Year 11 Mathematics Extension 1
Assessment Task 1
Term 2, Week 6, 2023

Name: ....

Teacher: ....

Monday 29th May 2023

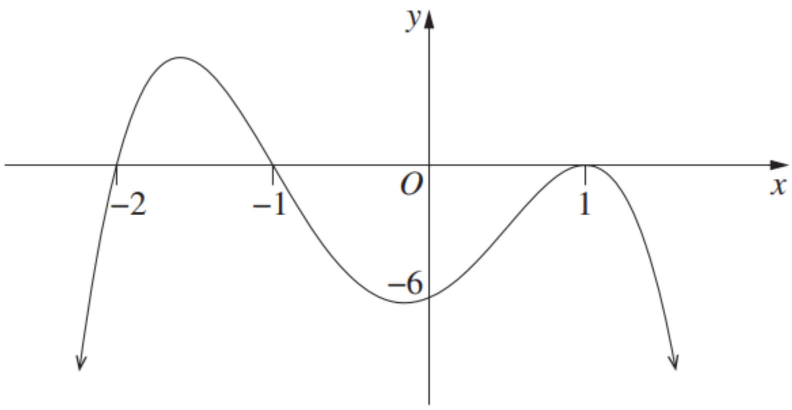
Setter: CHS

General Instructions

- **Working time – 45 minutes + 2 minutes reading time.**
- Answer all questions on the lined paper provided using black or blue pen.
- NESA approved calculators may be used.
- Show all relevant mathematical reasoning and/or calculations.
- **Total marks – 32**
- Attempt Questions 1 – 5

Question 1 (7 marks) **Marks**

(a) The diagram shows the graph of $y = a(x+b)(x+c)(x+d)^2$. **1**



What are the possible values of a, b, c and d ?

(A) $a=-6,\ b=-2,\ c=-1,\ d=1$

(B) $a=-6,\ b=2,\ c=1,\ d=-1$

(C) $a=-3,\ b=-2,\ c=-1,\ d=1$

(D) $a=-3,\ b=2,\ c=1,\ d=-1$

Question 1 continues on the next page

(b) The roots of $x^3 - 2x^2 + 5x + 1 = 0$ are α, β and γ. Find the value of:

(i) $5\alpha + 5\beta + 5\gamma$. **1**

(ii) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. **1**

(iii) $\alpha^2 + \beta^2 + \gamma^2$. **2**

(c) Let $P(x) = x^3 - ax^2 + x$ be a polynomial where a is a real number. When $P(x)$ is divided by $(x-3)$ the remainder is 12. Find the remainder when $P(x)$ is divided by $(x+1)$. **2**

End of Question 1

Question 2 (7 marks) (Start a new page) **Marks**

(a) Which inequality has the same solution as $|x+2|+|x+3|=5$? **1**

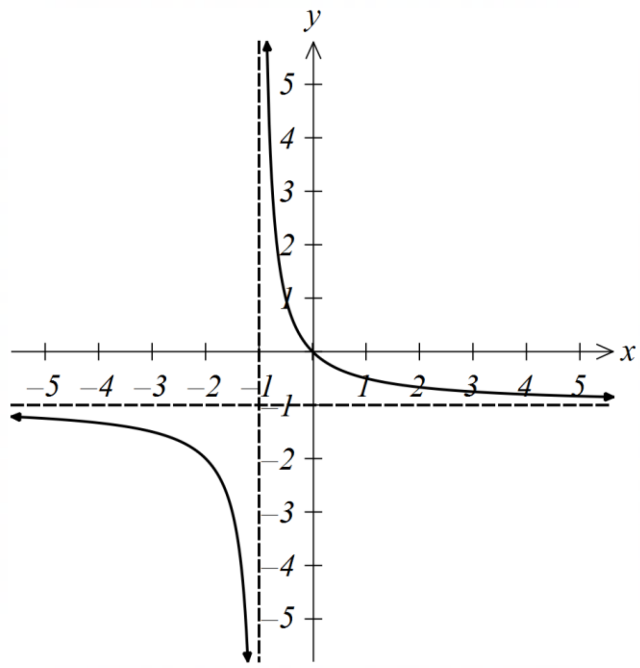
(A) $\dfrac{5}{3-x}\geq 1$

(B) $\dfrac{1}{x-3}-\dfrac{1}{x+2}\leq 0$

(C) $x^2-x-6\leq 0$

(D) $|2x-1|\geq 5$

(b) The diagram below is a sketch of the graph of the function $f(x)=-\dfrac{x}{x+1}$.



