Year 11 Mathematics Extension 1
Assessment Task 1
Term 2, Week 6, 2022

Name: ....

Teacher: ....

Monday 30th May, 2022

Setter: WIL

General Instructions

- **Working time – 45 minutes.**
- Answer all questions on the lined paper provided using black or blue pen.
- NESA approved calculators may be used.
- Show all relevant mathematical reasoning and/or calculations.
- Please remove the last page and submit it with your solutions for Questions 3.
- **Total marks – 32.**
- Attempt Questions $1-6$.

Question 1 (5 marks) **Marks**

(a) (i) Show that $(x-3)$ is a factor of the polynomial $P(x)=x^3-4x^2+x+6$. **1**

(ii) Hence or otherwise, express $P(x)$ in fully factored form. **1**

(b) Consider the functions $f(x)=\dfrac{1}{x+1}$ and $g(x)=\dfrac{1}{1-x}$.

(i) Show that $f\circ g(x)=\dfrac{1-x}{2-x}$. **1**

(ii) Without the use of calculus, sketch (using quarter of a page) $y=f\circ g(x)$ showing all the intercepts with the axes, asymptotes and any exclusions in its natural domain. **2**

End of Question 1

Question 2 (5 marks) (Start a new page) **Marks**

(a) Write the Cartesian equation given by the parametric equations $x = 4 + \operatorname{cosec}\theta$ and $y = 2 + \cot\theta$. **2**

(b) The roots of $x^3 - x^2 - 5x + 2 = 0$ are α, β and γ.

(i) Show that $\beta + \gamma = 1 - \alpha$. **1**

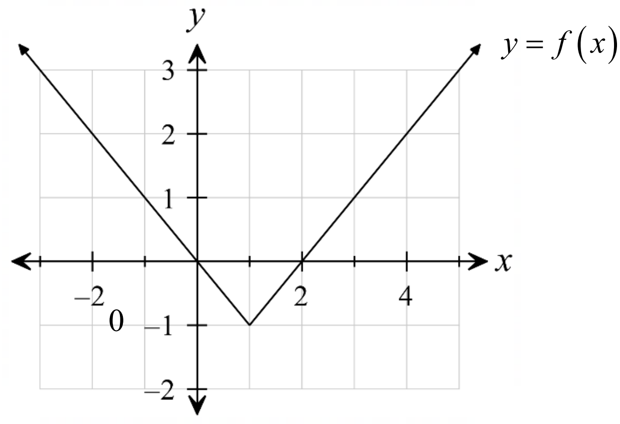
(ii) Using part (i), or otherwise, evaluate $\dfrac{\beta+\gamma}{\alpha} + \dfrac{\gamma+\alpha}{\beta} + \dfrac{\alpha+\beta}{\gamma}$. **2**

End of Question 2

Question 3 (5 marks) (Start a new page) **Marks**

Remove pages 8 and 9 (last page) from your test paper and use them to sketch the graphs and answer Question 3. When sketching graphs, show all intercepts with axes, asymptotes and changes to the curves' shape. Please go over $f(x)$ on the template with your pen to attract marks where necessary.

(a) The diagram below shows the graph of $f(x)=|x-1|-1$.

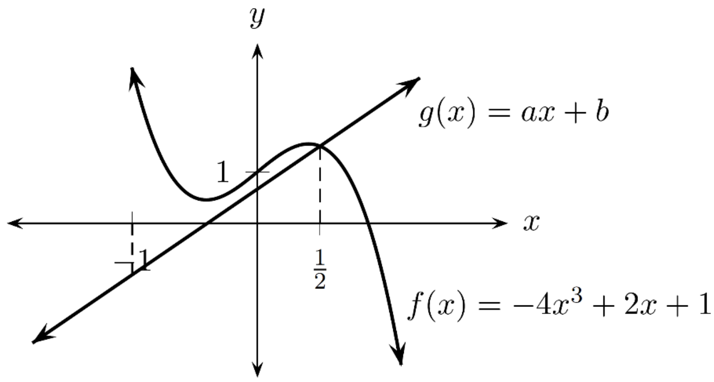


Sketch each of the following using the templates supplied on page 7:

(i) $y=\sqrt{f(x)}$ **1**

(ii) $y=\dfrac{1}{f(x)}$ **1**

(b) The graphs of $f(x)=-4x^3+2x+1$ and $g(x)=ax+b$ are shown below.



