solve $\sin 2\theta = -\tfrac{1}{2}\sqrt{3}$, for $-90° \le \theta \le 90°$. **2**

(c) (i) Without the use of calculus, sketch (using quarter of a page) the polynomial $y = x(x^2 - 1)$ showing all the intercepts with the axes. **1**

(ii) Hence, or otherwise, solve the inequation $\dfrac{x(x+1)}{(x-1)} \ge 0$. **2**

End of Question 2

Question 3 (6 marks) (Start a new page) **Marks**

For this **Multiple Choice** question, write down the correct option on the yellow sheet for Question 3.

(a) A curve is defined parametrically by the equations $x = \sin\theta$, $y = 2\cos\theta$.
What is the Cartesian equation of the curve? **1**

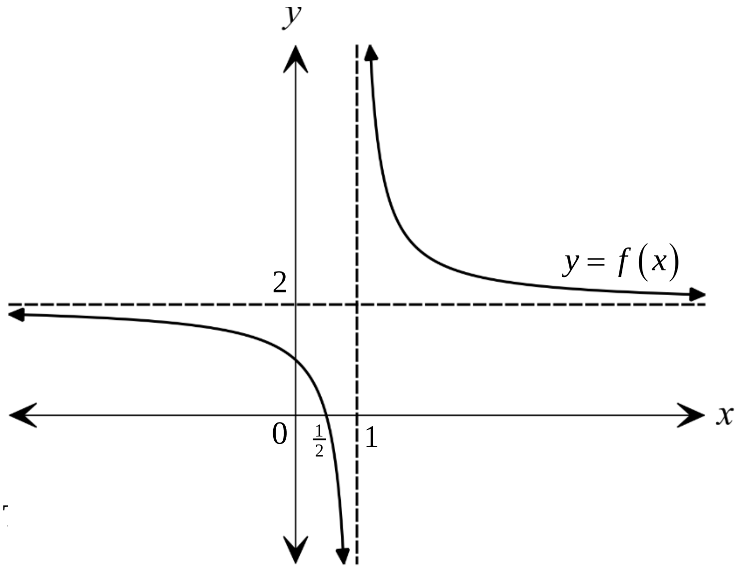
A. $4x^2 + y^2 = 4$

B. $4x^2 + y^2 = 1$

C. $x^2 + y^2 = 1$

D. $x^2 + y^2 = 4$.

(b)



The diagram above shows the graph of $y = f(x)$, which has a horizontal asymptote of $y = 2$ and a vertical asymptote of $x = 1$. Use the separate yellow sheet for Question 3 to sketch the following graphs, clearly showing any intercepts with axes and asymptotes. Please go over $f(x)$ on the template with your pen where it is part of your solution.

(i) $y = |f(x)|$ **1**

(ii) $y = f(|x|)$ **1**

(iii) $|y| = f(x)$ **1**

(iv) $y = f(x) + g(x)$, where $g(x) = x$. **2**

End of Question 3

Question 4 (7 marks) (Start a new page) **Marks**

For this **Multiple Choice** question, write down the correct option on the green sheet for <u>Question 4</u>.

(a) A function is defined by $f(x) = -\sqrt{4-x^2}$ for $0 \le x \le 2$. Which of the following correctly represents the inverse function of $f(x)$? **1**

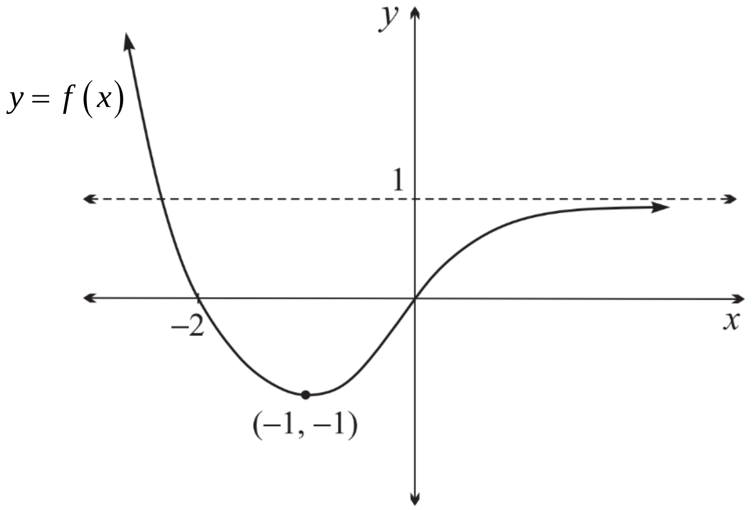
A. $f^{-1}(x) = -\sqrt{4-x^2}$ for $0 \le x \le 2$

B. $f^{-1}(x) = \sqrt{4-x^2}$ for $0 \le x \le 2$

C. $f^{-1}(x) = -\sqrt{4-x^2}$ for $-2 \le x \le 0$

D. $f^{-1}(x) = \sqrt{4-x^2}$ for $-2 \le x \le 0$.

(b)



The diagram above shows the graph of $y = f(x)$, which has a horizontal asymptote of $y = 1$. Use the separate green sheet for Question 4 to sketch the following graphs, clearly showing any intercepts with axes and asymptotes. Please go over $f(x)$ on the template with your pen, where it is part of your solution.

