Year 11 Mathematics Extension 1
Assessment Task 2
Term 3, Week 4, 2022

Name: ....

Teacher: ....

Thursday 11th August, 2022

Setter: NAI

General Instructions

- **Working time – 45 minutes plus 2 minutes reading time.**
- Answer all questions on the lined paper provided using black or blue pen.
- NESA approved calculators may be used.
- Show all relevant mathematical reasoning and/or calculations.
- **Total marks – 32.**
- Attempt Questions $1-6$.

Question 1 (5 marks) **Marks**

(a) (i) Factorise $3a^2 - 10a + 3$. **1**

(ii) Hence or otherwise solve, $3(\log_3 x)^2 - 10\log_3 x + 3 = 0$, for x. **2**

(b) Differentiate $e^{3x}(x^2+1)$ with respect to x, leaving your answer fully factorised. **2**

End of Question 1

Question 2 (5 marks) (Start a new page) **Marks**

(a) (i) Find the derivative of $y = e^x - x$. **1**

(ii) Hence find the equation of the tangent to the curve at the point where the curve cuts the y-axis. **2**

(b) Find the exact value of $\cos\left[\sin^{-1}\left(\frac{5}{13}\right) + \tan^{-1}\left(\frac{3}{5}\right)\right]$ **2**

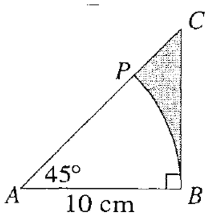
End of Question 2

Question 3 (5 marks) (Start a new page) **Marks**

(a) (i) Prove that $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$. **2**

(ii) Hence find the value for $\cot 67\frac{1}{2}^{\circ}$ in simplest surd form. **1**

(b) In the diagram $\triangle ABC$ is right-angled at B , $AB = 10$ cm and $\angle$A=45°. The circular arc BP has a centre A and radius AB . It meets the hypotenuse AC at P .



(i) Find the exact area of sector ABP . **1**

(ii) Hence find the exact area of the shaded portion BCP . **1**

End of Question 3

Question 4 (5 marks) (Start a new page) **Marks**

Ice-cream is taken out of a freezer at $-10^{\circ}C$ into a room where the air temperature is $24^{\circ}C$. The rate at which the ice-cream warms follows Newton's law, that is $\dfrac{dT}{dt} = -k(T-24)$, where k is a positive integer and t is measured in minutes and T in $^{\circ}C$.

(i) Show that the function $T = 24 - Ae^{-kt}$, where A is a constant, provides this rate of change. **1**

(ii) Hence find the value of A . **1**

(iii) The temperature of the ice-cream reaches $2^{\circ}C$ in 1.5 hours. Find the exact value of k . **2**

(iii) Find the temperature of the ice-cream after a further1.5 hours correct to 2 decimal places. **1**

End of Question 4

Question 5 (6 marks) (Start a new page) **Marks**

(a) (i) Prove that: $\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{x+h}}=\frac{h}{(x+h)\sqrt{x}+x\sqrt{x+h}}$. **3**

(ii) Hence, find $\frac{d}{dx}\sqrt{\frac{2}{x}}$ from first principles. **3**

End of Question 5

Question 6 (6 marks) (Start a new page) **Marks**

(a) Solve $\sin x+\cos x=-1$, for $0\le x\le 2\pi$, by using the substitution $t=\tan\frac{x}{2}$. **2**

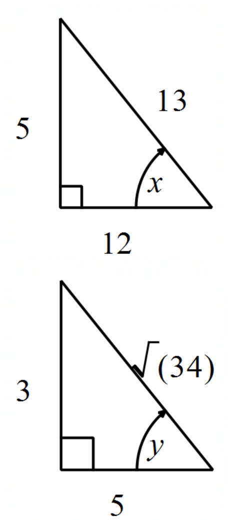
(b) Using *t*-formula, prove that: $\tan 2\theta(\sin 2\theta-\tan\theta)=\sin 2\theta\tan\theta$. **2**

(c) Using at least a third of a page, sketch the function $y=\sin\left(\cos^{-1}x\right)$. **2**