(i) Sketch the graph for $h(x)=-4|x|^3+2|x|+1$ using the template supplied on page 8. **1**

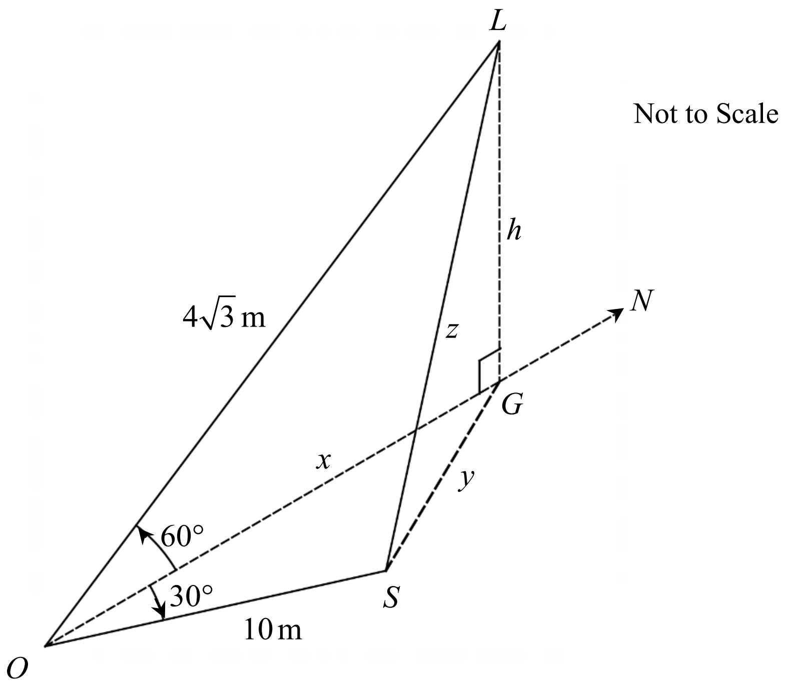
(ii) The solution to the inequality $ax+b>-4|x|^3+2|x|+1$ is $x\in(-\infty,-1)\cup\left(\dfrac{1}{2},\infty\right)$.

Find the values of a and b. **2**

End of Question 3

Question 4 (5 marks) (Start a new page) **Marks**

(a) During the Festival of Lanterns, Owen releases a lantern from point O. It floats up into the sky due north in a straight line with an angle of elevation of $60°$. Sam is located at S, 10 metres from Owen on a bearing of $N30°E$. The lantern has travelled $4\sqrt{3}$ metres from O when Sam first observes it at L. The point G is on the ground directly below L.



Given that $OG = x$, $GL = h$, $SG = y$ and $SL = z$. Find the distance SL, the distance between Sam and the lantern. Leave your answer correct to two decimal places. **3**

(b) The function $g(x) = x^2 - 6x + 8$ has been restricted to the domain $x \le 3$, so that it has an inverse function. By the use of the quadratic formula, or otherwise, find the equation of the inverse function $g^{-1}(x)$. **2**

End of Question 4

Question 5 (6 marks) (Start a new page) **Marks**

(a) (i) Solve the inequality $\dfrac{x}{1-x^2} \geq 0$ for x. **3**

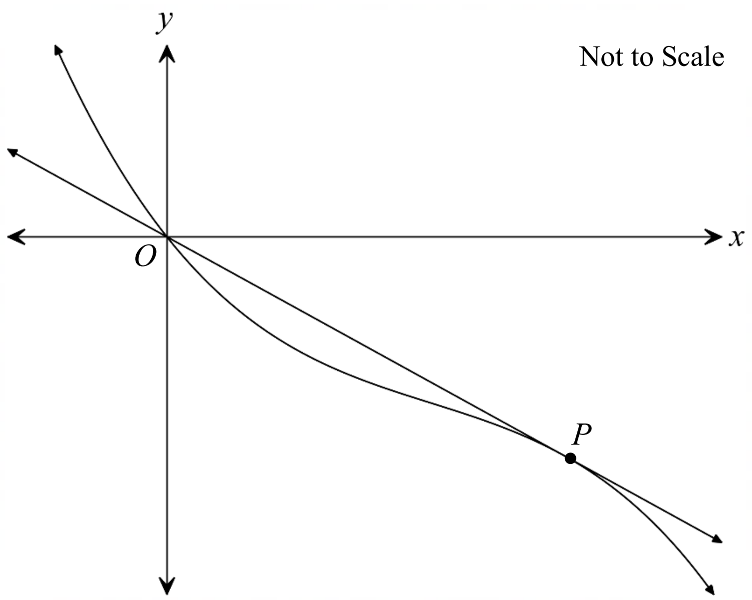
(ii) Show that $\sec x \tan x = \dfrac{\sin x}{1-\sin^2 x}$. **1**

(iii) Hence, or otherwise, find all solutions of $\sec x \tan x \geq 0$ for $0 \leq x \leq 360°$. **2**

End of Question 5

Question 6 (6 marks) (Start a new page) **Marks**

(a) The line $y = mx$ is a tangent to the curve $y = -x^3 + 3x^2 - 4x$ at the point P.