(i) $y = \dfrac{1}{f(x)}$ **1**

(ii) $y^2 = f(x)$ **2**

(c) (i) A polynomial $Q(x)$ is defined by $Q(x) = x^3 - b^3$, where b is a constant. Divide $Q(x)$ by $(x-b)$, and hence express $Q(x)$ as the product of a linear and a quadratic factor. **1**

(ii) Using your answer to part (i) or otherwise, simplify $\dfrac{(\tan^2\theta - 1)(\sin\theta\cos\theta + 1)}{\cos^2\theta(\tan^3\theta - 1)}$. **2**

End of Question 4

Question 5 (7 marks) (Start a new page) **Marks**

For **Multiple Choice** questions, write down the correct option on your writing paper.

(a) If $\cos\alpha = \frac{a}{b}$, where α is an acute angle, what is the value of $\sec(180° + \alpha)$? **1**

A. $\frac{\sqrt{b^2 - a^2}}{b}$

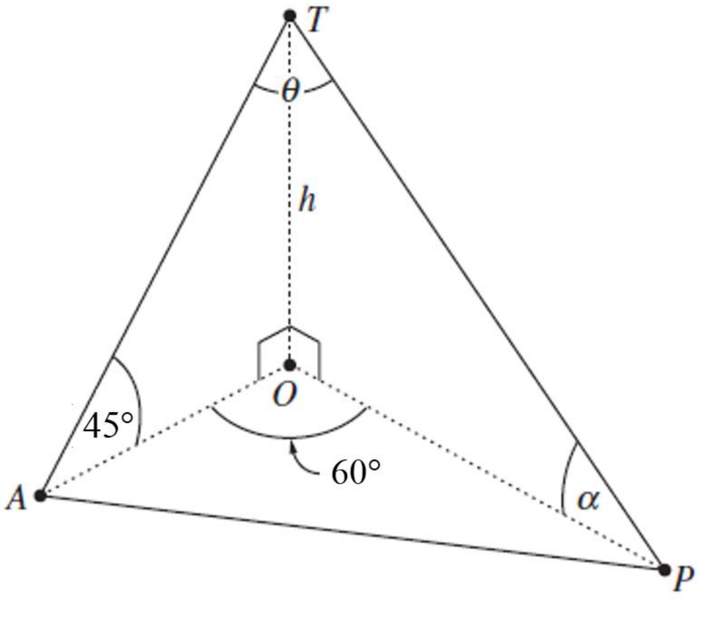
B. $\frac{a}{b}$

C. $-\frac{a}{b}$

D. $-\frac{b}{a}$

(b) Find the domain of the function $f(x) = \sqrt{9 - x^2} + \frac{1}{x+3}$. **1**

(c) Consider the diagram below, which shows a vertical tower OT of height h metres, a fixed point A, and a variable point P that is constrained to move so that angle AOP is $60°$. The angle of elevation of T from A is $45°$. Let the angle of elevation of T from P be α and let angle ATP be θ.



(i) By considering triangle AOP, show that $AP^2 = h^2 + h^2\cot^2\alpha - h^2\cot\alpha$. **2**

(ii) By finding a second expression for AP^2, deduce that $\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$. **3**

End of Assessment Task

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Comments

Question 1

(a) Ⓑ ✓

(b)(i) $\alpha + \beta = -\frac{b}{a}$

$= -5$ ✓

(ii) $\alpha\beta = \frac{c}{a}$

$= -1$ ✓

(iii) $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= \alpha^2 + \beta^2 - 2\alpha\beta$

$= (\alpha + \beta)^2 - 4\alpha\beta$ ✓

$= (-5)^2 - 4(-1)$

$= 29$ ✓

(iv) $|\alpha - \beta| = \sqrt{(\alpha - \beta)^2}$

$= \sqrt{29}$ ✓

Question 2

(a) Ⓒ ✓

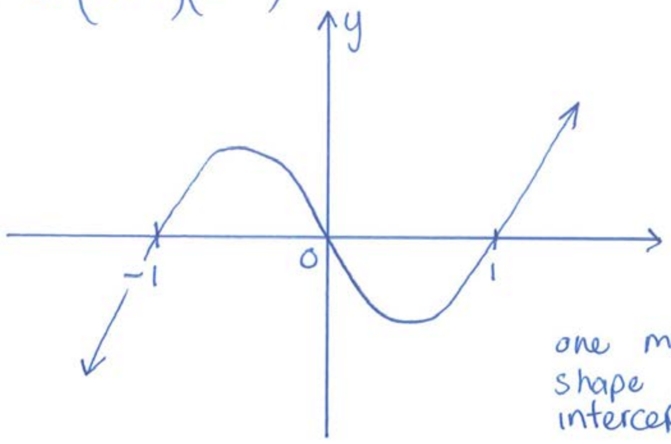
(b) $\text{ref} \angle = 60^\circ$ $\begin{array}{c|c} S & A \\ \hline \checkmark T & C\checkmark \end{array}$ one for correct placement one for answers