(i) Sketch the graph $y=\left(f(x)\right)^2$, showing all the asymptotes and intercepts. **2**

(ii) Sketch the graph $y=x+f(x)$, showing all asymptotes and intercepts. **2**

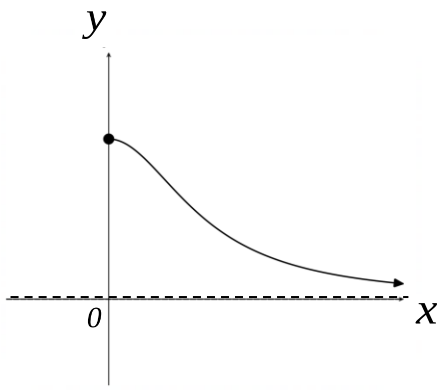
(iii) Solve the equation $\left(f(x)\right)^2=f(x)$. **2**

End of Question 2

Question 3 (6 marks) (Start a new page) **Marks**

(a) Solve $\dfrac{x^2+5}{x} > 6.$ **2**

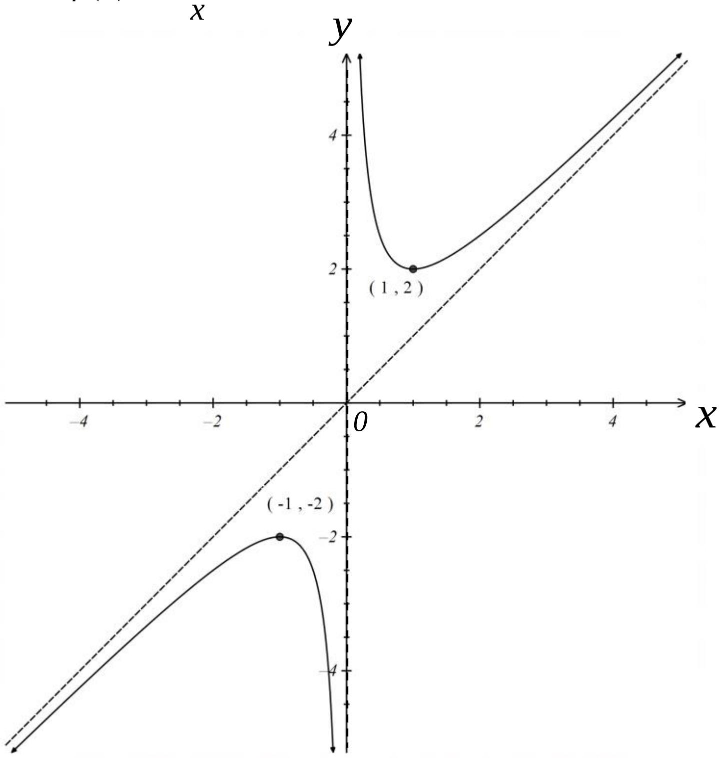
(b) **2**



The diagram above shows the function $f(x) = \dfrac{1}{x^2+1}$ for $x \geq 0.$

State the domain and range of the inverse function $f^{-1}(x).$

(c) The graph of the function $f(x) = \dfrac{x^2+1}{x}$ is shown. **2**



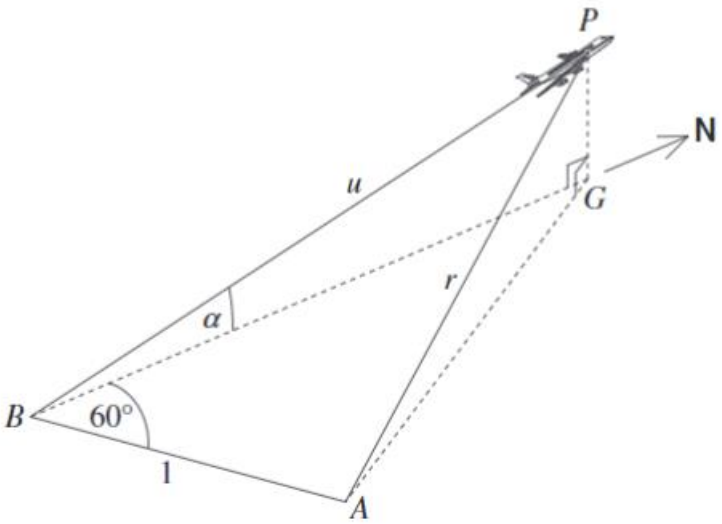
Sketch the graph of $y = \dfrac{1}{\sqrt{f(x)}}$, showing all important features including turning point(s), intercept(s) and asymptote(s). Your sketch is required to be at least a third of the page.

End of Question 3

Question 4 (6 marks) (Start a new page) **Marks**

(a) A monic polynomial $P(x)$ of degree 4 has one zero equal to 2 and a double zero equal to -1. **3**
If the polynomial passes through the point $(0,6)$, find the equation of $P(x)$.

(b) A plane *P* takes off from a point *B*. It flies due north at a constant angle α to the horizontal. **3**