Determine the the coordinates of the point P and the equation of the tangent. **3**

(b) The polynomial $P(x) = px^3 + qx + s$ has a double root at $x = -1$ and a remainder is 6 when it is divided by $(x-1)$. If p, q and s are real, find their values. **3**

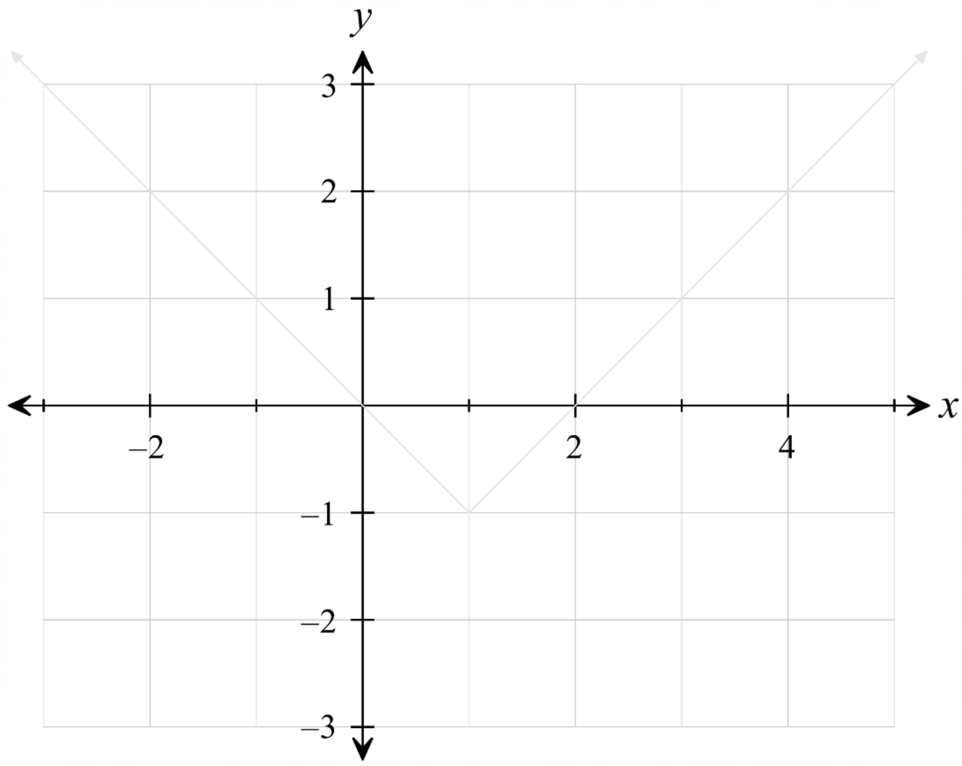
End of Assessment Task

Name: ....

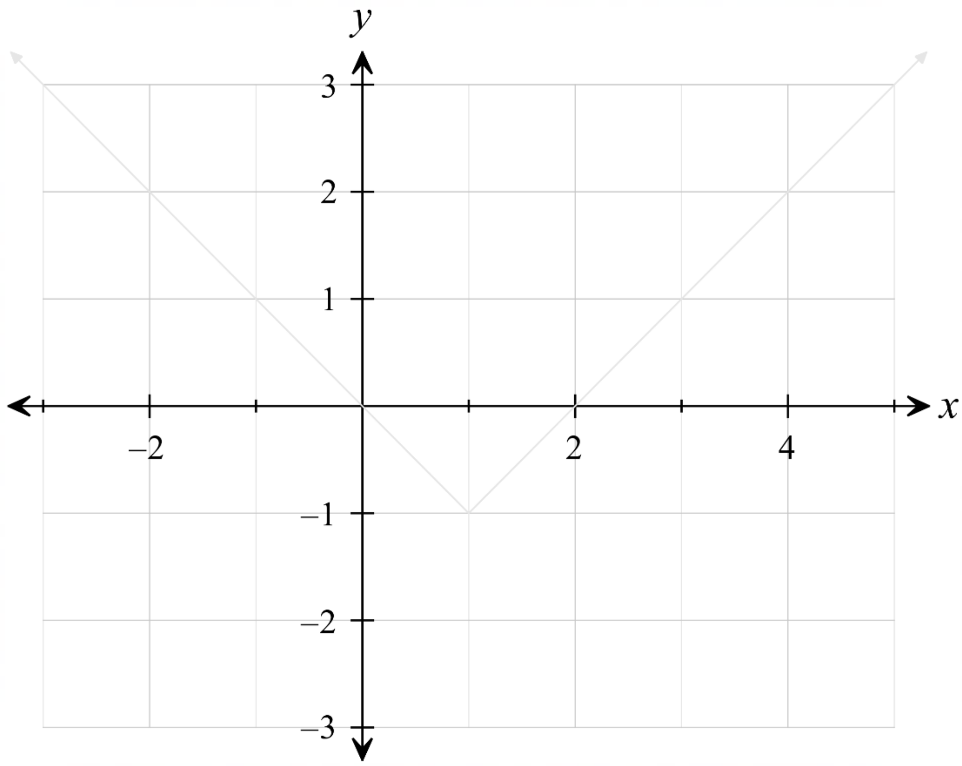
Teacher: ....