$-180^\circ \leq 2\theta \leq 180^\circ$

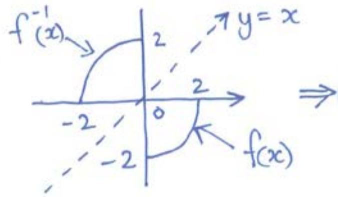
$2\theta = -120^\circ, -60^\circ$

$\therefore \theta = -60^\circ, -30^\circ$

(c)(i) $y = x(x+1)(x-1)$



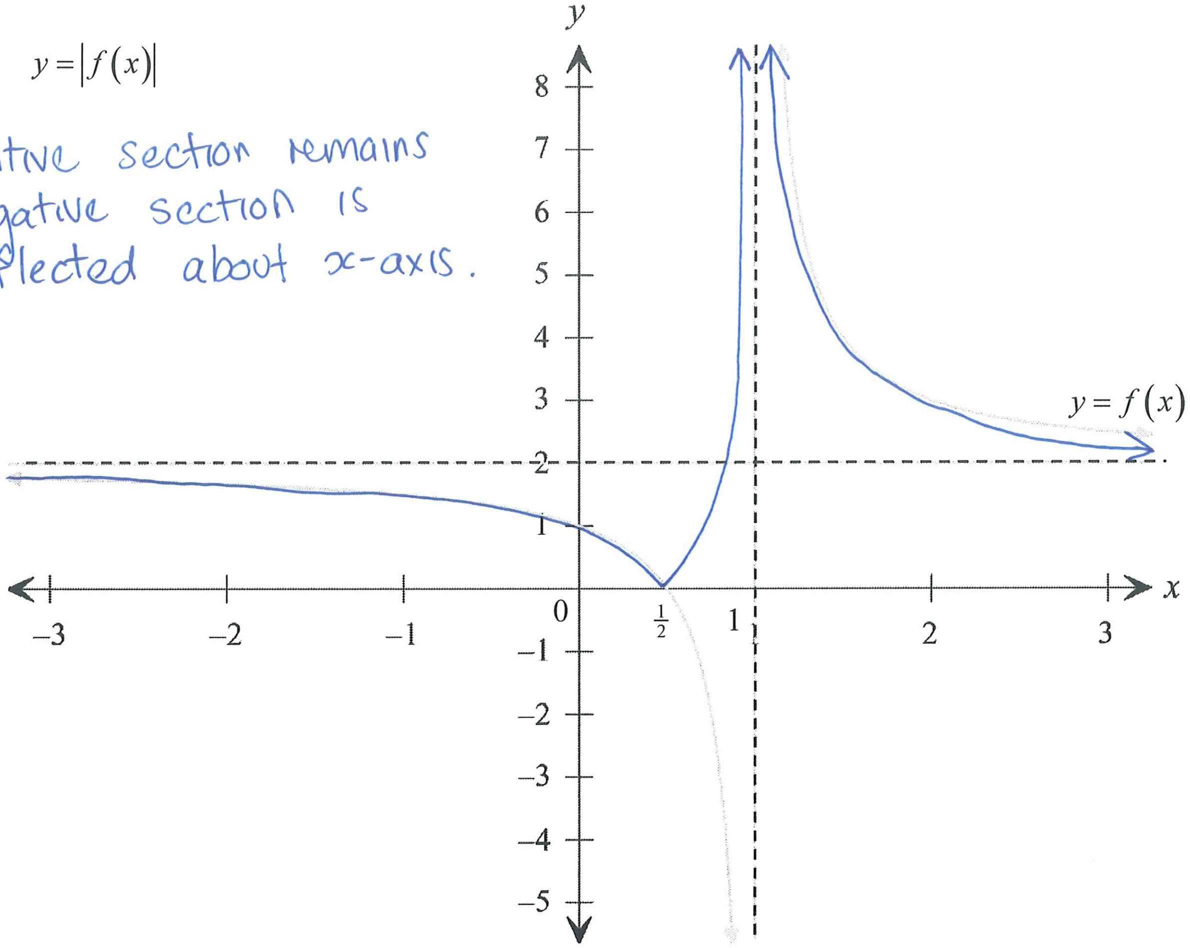
one mark for shape and intercepts.