An observer is located at *A*, 1 km from *B*, at a bearing 060° from *B*. Let *u* km be the distance from *B* to the plane and let *r* km be the distance from the observer to the plane. The point *G* is on the ground directly below the plane.



Show that $r = \sqrt{1+u^2-u\cos\alpha}$.

End of Question 4

Question 5 (6 marks) (Start a new page) **Marks**

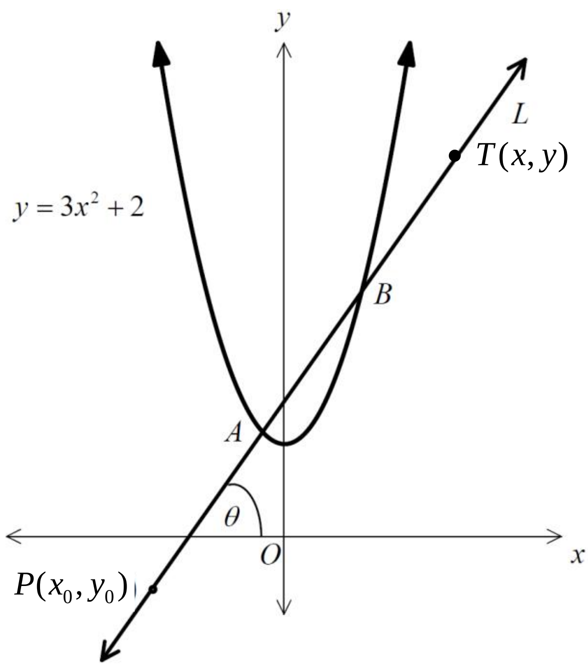
Let L be the straight line passing through a fixed point $P\left(x_0, y_0\right)$ with an angle of inclination θ to the positive x-axis. The line L is defined parametrically by:

$$x = x_0 + t\cos\theta$$

$$y = y_0 + t\sin\theta$$

Where t is real and $0 \le \theta \le 180°$.

In the diagram below the line L cuts the parabola $y = 3x^2 + 2$ at points A and B, where P is a fixed point taking the coordinates $\left(-1, -\frac{1}{3}\right)$.



(i) $|PA| = t_1$ and $|PB| = t_2$. Show that for any point T on the line L that $|PT| = |t|$. **1**

(ii) Show that t_1 and t_2 are the roots of the equation: **2**

$$9t^2\cos^2\theta - 3t(\sin\theta + 6\cos\theta) + 16 = 0\,.$$

(iii) Given that $|AB|^2 = \dfrac{\sin^2\theta + 12\sin\theta\cos\theta - 28\cos^2\theta}{9\cos^4\theta}$ (Do **NOT** prove this). **2**

and L is a tangent to the parabola $y = 3x^2 + 2$, show that the two possible gradients of L are 2 and -14.