Answer Sheet for Question 3 (a) (Use the light grey as a guide, only lines made in pen will be marked).

(a) (i) $y=\sqrt{f(x)}$



(ii) $y=\dfrac{1}{f(x)}$



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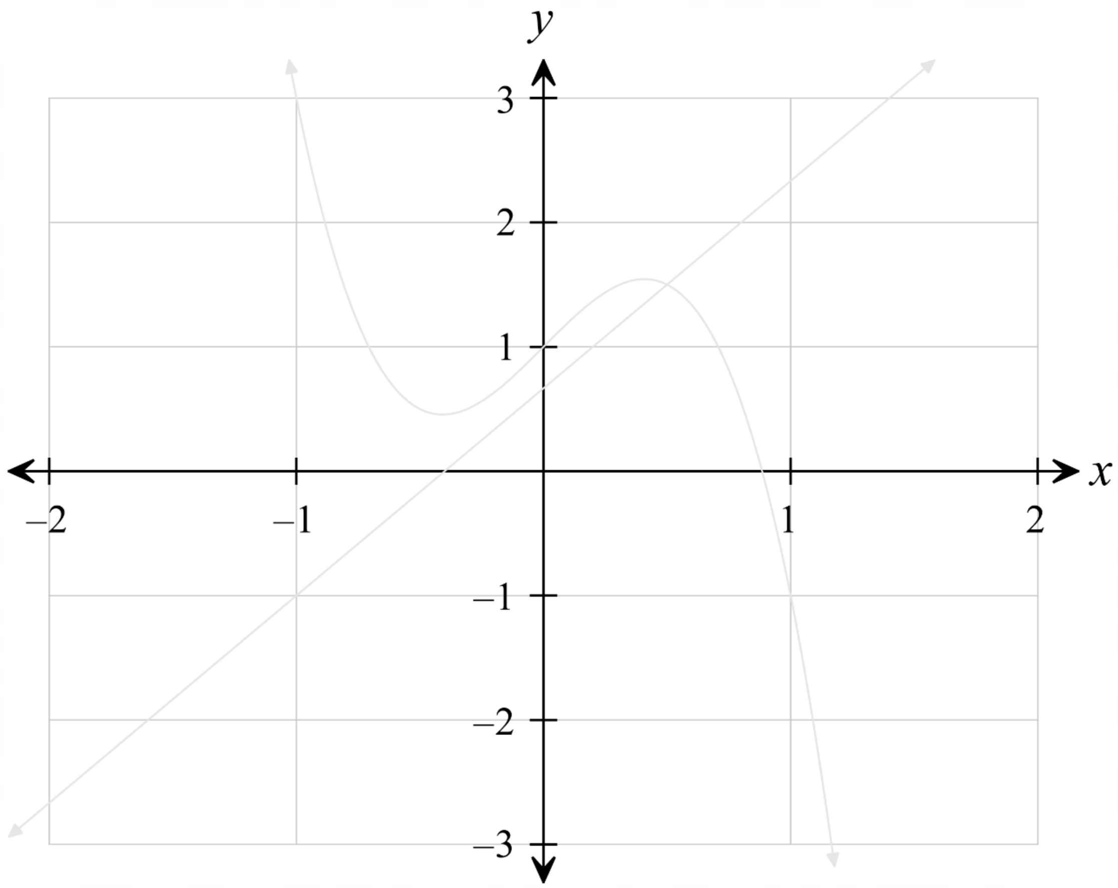
Answer Sheet for Question 3 (b) (Use the light grey as a guide, only lines made in pen will be marked).

(b) (i) $h(x) = -4|x|^3 + 2|x| + 1$



(ii) Write your answer here.

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Suggested Solution (s)	Comments

Question 1.

(a)(i) $P(3) = 3^3 - 4(3^2) + 3 + 6$

$= 27 - 36 + 3 + 6$

$= 0$

either line for one mark.

(ii) method 1

$$\begin{array}{r} x^2 - x - 2 \\ x-3 \overline{\big)\, x^3 - 4x^2 + x + 6} \\ -(x^3 - 3x^2) \downarrow \\ \hline -x^2 + x \\ -(-x^2 + 3x) \\ \hline -2x + 6 \\ (-2x + 6) \\ \hline 0 \end{array}$$

and then factor quotient $(x-2)(x+1)$

method 2

look at possible factors considering the constant term.

try $P(2)$ and then $P(-1)$ or $P(-2)$ and $P(1)$

final answer:

$P(x) = (x-3)(x-2)(x+1)$ ✓

(b)(i) $f \circ g(x) = \dfrac{1}{\frac{1}{1-x} + 1}$

$= \dfrac{1}{\frac{1+1-x}{1-x}}$

$= \dfrac{1-x}{2-x}$

$= \dfrac{2-x-1}{2-x}$

$= 1 - \dfrac{1}{2-x}$

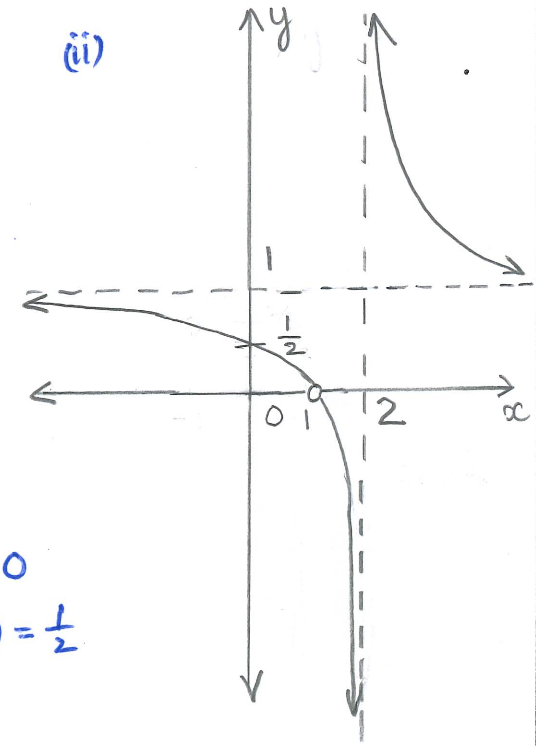
$= 1 + \dfrac{1}{x-2}$

at $x = 1$ $f \circ g(x) = 0$

at $x = 0$ $f \circ g(x) = \frac{1}{2}$

(i) one mark for correct substitution.

(ii)



(ii) key inclusions
- y-intercept
- exclusions at $x = 1$
- shape in correct quadrants
- x and y asymptotes

all 4 required for 2 marks

2 required for one mark.

	Comments

Question 2.

(a) $(x-4)^2 = \operatorname{cosec}^2\theta$, $(y-2)^2 = \cot^2\theta$ } both 1 mark.

... recognising $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ no mark alone.

by subtraction:

$(x-4)^2 - (y-2)^2 = \operatorname{cosec}^2\theta - \cot^2\theta$

$(x-4)^2 - (y-2)^2 = 1$ ✓

(b) (i) investigating the sum gives

$\alpha + \beta + \gamma = 1$ ✓

$\therefore \quad \beta + \gamma = 1 - \alpha$ as required

(ii) rearranging this expression also gives

$\alpha + \beta = 1 - \gamma$ and $\alpha + \gamma = 1 - \beta$

so that: $\dfrac{\beta+\gamma}{\alpha} + \dfrac{\gamma+\alpha}{\beta} + \dfrac{\alpha+\beta}{\gamma}$

$= \dfrac{1-\alpha}{\alpha} + \dfrac{1-\beta}{\beta} + \dfrac{1-\gamma}{\gamma}$

$= \dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} - 3$ ✓

$= \dfrac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} - 3$

$= \dfrac{-5}{-2} - 3$

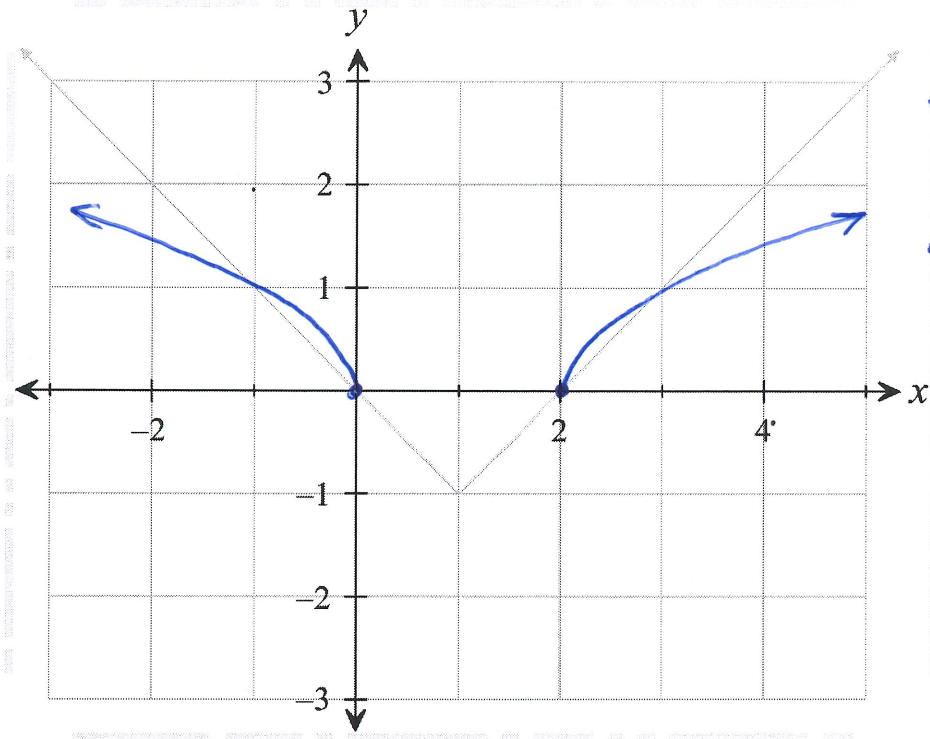
$= -\dfrac{1}{2}$ ✓

Name: solutions

Teacher:

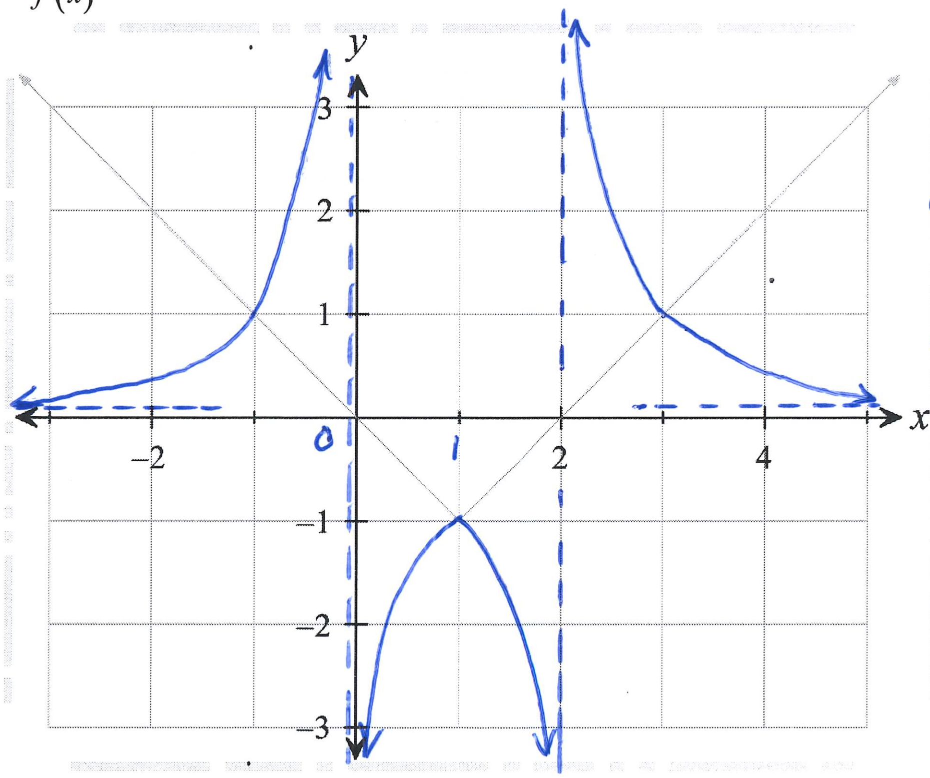
Answer Sheet for Question 3 (a) (Use the light grey as a guide, only lines made in pen will be marked).

(a) (i) $y=\sqrt{f(x)}$



- lines must cut $(-1,1)$ and $(3,1)$
- must be above $f(x)$ for $(-1,0)$ and $(2,3)$ and below for $(-\infty,-1)$ and $(3,\infty)$

(ii) $y=\dfrac{1}{f(x)}$



- lines must cut at $(-1,1)$ and $(3,1)$
- show vert. asymptotes of $x=0$, $x=2$
- horizontal asymptote at $y=0$
- nature of graph at $x=1$