Suggested Solution (s)	Comments
(ii) $\frac{x(x+1)(x-1)^2}{(x-1)} \geqslant 0$ multiplying by square of denominator $\Rightarrow x(x+1)(x-1) \geqslant 0$ using part (i) $-1 \leqslant x \leqslant 0$ and $x > 1$ one mark for each inequality. Question 3 (a) $x^2 = \sin^2\theta$ — ① $y^2 = 4\cos^2\theta$ $\frac{y^2}{4} = \cos^2\theta$ — ② ① + ② $\Rightarrow x^2 + \frac{y^2}{4} = 1$ or $4x^2 + y^2 = 4 \Rightarrow$ Ⓐ (b) separate solutions sheet Question 4 (a)  $\Rightarrow$ Ⓓ ✓ (b) separate solutions sheet	

Answer Sheet for Question 3 (b) (Use the light grey as a guide, only lines made in pen will be marked).

(b) (i) $y=\left|f(x)\right|$

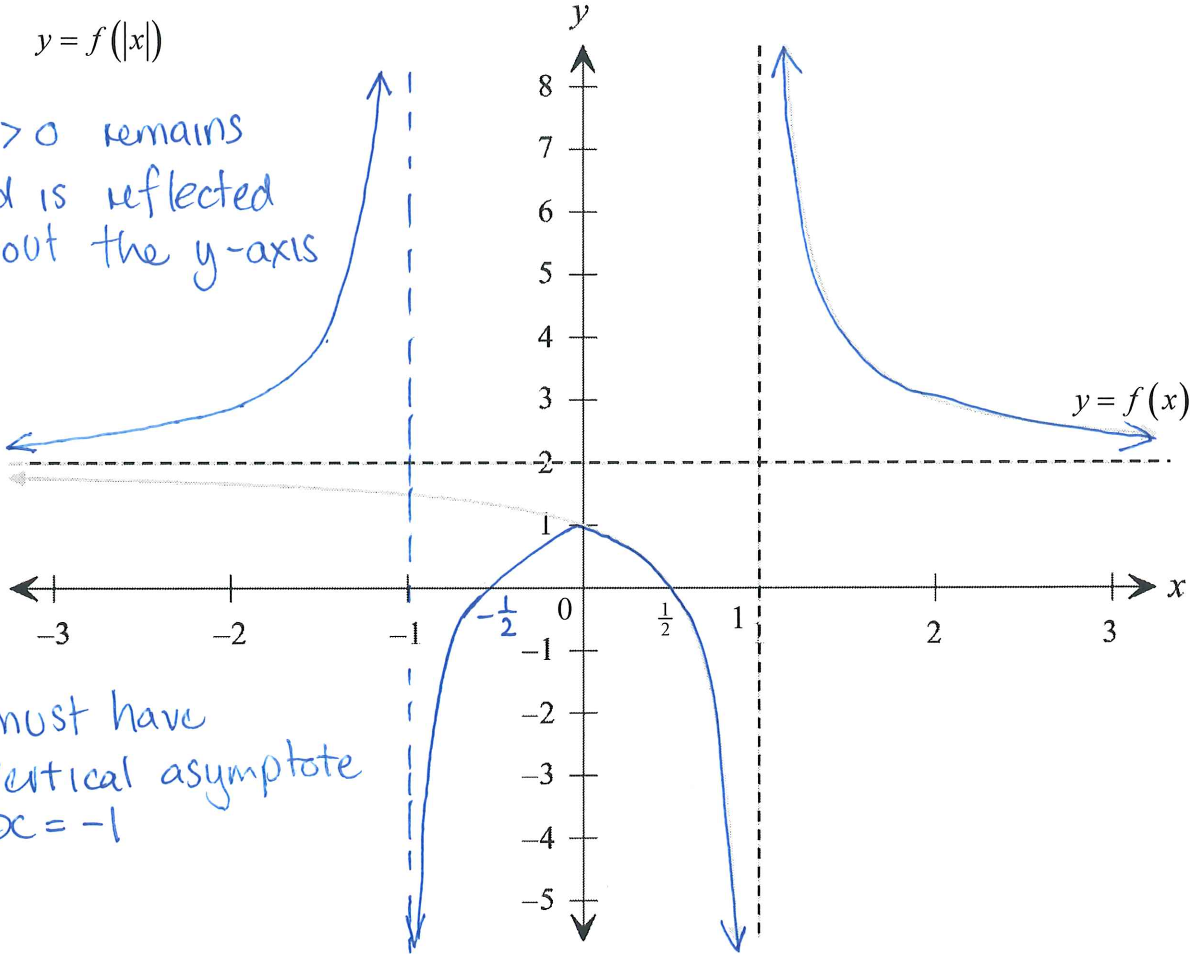
- positive section remains
- negative section is reflected about x-axis.



