Year 11 Mathematics Extension 1
Assessment Task 2
Term 3, Week 4, 2023

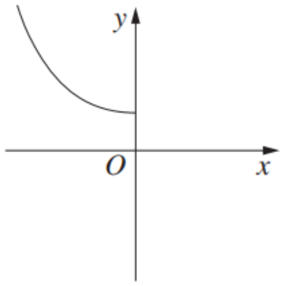
Name: ____________________

Monday, 7th August 2023

- Start each question on a new page.
- NESA approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- Total possible mark: **32**
- Time allowed**: 45 minutes + 2 minutes reading time**

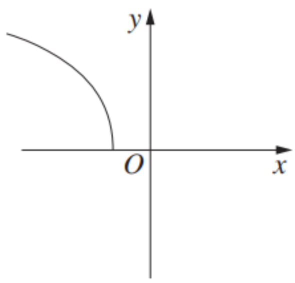
Question 1 (7 marks) **Marks**

(a) The diagram shows the graph $y = f(x)$. **1**

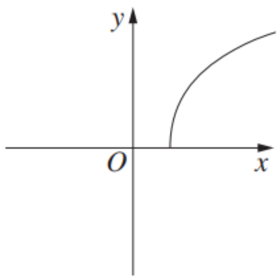


Which diagram shows the graph $y = f^{-1}(x)$?

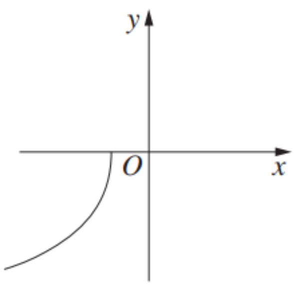
(A)



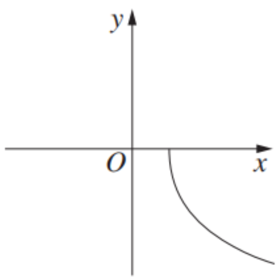
(B)



(C)



(D)



Question 1 continues on the next page

(b) Show that $2\cos\left(x+\frac{\pi}{6}\right)=\sqrt{3}\cos x-\sin x.$ **1**

(c) Solve $4^x-2^{x+2}=32.$ **2**

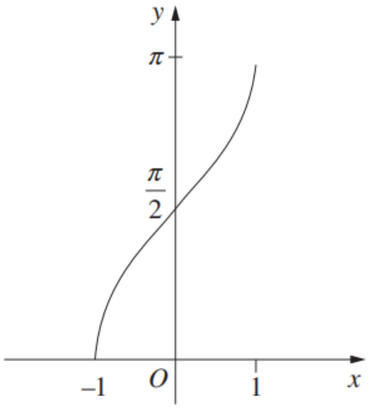
(d) Find the equation of the normal to the curve $y=xe^{2x}$ at the point where $x=\frac{\pi}{2}.$ **3**

End of Question 1

Question 2 starts on the next page

Question 2 (7 marks) *Start a new page* **Marks**

(a) The diagram shows the graph of a function. **1**



Which function does the graph represent?

(A) $y = \cos^{-1} x$

(B) $y = \dfrac{\pi}{2} + \sin^{-1} x$

(C) $y = -\cos^{-1} x$

(D) $y = -\dfrac{\pi}{2} - \sin^{-1} x$

(b) After time t years from the start of the year 2023 in Icebox, the number of people in the population is given by:

$$N = 70 + Ae^{0.1t}, \text{ where } A > 0.$$

(i) Show that $\dfrac{dN}{dt} = 0.1(N - 70)$. **1**

(ii) There were 100 people in the population at the start of the year 2023. Find the first year where all months will have a population size that exceeds 190. **2**

Question 2 continues on the next page

(c) A piecewise function is defined by:

$$f(x)=\begin{cases} x^2+2x, & \text{for } x\le 1 \\ -\dfrac{2}{2x-1}+5, & \text{for } x>1 \end{cases}$$

Providing justification for your answer:

(i) Determine if $f(x)$ is continuous at $x=1$. **1**

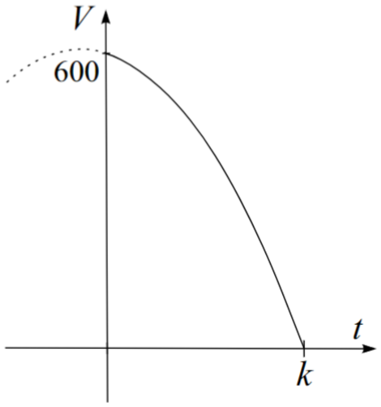
(ii) Determine if $f(x)$ is differentiable at $x=1$. **2**

End of Question 2

Question 3 starts on the next page

Question 3 (6 marks) *Start a new page* **Marks**

(a)



The diagram above represents the volume of water in a water tank as it is being drained. The volume of water in the tank is initially 600 Litres (L), and it take k minutes for the water to completely drain from the tank. It is known that the volume of water in litres after t minutes is given by $V = -30t^2 - 30t + 600$ for $0 \le t \le k$.