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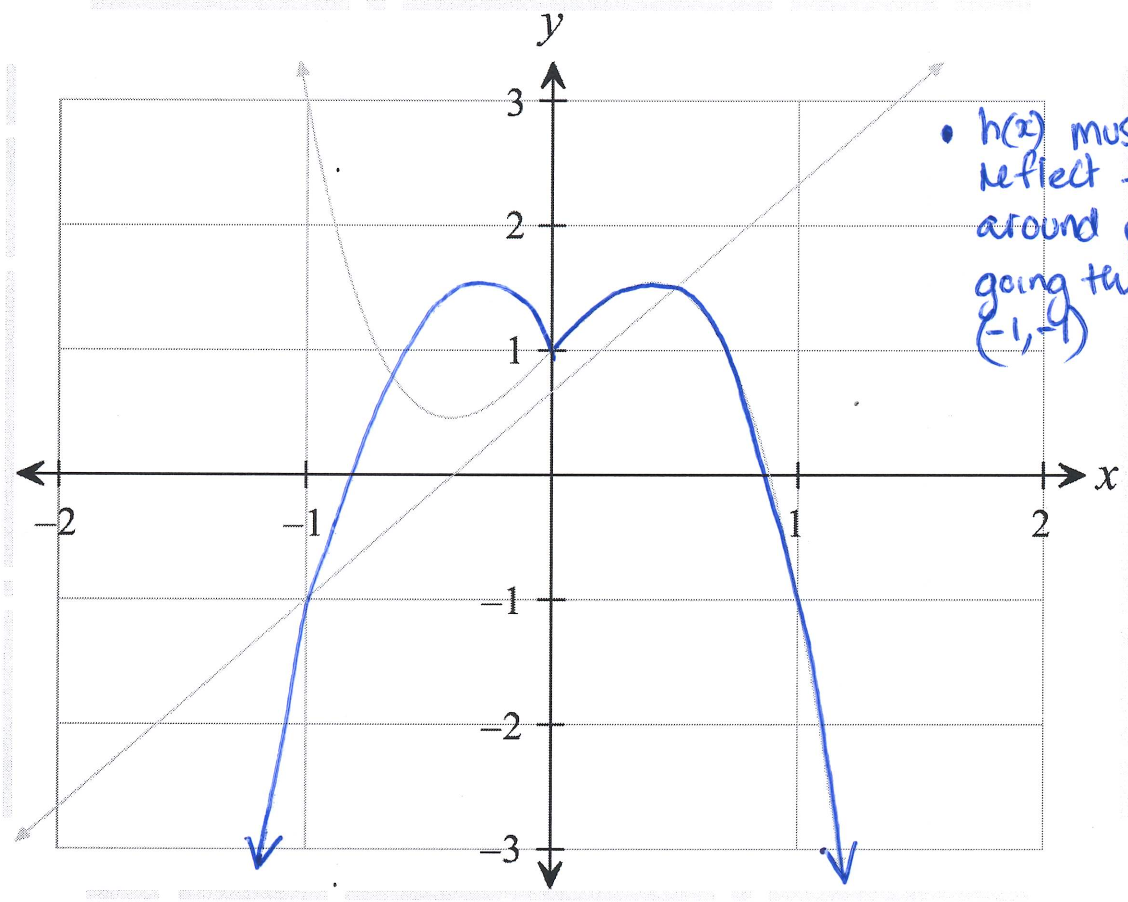
Answer Sheet for Question 3 (b) (Use the light grey as a guide, only lines made in pen will be marked).

(b) (i) $h(x) = -4|x|^3 + 2|x| + 1$



• h(x) must reflect f(x) around y-axis going through (-1,-1)

(ii) P and Q are the pts of intersection of g(x) and h(x)

$h(\tfrac{1}{2}) = -4\left|\tfrac{1}{2}\right|^3 + 2\left|\tfrac{1}{2}\right| + 1$ $\qquad h(-1) = -4|-1|^3 + 2|-1| + 1$

$= \tfrac{3}{2} \quad \therefore P\left(\tfrac{1}{2}, \tfrac{3}{2}\right)$ $\qquad = -1 \quad \therefore Q(-1,1)$

using gradient formula $a = \dfrac{-1 - \frac{3}{2}}{-1 - \frac{1}{2}}$

$= \tfrac{5}{3}$ ✓

substituting $Q(-1,-1)$ into $g(x)$ to find b

$-1 = \tfrac{5}{3}(-1) + b$

$b = \tfrac{5}{3} - 1$

$b = \tfrac{2}{3}$ ✓

	Comments

Question 4.

(a) $SL = Z$, we know that $z^2 = y^2 + h^2$ from ΔSGL

and $y^2 = x^2 + 10^2 - 2 \times 10 \times \cos 30°$

$\therefore z^2 = x^2 + 10^2 - 10\sqrt{3}x + h^2$

> correctly recognising and utilising Pythag and cosine rule $\Rightarrow$ 1 mark.

$\sin 60 = \dfrac{h}{4\sqrt{3}}$ $\quad \therefore h = 6$

$\cos 60 = \dfrac{x}{4\sqrt{3}}$ $\quad \therefore x = 2\sqrt{3}$

> correct values of both x and h $\Rightarrow$ one mark.

$z^2 = (2\sqrt{3})^2 + 10^2 - 10\sqrt{3} \times 2\sqrt{3} + 6^2$

$= 12 + 100 - 60 + 36$

$= 88$ $\quad \therefore z = \sqrt{88}$ or $2\sqrt{22}$

$z = 9.38\text{ m}$ ✓

(b) inverse relation is $x = y^2 - 6y + 8$

<u>method 1</u> $y^2 - 6y + 8 - x = 0$

$y = \dfrac{6 \pm \sqrt{36 - 4(1)(8-x)}}{2}$

$y = \dfrac{6 \pm \sqrt{4(1+x)}}{2}$

$y = 3 \pm \sqrt{1+x}$

<u>method 2</u> (completing the square).