(b) (ii) $y=f\left(|x|\right)$

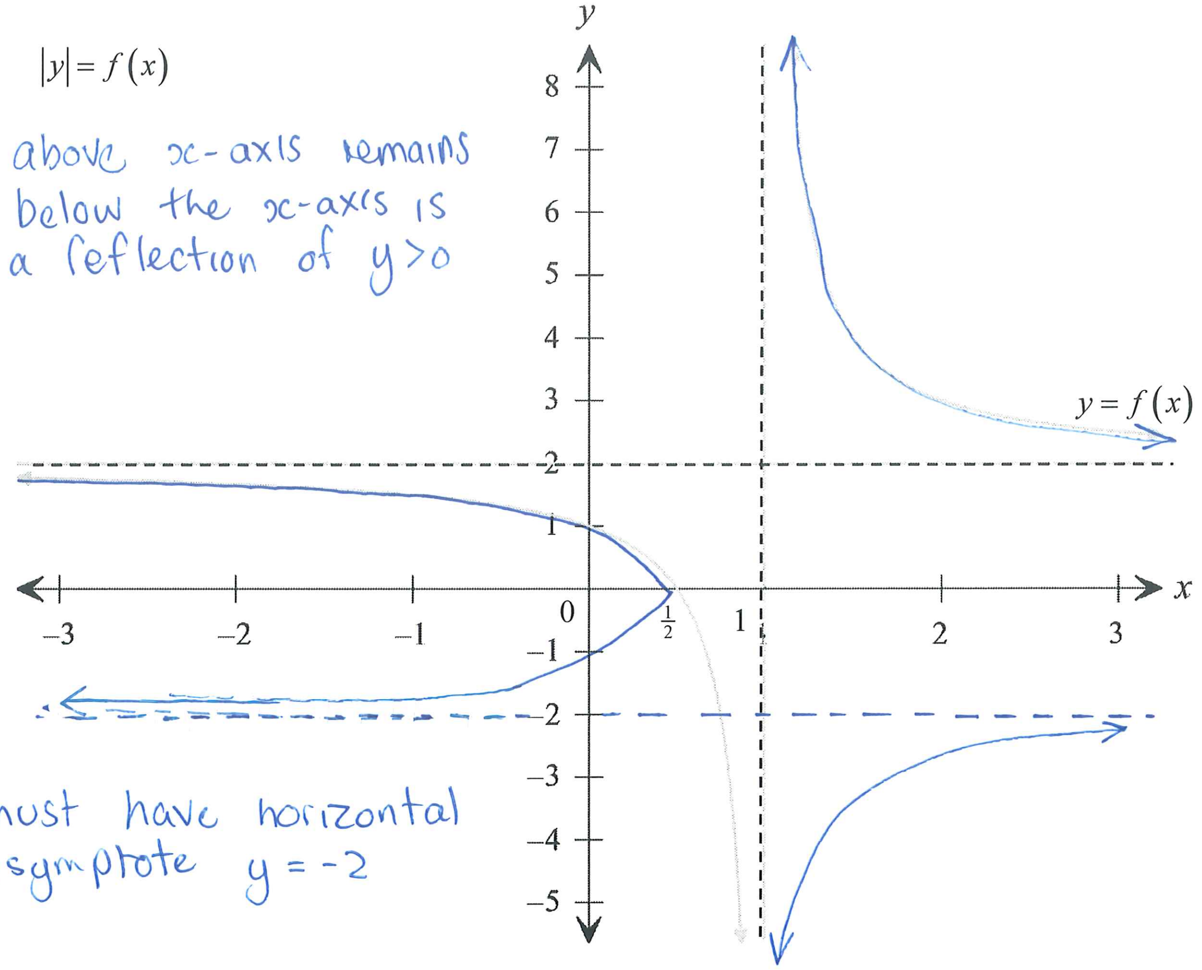
- $x>0$ remains and is reflected about the y-axis