End of Assessment Task

Blank page

2022 Year 11 Mathematics Ext1 AT2 Solutions

Suggested Solution (s)	Comments
Question 1	
ai) $(3a-1)(a-3)$	1 mark
ii) $(3\log_3 x-1)(\log_3 x-3)=0$ $3\log_3 x-1=0$ or $\log_3 x-3=0$	1 mark
$\log_3 x=\frac{1}{3}$ or $\log_3 x=3$ $x=3^{\frac{1}{3}}=\sqrt[3]{3}$ or $x=3^3=27$	1 mark for both answers
b) let $y=e^{3x}(x^2+1)$ let $u=e^{3x}$ and $v=x^2+1$ $u'=3e^{3x}$ and $v'=2x$ $y'=3e^{3x}(x^2+1)+e^{3x}\times 2x$	1 mark
$y'=e^{3x}(3x^2+2x+3)$	1 mark
Question 2	
2ai) $y'=e^x-1$	1 mark
aii) when $x=0, y=e^0-0=1$ $\therefore$ the curve cuts the $y-axis$ at $(0,1)$	1 mark
$\therefore$ the equation of the tangent is: $y-1=(e^0-1)(x-0)$ $y=1$	1 mark
b)  	1 mark for both the diagrams

Suggested Solution (s)	Comments
$\cos\left(\sin^{-1}\left(\frac{5}{13}\right)+\tan^{-1}\left(\frac{3}{5}\right)\right)$ $=\cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right)\cos\left(\tan^{-1}\left(\frac{3}{5}\right)\right)-\sin\left(\sin^{-1}\left(\frac{5}{13}\right)\right)\sin\left(\tan^{-1}\left(\frac{3}{5}\right)\right)$ $=\frac{12}{13}\times\frac{5}{\sqrt{34}}-\frac{5}{13}\times\frac{3}{\sqrt{34}}$ $=\frac{45}{13\sqrt{34}}\times\frac{\sqrt{34}}{\sqrt{34}}$ $=\frac{45\sqrt{34}}{442}$	1 mark (no penalty for not rationalising)
Alternate Solution Let $x=\sin^{-1}\left(\frac{5}{13}\right)$, $\sin x=\frac{5}{13}$ $y=\tan^{-1}\left(\frac{3}{5}\right)$, $\tan y=\frac{3}{5}$	Use diagrams as starting point
$\cos(x+y)=\cos x\cos y-\sin x\sin y$ $=\frac{12}{13}\times\frac{5}{\sqrt{34}}-\frac{5}{13}\times\frac{3}{\sqrt{34}}$ $=\frac{45}{13\sqrt{34}}\times\frac{\sqrt{34}}{\sqrt{34}}$ $=\frac{45\sqrt{34}}{442}$	1 mark for the final answer
Question 3	
3ai) $LHS=\frac{\sin 2x}{1-\cos 2x}$ $=\frac{2\sin x\cos x}{1-(\cos^2 x-\sin^2 x)}$ $=\frac{2\sin x\cos x}{1-\cos^2 x+\sin^2 x}$	1 mark
$=\frac{2\sin x\cos x}{\cos^2 x+\sin^2 x-\cos^2 x+\sin^2 x}$ $=\frac{2\sin x\cos x}{2\sin^2 x}$ $=\frac{\cos x}{\sin x}$ $=\cot x$	1 mark