(iv) If Z is the point of contact of the tangent L with the parabola $y = 3x^2 + 2$ and L has a gradient of 2, show that $|PZ| = \dfrac{4\sqrt{5}}{3}$. **1**

End of Assessment Task

2023 Year 11 Mathematics Extension 1 Task 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

QUESTION 1

a) single root @ $x = -2, -1$

double root @ $x = 1$.

possible values of 'b' & 'c'

$b = 2$

$c = 1$

$d = -1$

$y = a(x+2)(x+1)(x-1)^2$

let $x = 0, y = -6$ (y int).

$-6 = a(2)(1)(-1)^2$

$\boxed{a = -3}$

$\therefore$ Answer : D

b) i) $5(\alpha+\beta+\gamma)$

$\frac{-b}{a} = 2 \Rightarrow 5(2)$

$= 10$ ✓

① answer

ii) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$\alpha\beta\gamma = -\frac{d}{a}$

$= -1$

$\alpha\beta\gamma(\alpha+\beta+\gamma) = (-1)(2)$

$= -2$ ✓

① answer.

iii) $\alpha^2+\beta^2+\gamma^2$

$(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$(2)^2 - 2(5)$

$4 - 10$

-6 ✓

① answer

c) $P(x) = x^3 - ax^2 + x$

$P(3) = 12$

$12 = (3)^3 - a(3)^2 + 3$

$12 = 30 - 9a$

$\boxed{a = 2}$ ✓

① solving 'a'

@ $P(-1) = (-1)^3 - 2(-1)^2 + (-1)$

$= -1 - 2 - 1$

$= -4$ ✓

① answer.

QUESTION 2

a) if $x < -2$, $|x-3| > 5$

$\therefore$ no solution for $x < -2$

if $x > 3$, $|x+2| > 5$

no solution for $x > 3$

if $-2 \le x \le 3$

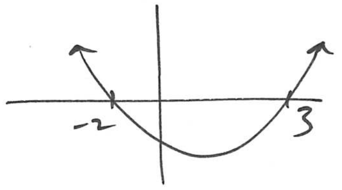
$x+2-(x-3) = 5$

$x+2-x+3 = 5$

$5 = 5$

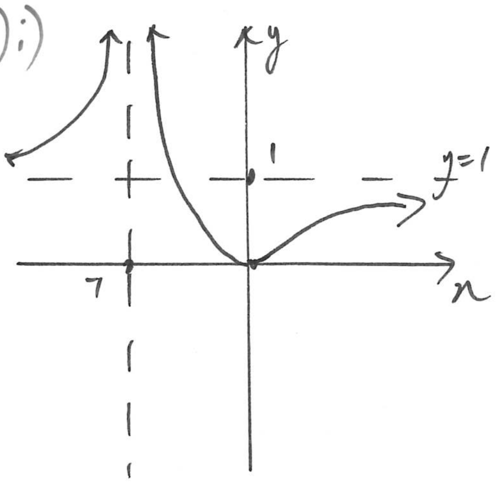
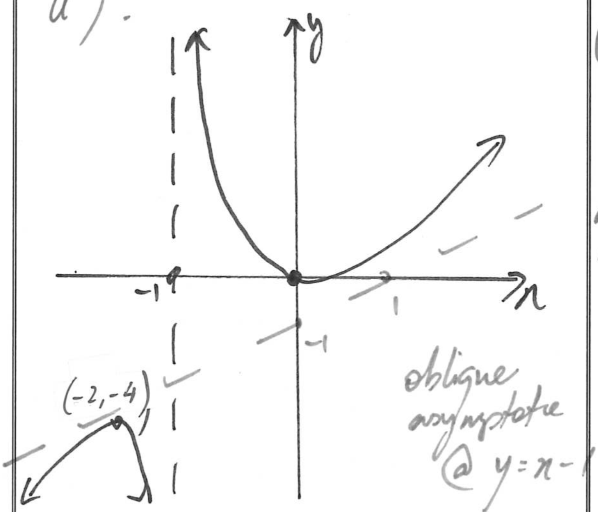
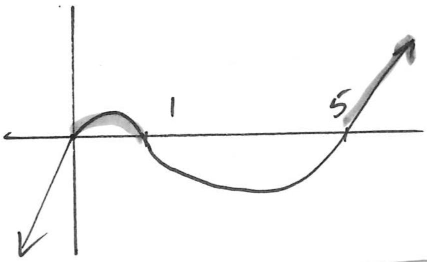
looking at $x^2 - x - 6 \le 0$