- must have vertical asymptote $x=-1$



(b) (iii) $|y| = f(x)$

- above x-axis remains
- below the x-axis is a reflection of $y>0$

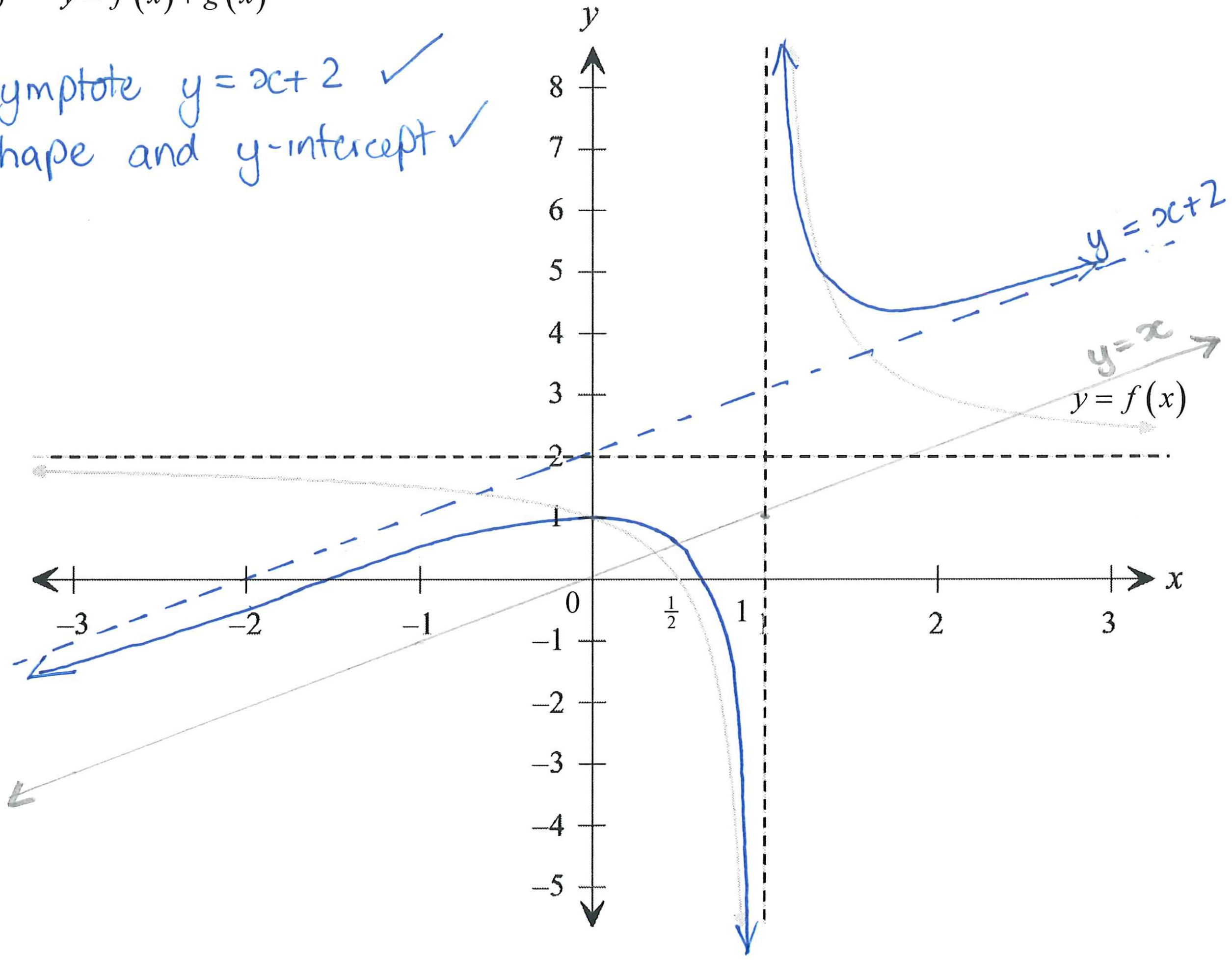


- must have horizontal asymptote $y = -2$

(b) (iv) $y = f(x) + g(x)$

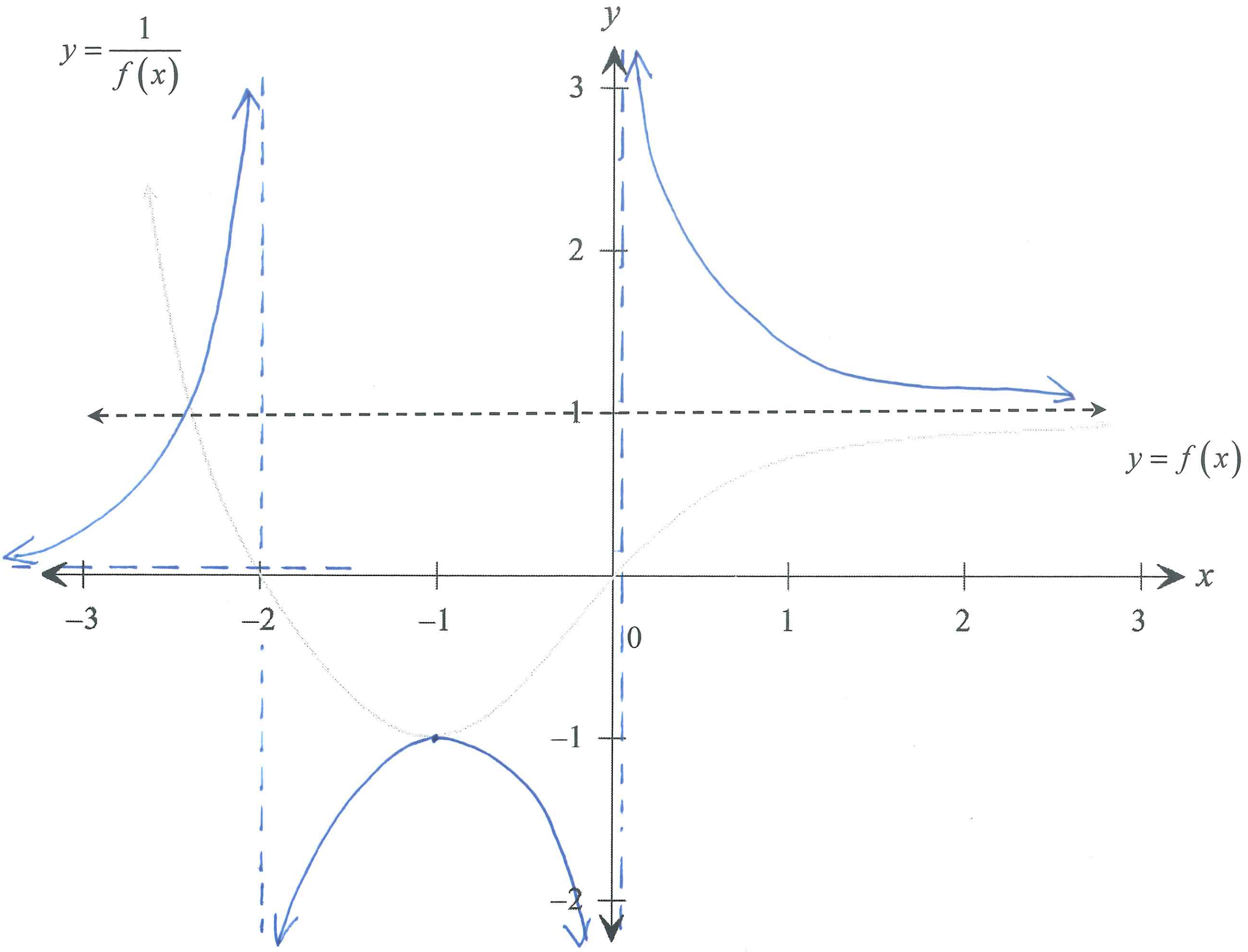
asymptote $y = x + 2$ ✓

Shape and y-intercept ✓

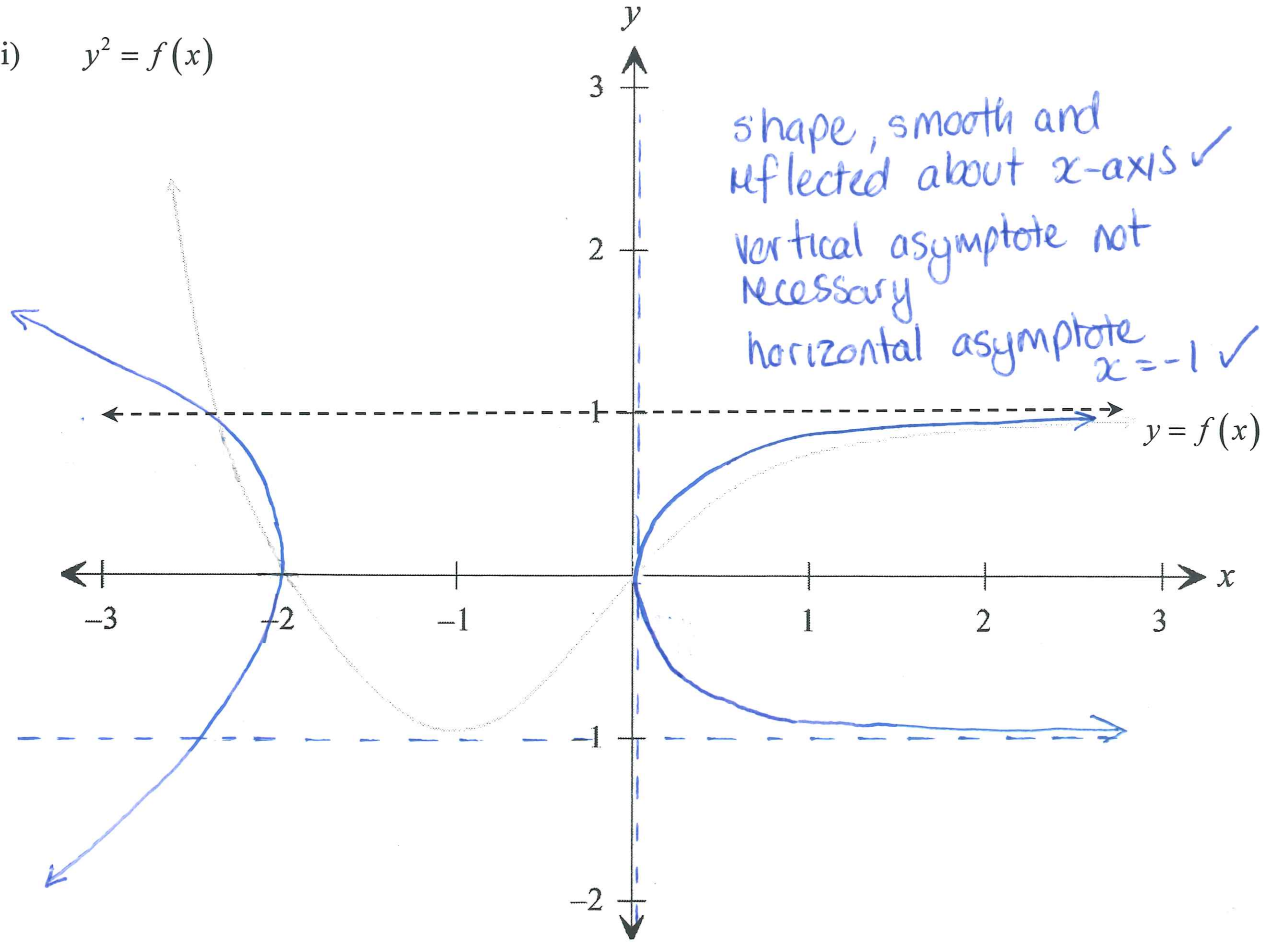


Answer Sheet for Question 4 (b) (Use the light grey as a guide, only lines made in pen will be marked).

(b) (i) $y = \dfrac{1}{f(x)}$



(b) (ii) $y^2 = f(x)$



shape, smooth and reflected about x-axis ✓

vertical asymptote not necessary

horizontal asymptote $x = -1$ ✓

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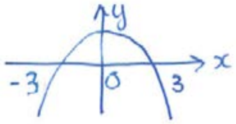
Suggested Solution (s)	Comments
(c)(i) x^2+bx+b^2 ✓ $x-b\ \overline{)\,x^3+0x^2+0x-b^3}$ $-(x^3-bx^2)$ ↓ bx^2+0x $-(bx^2-b^2x)$ ↓ b^2x-b^3 $-(b^2x-b^3)$ 0 $\therefore x^3-b^3=(x-b)(x^2+bx+b^2)$ ✓ (ii) $\frac{(\tan^2\theta-1)(\sin\theta\cos\theta+1)}{\cos^2\theta(\tan^3\theta-1)}=\frac{(\tan\theta+1)(\tan\theta-1)(\sin\theta\cos\theta+1)}{\cos^2\theta(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}$ ✓ $=\frac{(\tan\theta+1)(\sin\theta\cos\theta+1)}{\cos^2\theta(\sec^2\theta+\tan\theta)}$ ✓ $=\frac{(\tan\theta+1)(\sin\theta\cos\theta+1)}{1+\sin\theta\cos\theta}$ $=\tan\theta+1$ **Question 5** (a) $\begin{array}{cc} S & A \\ \hline \checkmark T & C \end{array}$ $\sec(180^\circ+\alpha)$ $=\frac{1}{\cos(180^\circ+\alpha)}$ $=\frac{-1}{\cos\alpha}$ $=-\frac{b}{a}\Rightarrow$ Ⓓ ✓ (b) $9-x^2\geqslant 0$  $-3\leqslant x\leqslant 3$ but $x\neq-3$ $\therefore$ D: $-3<x\leqslant 3$ ✓	

Suggested Solution (s)	Comments
(c)(i) $\tan 45° = \frac{h}{AO}$ $\quad \tan\alpha = \frac{h}{OP}$ $AO = h$, $\quad OP = h\cot\alpha$ both ✓ $\left.