2022 Year 11 Mathematics Ext1 AT2 Solutions

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments	
3aii) $\cot 67\frac{1}{2}^{\circ} = \dfrac{\sin 2\times 67\frac{1}{2}^{\circ}}{1-\cos 2\times 67\frac{1}{2}^{\circ}}$ $\cot 67\frac{1}{2}^{\circ} = \dfrac{\sin 135^{\circ}}{1-\cos 135^{\circ}}$ $\cot 67\frac{1}{2}^{\circ} = \dfrac{1}{\sqrt{2}} \div \left(1-\left(-\dfrac{1}{\sqrt{2}}\right)\right)$ $\cot 67\frac{1}{2}^{\circ} = \dfrac{1}{\sqrt{2}} \div \left(\dfrac{\sqrt{2}+1}{\sqrt{2}}\right)$ $\cot 67\frac{1}{2}^{\circ} = \dfrac{1}{\sqrt{2}+1} = \sqrt{2}-1$ 3bi) Area of Sector = $\dfrac{45^{\circ}}{360^{\circ}} \times \pi \times 10^2$ Area of Sector = $\dfrac{100\pi}{8}$ Area of Sector = $\dfrac{25\pi}{2}$ bii) Area of shaded portion=$\dfrac{1}{2}\times 10\times 10 - \dfrac{25\pi}{2}$ Area of shaded portion=$\dfrac{100-25\pi}{2}$	1 mark No penalty for not rationalising 1 mark 1 mark	**Question 4** i) $T = 24 - Ae^{-kt}$ $\dfrac{dT}{dt} = -k \times -Ae^{-kt}$ $\dfrac{dT}{dt} = -k\left(24 - Ae^{-kt} - 24\right)$ $\dfrac{dT}{dt} = -k\left(T-24\right)$ $\therefore T = 24 - Ae^{-kt}$ is a solution ii) when $t=0,\ T=-10$ $-10 = 24 - Ae^{\circ}$ $\therefore A = 34$ iii) when $t=90,\ T=2^{\circ}C$ $2 = 24 - 34e^{-k\times 90}$ $\therefore \dfrac{-22}{-34} = e^{-90k}$ $\log_e\left(\dfrac{11}{17}\right) = -90k$ $k = -\dfrac{1}{90}\log_e\left(\dfrac{11}{17}\right)$ iv) when t=180min $T = 24 - 34e^{2\log_e\left(\frac{11}{17}\right)} = 9.76$	1 mark for complete answer 1 mark 1 mark 1 mark 1 mark	

2022 Year 11 Mathematics Ext1 AT2 Solutions

Question 5

ai) $LHS = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+h}}$

$= \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x}\sqrt{x+h}} \times \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$

2 marks

$= \frac{x+h-x}{x\sqrt{x+h}+\sqrt{x}(x+h)}$

1 mark

$= \frac{h}{\sqrt{x}(x+h)+x\sqrt{x+h}}$

ii) $\frac{d}{dx}\sqrt{\frac{2}{x}} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \sqrt{2}\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)$

1 mark

$= \sqrt{2}\lim_{h\to 0}\frac{1}{h}\left(\frac{-h}{\sqrt{x}(x+h)+x\sqrt{x+h}}\right)$

1 mark

$= \sqrt{2}\left(\frac{-1}{x\sqrt{x}+x\sqrt{x}}\right)$

$= \frac{-\sqrt{2}}{2x\sqrt{x}} = \frac{-\sqrt{2x}}{2x^2}$

1 mark (no penalty for not rationalising)

Question 6

a) $\sin x + \cos x = -1$

$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = -1$

1 mark

$\therefore 2t + 1 - t^2 = -1 - t^2$

$2t = -2$

$t = -1$

$\therefore \frac{x}{2} = \pi - \frac{\pi}{4}$

$x = \frac{3\pi}{2}$

test: $x = \pi$

$\sin\pi + \cos\pi = 0 - 1 = -1$

$\therefore x = \pi$ and $x = \frac{3\pi}{2}$

1 mark

b) $LHS = \tan 2\sigma(\sin 2\sigma - \tan\sigma)$

let $t = \tan\sigma$

$\therefore LHS = \frac{2t}{1-t^2}\left(\frac{2t}{1+t^2} - t\right)$

1 mark

$= \frac{2t}{1-t^2}\left(\frac{2t-t-t^3}{1+t^2}\right)$

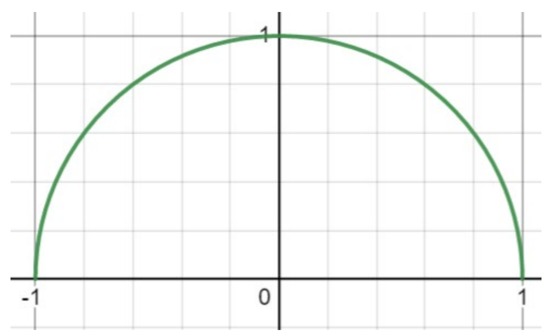
$= \frac{2t}{1-t^2} \times \frac{t(1-t^2)}{1+t^2}$

$= \frac{2t}{1+t^2} \times t$

$= \tan\sigma\sin 2\sigma$

1 mark

c)



1 mark for intercepts

1 mark for the correct shape