$(x-3)(x+2) \le 0$



Answer : C

2023 Year 11 Mathematics Extension 1 Task 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
b) i) 	① correct shape ① correct asymptotes -1 for any incorrect/missing features.	$\boxed{x=0,\ y=0}$, $x\neq -1$, $\boxed{x=-\frac{1}{2},\ y=1}$	① soln must include $x\neq -1$.
ii). oblique asymptote @ $y=x-1$	① correct shape ① key points &/ asymptotes.	QUESTION 3 a) $\frac{x^2+5}{x}>6$ $x(x^2+5)>6x^2$ $x^3-6x^2+5x>0$ $x(x^2-6x+5)>0$ $x(x-1)(x-5)>0$ $\therefore \boxed{0<x<1}$ or $\boxed{x>5}$	① working ① answer
iii) $\left(\frac{-x}{x+1}\right)^2 = -\frac{x}{x+1}$ $\frac{x^2}{(x+1)^2} = -\frac{x}{x+1}$ $x^2(x+1) = -x(x+1)^2$ $x^3+x^2 = -x^3-2x^2-x$ $2x^3+3x^2+x=0$ $x(2x^2+3x+1)=0$ $x(x+1)(2x+1)=0$	① factorise	b). D: $0<x\le 1$ ✓ R: $y\ge 0$ ✓ c) at $x=1 \Rightarrow f(1)=2$ $(1,2)$ @ $y=\frac{1}{\sqrt{f(x)}} \Rightarrow \frac{1}{\sqrt{2}} = (1, 0.707)$ at $x=-1 \Rightarrow f(-1)$ no soln at $x=0 \Rightarrow$ no soln $x>0$ for graph to exist as $x\to\infty$, $y\to 0^+$	① each correct

2023 Year 11 Mathematics Extension 1 Task 1 SOLUTIONS

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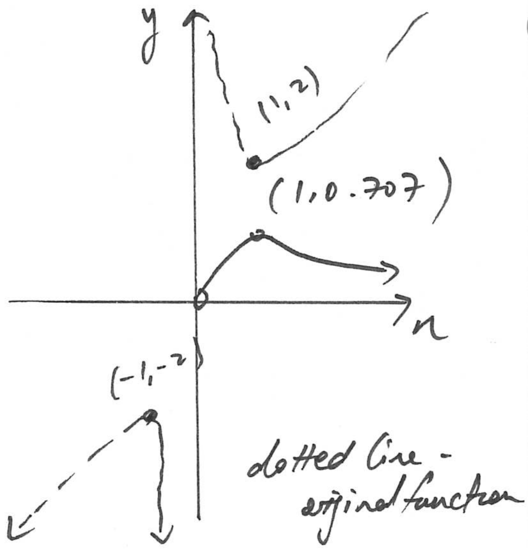