$x - 8 + 9 = y^2 - 6y + 9$

$x + 1 = (y-3)^2$

$y - 3 = \pm\sqrt{1+x}$

$y = 3 \pm \sqrt{1+x}$

> either method one mark.

but domain of $g(x)$ is $x \leq 3$

$\therefore$ range of $g^{-1}(x)$ is $y \leq 3$

$\therefore g^{-1}(x) = 3 - \sqrt{1+x}$ ✓

> students must consider the range/domain for second mark.

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Comments

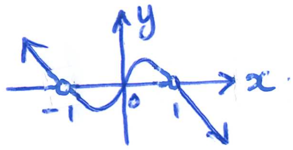
Question 5

(a) (i) $\frac{x}{\cancel{1-x^2}} \times (1-x^2)^{\cancel{2}} \geqslant (1-x^2)^2 \times 0 \qquad x \neq \pm 1$

one mark for correct method.

$x(1-x^2) \geqslant 0$

$x(1-x)(1+x) \geqslant 0$



$\left.\begin{array}{l} x < -1 \text{ or } 0 \leqslant x < 1 \\ (-\infty, -1) \cup [0, 1) \end{array}\right\}$ either

2 marks for considering exclusions in domain

(ii) $\sec x \tan x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$= \frac{\sin x}{\cos^2 x}$ ✓

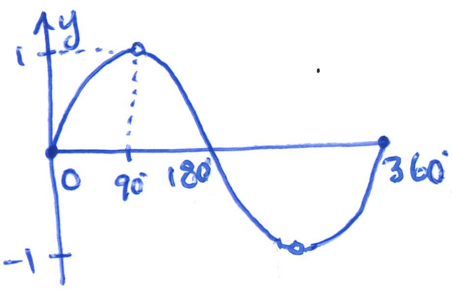
$= \frac{\sin x}{1-\sin^2 x} \quad \because \sin^2 x + \cos^2 x = 1$

(iii) $\frac{\sin x}{1-\sin^2 x} \geqslant 0$ will give same solution as $\sec x \tan x \geqslant 0$ (part (ii))

from part (i) $\sin x < -1$ or $0 \leqslant \sin x < 1$

$\downarrow$

no solution



$0 \leqslant x < 90°$ or $90° < x \leqslant 180°$ or $x = 360°$

2 marks all 3 solutions and 1 mark for 2 correct solutions

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Suggested Solution (s)	Comments
Question 6.	
(a). $mx = -x^3 + 3x^2 - 4x$	
Rearranging... $-x^3 + 3x^2 - (4+m)x = 0$	
factoring... $x(x^2 - 3x + (4+m)) = 0$ ✓	one mark for this factored form.
$x = 0$ correlates to passing through origin.	
so, using sum and product of roots:	
let roots be α, α (only one root $\Rightarrow$ tangent)	
Sum: $2\alpha = 3$, $\alpha = 3/2$	
$y = -\left(\frac{3}{2}\right)^3 + 3\left(\frac{3}{2}\right)^2 - 4\left(\frac{3}{2}\right)$	
$y = -\frac{27}{8} + \frac{27}{4} - \frac{12}{2}$	
$y = -21/8 \quad \therefore P\left(\frac{3}{2}, -21/8\right)$	
or using $\Delta = 0$, $(-3)^2 - 4(4+m) = 0$	one for either method.
$9 - 16 - 4m = 0$	
$4m = -7$	
$m = -7/4$	
So $x^2 - 3x + \left(4 - \frac{7}{4}\right) = 0$	
$4x^2 - 12x + 9 = 0 \quad \therefore (2x-3)^2 = 0 \quad x = 3/2$	
$P\left(3/2, -\frac{21}{8}\right)$	
if using method 1	
$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 4 + m = 0$	
$\frac{9}{4} - \frac{9}{2} + 4 + m = 0 \quad \therefore$ tangent $y = -\frac{7}{4}x$ ✓	
$m = -\frac{7}{4}$	
(b) $P(-1) = P'(-1) = 0$ and $P(1) = 6$	1 mark for forming 2 eqns correctly.
$P'(x) = 3px^2 + q$	
$P'(-1) = 3p + q$	
$\therefore 3p + q = 0$ — (1)	
$P(-1) = -p - q + s = 0$	
$p + q - s = 0$ — (2)	2 marks for all correct and making attempt to solve correctly.
$P(1) = p + q + s = 6$	
$p + q + s = 6$ — (3)	
solve (1), (2) and (3) simultaneously	

(2) + (3): $p + q = 3$ — (4)

(1) − (4) $2p = -3$

$p = -3/2$

substitute p into (4)

$q = 3 + \frac{3}{2}$, $q = \frac{9}{2}$

$s = \frac{9}{2} - \frac{3}{2}$, $s = 3$

3 marks for finding p, q and s correctly.