\begin{aligned} AP^2 &= AO^2 + OP^2 - 2(AO)(OP)\cos 60° \\ AP^2 &= h^2 + h^2\cot^2\alpha - 2h^2\cot\alpha\left(\tfrac{1}{2}\right) \end{aligned}\right\}$ either ✓ (ii) $\sin 45° = \frac{h}{AT}$, $\quad \sin\alpha = \frac{h}{TP}$ $AT = \sqrt{2}h$, $\quad TP = h\,\mathrm{cosec}\,\alpha$ $AP^2 = (\sqrt{2}h)^2 + (h\,\mathrm{cosec}\,\alpha)^2 - 2(\sqrt{2}h)h\,\mathrm{cosec}\,\alpha\cos\theta$ $= 2h^2 + h^2\mathrm{cosec}^2\alpha - 2\sqrt{2}h^2\mathrm{cosec}\,\alpha\cos\theta$ ✓ $h^2 + h^2\cot^2\alpha - h^2\cot\alpha = 2h^2 + h^2\mathrm{cosec}^2\alpha - 2\sqrt{2}h^2\mathrm{cosec}\,\alpha\cos\theta$ dividing through by h^2 and using identity $1+\cot^2\alpha = \mathrm{cosec}^2\alpha$ ✓ $1 + \cot^2\alpha - \cot\alpha = 2 + \mathrm{cosec}^2\alpha - 2\sqrt{2}\,\mathrm{cosec}\,\alpha\cos\theta$ $-\cot\alpha = 2 - 2\sqrt{2}\,\mathrm{cosec}\,\alpha\cos\theta$ $\left.\begin{aligned} \frac{2\sqrt{2}\cos\theta}{\sin\alpha} &= 2 + \frac{\cos\alpha}{\sin\alpha} \\ 2\sqrt{2}\cos\theta &= 2\sin\alpha + \cos\alpha \end{aligned}\right\}$ ✓ $\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$	