①open circle @ $x=0$

①correct shape & approaching $y \to 0^+$

QUESTION 4

a) one zero @ $x=2$

double zero @ $x=-1$

$P(x)=(x-2)(x+1)^2(x-\alpha)$ ①use of 2 roots

passes thru $(0,6)$

$6=(0-2)(0+1)^2(0-\alpha)$ ①correct sub

$6=(-2)(1)(-\alpha)$

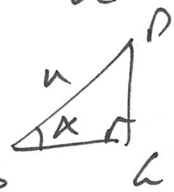
$6=2\alpha$

$\alpha=3$

$\therefore P(x)=(x-2)(x+1)^2(x-3)$ ①$P(x)$

b) $\triangle PBG$: $\sin\alpha = \dfrac{PG}{u}$

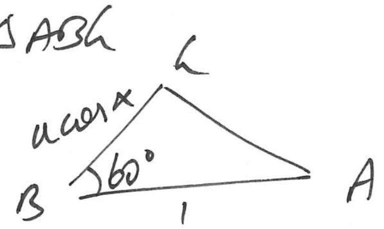
$\boxed{PG = u\sin\alpha}$



$\cos\alpha = \dfrac{BG}{u}$

$\boxed{BG = u\cos\alpha}$

$\triangle ABG$

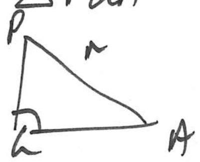


$AG^2 = BG^2 + AB^2 - 2(BG)(AB)\cos A$

$AG^2 = (u\cos\alpha)^2 + 1 - 2(u\cos\alpha)(1)\times\frac{1}{2}$

$\boxed{= u^2\cos^2\alpha + 1 - u\cos\alpha}$ ①cosine rule AG

$\triangle PGA$



$PA^2 = PG^2 + AG^2$

$r^2 = (u\sin\alpha)^2 + u^2\cos^2\alpha + 1 - u\cos\alpha$ ①using $\triangle PGA$ for PA^2

$= u^2\sin^2\alpha + u^2\cos^2\alpha + 1 - u\cos\alpha$

$= u^2(\sin^2\alpha + \cos^2\alpha) + 1 - u\cos\alpha$

$= u^2 + 1 - u\cos\alpha$

$\boxed{r = \sqrt{1+u^2-u\cos\alpha}}$ ①answer

2023 Year 11 Mathematics Extension 1 Task 1 SOLUTIONS

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QUESTION 5

(i) $P(x_0, y_0)$ $T(x, y)$

$PT = \sqrt{(x-x_0)^2+(y-y_0)^2}$

$PT = \sqrt{t^2(\cos^2\theta+\sin^2\theta)}$

$= \sqrt{t^2}$

$PT = |t|$

① correct use of parametrics

ii) let $x_0 = -1$, $y_0 = \frac{1}{3}$

$y = 3x^2+2 \Rightarrow$ sub $x = x_0 + t\cos\theta$
$y = y_0 + t\sin\theta$

$y_0 + t\sin\theta = 3(x_0 + t\cos\theta)^2 + 2$

sub x_0, y_0

$\frac{-1}{3} + t\sin\theta = 3(-1+t\cos\theta)^2 + 2$ ✓

① correct sub x_0, y_0

$\times 3$ $\times 3$

$-1 + 3t\sin\theta = 9 - 18t\cos\theta + 9t^2\cos^2\theta + 6$

$0 = 9t^2\cos^2\theta - 18t\cos\theta - 3t\sin\theta + 16$

$0 = 9t^2\cos^2\theta - 3t(\sin\theta + 6\cos\theta) + 16$

① answer

iii) gradient $m_L = \frac{y-y_0}{x-x_0} \Rightarrow \frac{t\sin\theta}{t\cos\theta}$

$m_L = \tan\theta$

① gradient m_L

L is a tangent $\Rightarrow$ A & B at the same point

$AB^2 = 0$

$\sin^2\theta + 12\sin\theta\cos\theta - 28\cos^2\theta = 0$

$(\sin\theta - 2\cos\theta)(\sin\theta + 14\cos\theta) = 0$

$\sin\theta = 2\cos\theta$ | $\sin\theta = -14\cos\theta$

$\boxed{\tan\theta = 2}$ | $\boxed{\tan\theta = -14}$

① use of AB^2 to solve quadratic

$\therefore$ angle of inclination $\tan\theta = m$

iv)

let $m = 2$

$\tan\theta = \frac{2}{1}$

$\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$

- due to tangent. sum of roots $= \frac{-b}{a}$ $2t_2 = \frac{-b}{a}$.

$2t_2 = \frac{\sin\theta + 6\cos\theta}{3\cos^2\theta}$

$\div 2$

$t_2 = \frac{\sin\theta + 6\cos\theta}{6\cos^2\theta}$

$= \frac{\left(\frac{2}{\sqrt{5}}\right) + 6\left(\frac{1}{\sqrt{5}}\right)}{6\left(\frac{1}{\sqrt{5}}\right)^2}$

$= \frac{\left(\frac{8}{\sqrt{5}}\right)}{\left(\frac{6}{5}\right)} \Rightarrow \frac{4\sqrt{5}}{3}$

① working to answer