(i) Find the value of k. **1**

(ii) Find the average rate of change of the volume of water over the time that the tank is drained. **1**

(iii) Find the time at which the instantaneous rate of change in volume, $\frac{dV}{dt}$, is equal to the average rate of change of the volume for the time it takes the tank to drain. **2**

(b) Solve the pair of simultaneous equations: **2**

$$\log_2 x + \log_2 y = 3$$
$$\log_x y = 2$$

End of Question 3

Question 4 starts on the next page

Question 4 (6 marks) *Start a new page* **Marks**

(a) The length of a rectangle remains twice as its width at all times when the rectangle is expanding. The area of the rectangle is increasing at a rate of $16 cm^2 / \sec$. **3**

Find the rate at which the perimeter of the rectangle is increasing at the instant when its width is 200 cm.

(b) Find the exact value of $\sin\left[\cos^{-1}\frac{3}{7}+\tan^{-1}\left(\frac{-3}{4}\right)\right]$. **3**

End of Question 4

Question 5 (6 marks) *Start a new page* **Marks**

(a) Write the expression $\cos x+\cos 5x$ as the product of trigonometric ratios. **1**

(b) (i) Show that $\tan(\alpha+\beta+\gamma)=\dfrac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}$. **3**

(ii) If $\tan\alpha, \tan\beta, \tan\gamma$ are the roots of the equation $x^3-(a+1)x^2+(b-a)x-b=0$, **2**

hence show that $\tan(\alpha+\beta+\gamma)=1$.

End of Assessment

2023 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
QUESTION 1 a) ANSWER = D. b) CHS: $2\cos(x+\pi/6)$ $2(\cos x\cos\frac{\pi}{6}-\sin x\sin\frac{\pi}{6})$ $2(\cos x\times(\frac{\sqrt3}{2})-\sin x\times(\frac12))$ $\sqrt3\cos x-\sin x$	① correct expansion & sub of exact values	normal: $m_1\times m_2=-1$ $m_2=-\frac{1}{e^{\pi}(\pi+1)}$ when $x=\frac{\pi}{2}\Rightarrow$ sub into y $y=\frac{\pi}{2}e^{\pi}$ $(\frac{\pi}{2},\frac{\pi}{2}e^{\pi})$ eqn of normal $y-\frac{\pi}{2}e^{\pi}=\frac{-1}{e^{\pi}(\pi+1)}(x-\pi/2)$	① normal ① eqn of normal
c) $(2^2)^x-(2^2)(2^x)-32=0$ let $u=2^x$ $u^2-4u-32=0$ $(u-8)(u+4)=0$ $u=8$, $2^x=8$, $\boxed{x=3}$ | $u=-4$, $2^x=-4$ no soln.	① correct sub & factorise ① soln (-1) for not stating no soln.	QUESTION 2 a) ANSWER: B. b) i) $N=70+Ae^{0.1t}$ $\frac{dN}{dt}=0.1Ae^{0.1t}$ $=0.1(N-70)$ ii) when $t=0$, $N=100$ $100=70+Ae^{0.1\times0}$ $\boxed{30=A}$	① working & answer ① finding A
d) $y=x\cdot e^{2x}$ $u=x$, $u'=1$ | $v=e^{2x}$, $v'=2e^{2x}$ $y'=2xe^{2x}+e^{2x}$ $y'=e^{2x}(2x+1)$ @ $x=\frac{\pi}{2}$ $y'=e^{\pi}(\pi+1)$ $\therefore m_1=e^{\pi}(\pi+1)$ @ $x=\frac{\pi}{2}$.	① correct dy/dx	let $N=190$. $190=70+30e^{0.1t}$ $120=30e^{0.1t}$ $4=e^{0.1t}$ $t=\frac{\ln 4}{0.1}$ $\doteqdot 13.8629$ years $\Rightarrow$ 14 years $\therefore$ The year will be 2037.	① t years → need to find year

2023 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

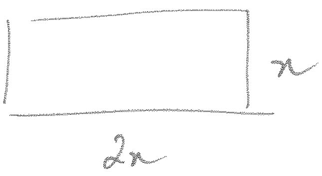
Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
c) i) $f(1) = (1)^2 + 2\times 1$ $= 3$. As $x \to 1^+$, $f(x) = \frac{-2}{2(1)-1} + 5$ $= 3$ $\therefore f(x)$ is continuous at $x=1$ $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ $= f(1)$ ii) $\frac{d}{dx}(x^2+2x) = 2x+2$ when $x=1 \Rightarrow 2(1)+2$ $= 4$ $\frac{d}{dx}\left(-\frac{2}{2x-1}+5\right) : -2(2x-1)^{-1}+5$ $= 2(2x-1)^{-2}\times 2$ $= \frac{4}{(2x-1)^2}$ When $x=1$: $\frac{4}{(2(1)-1)^2} = 4$ As $x \to 1^-$: $f'(x) \to 4$ " $x \to 1^+$; $f'(x) \to 4$	① need to show appropriate working of state $\lim_{x\to 1^-} = \lim_{x\to 1^+}$ ① correct derivative of both ① show both limits are equal	QUESTION 3 a) i) let $V=0$ $-30t^2-30t+600=0$ $t^2+t-20=0$ $(t+5)(t-4)=0$ $t=-5$, $t=4$ but $t>0$. $\therefore h=4$ as $h>0$. ii) Average rate of change $= \frac{-600}{4} \Rightarrow -150L/min$ iii) $\frac{dV}{dt} = -60t-30$ $-150L = -60t-30$ $-60t = -120$ $t=2$. b) $\log_x y = 2 \Rightarrow y = x^2$ $\log_2 x + \log_2 y = 3$ ① $\log_2 xy = 3$. sub $y = x^2$ $\log_2 x^3 = 3$ $x^3 = 2^3$ $x^3 = 8$ $x = 2$. / $y = 4$	① all correct ① answer ① correct $\frac{dV}{dt}$ ① answer ① use of any log laws correctly ① soln

2023 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

QUESTION 4

a) i)



$A = 2x^2 \Rightarrow \frac{dA}{dx} = 4x$

$\frac{dA}{dt} \Rightarrow 4x \times \frac{dx}{dt} = 16$ — (1) rate of change

$x\frac{dx}{dt} = 4 \quad \Big| \quad \frac{dx}{dt} = \frac{4}{x}.$

when $x = 200$ cm

$\frac{dx}{dt} = \frac{4}{200} \Rightarrow \frac{1}{50}$ cm/sec — (1) sub $x=200$

$P = 6x$

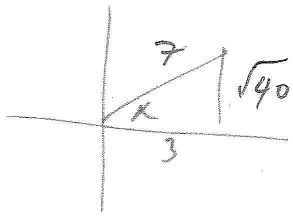
$\frac{dP}{dt} = 6 \times 1 \times \frac{dx}{dt}$ — (1) solving $\frac{dP}{dt}$.

$= 6 \times 1 \times \frac{1}{50}$

$= \frac{3}{25}$ cm/sec

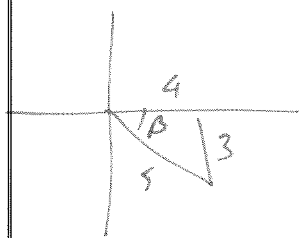
b) let $\alpha = \cos^{-1}\frac{3}{7}$

$\cos\alpha = \frac{3}{7}. \quad 0 \le \alpha \le \pi$ — (1) finding $\cos\alpha$



$\beta = \tan^{-1}\left(\frac{-3}{4}\right)$

$\tan\beta = \frac{-3}{4}, \quad \frac{-\pi}{2} < \beta < \frac{\pi}{2}$ — (1) finding $\tan\beta$



$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{\sqrt{40}}{7} \times \frac{4}{5} + \frac{3}{7} \times \frac{-3}{5}$ — (1) answer

$= \frac{4\sqrt{40}}{35} - \frac{9}{35}.$

$= \frac{4\sqrt{40} - 9}{35}$

2023 Year 11 Mathematics Extension 1 Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
QUESTION 5 a) $\cos x + \cos 5x$ $= 2\cos 3x \cos 2x$ when $A+B=5$ $A-B=1$ b) i) $\tan(\alpha+\beta+\gamma) = \dfrac{\tan\alpha + \tan(\beta+\gamma)}{1-\tan\alpha\tan(\beta+\gamma)}$ $\dfrac{\tan\alpha + \left(\dfrac{\tan\beta+\tan\gamma}{1-\tan\beta\tan\gamma}\right)}{1-\tan\alpha\left(\dfrac{\tan\beta+\tan\gamma}{1-\tan\beta\tan\gamma}\right)} \begin{matrix}\times(1-\tan\beta\tan\gamma)\\ \times(1-\tan\beta\tan\gamma)\end{matrix}$ $\dfrac{\tan\alpha(1-\tan\beta\tan\gamma)+\tan\beta+\tan\gamma}{1(1-\tan\beta\tan\gamma)-\tan\alpha(\tan\beta+\tan\gamma)}$ $\dfrac{\tan\alpha - \tan\alpha\tan\beta\tan\gamma + \tan\beta+\tan\gamma}{1-\tan\beta\tan\gamma - \tan\alpha\tan\beta - \tan\alpha\tan\gamma}$ $\dfrac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\beta\tan\gamma-\tan\alpha\tan\gamma}$	(1) answer (1) (1) working (1) answer	ii) sum of roots: $\tan\alpha+\tan\beta+\tan\gamma = \frac{-b}{a}$ $= a-1$ product of pairs sum $\tan\alpha\tan\beta+\tan\beta\tan\gamma+\tan\alpha\tan\gamma = \frac{c}{a}$ $= b-a$ product of roots $\tan\alpha\tan\beta\tan\gamma = -\frac{d}{a}$ $= b$ from part (i) $\tan(\alpha+\beta+\gamma) = \dfrac{a+1-b}{1-(b-a)}$ $= \dfrac{a+1-b}{1+a-b}$ $= 1.$	(1) for all 3. (1) correct sub