



Northern Beaches Secondary College
Manly Campus

2021

HIGHER SCHOOL CERTIFICATE

Year 11 Assessment 1

Mathematics Extension 1

General

Instructions

- Reading time – 2 minutes
- Working time – 55 minutes
- Write using black/blue pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks:

36

Section I – 3 marks (page 2)

- Attempt Questions 1–3
- Allow about 5 minutes for this section
- Answer on Multiple Choice Answer Sheet provided

Section II – 33 marks (pages 4–9)

- Attempt Questions 4–14
- Allow about 50 minutes for this section
- Answer in the spaces provided

This paper MUST NOT be removed from the examination room

MARKS	MC	Q4-7	Q8-11	Q12-14
STUDENT MARK				
MAXIMUM	3	12	11	10



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Multiple-Choice Answer Sheet

Select the alternative A, B, C, or D that best answers the question by placing a **X** in the box.

	A	B	C	D
1				
2				
3				

Section I - 3 marks

Attempt Questions 1 – 3

Allow about 5 minutes for this section

Use the multiple-choice answer sheet for Questions 1-3

1. The solutions to $|2x - 3| \leq 11$ are:
 - A. $-7 \leq x \leq 4$
 - B. $-4 \leq x \leq 7$
 - C. $x \leq -4$ or $x \geq 7$
 - D. $x \leq -7$ or $x \geq 4$

2. A 3-digit number is formed using the digits $\{2, 3, 5, 6, 7, 9\}$. If digits can be repeated, what is the probability that the number is odd and greater than 600?
 - A. $\frac{1}{2}$
 - B. $\frac{1}{3}$
 - C. $\frac{1}{6}$
 - D. $\frac{1}{20}$

3. Which expression is not equivalent to ${}^n\mathbf{P}_2$?
 - A. $n(n - 1)$
 - B. ${}^nC_2 \times 2!$
 - C. ${}^n\mathbf{P}_{n-2}$
 - D. ${}^nC_{n-2} \times 2!$

End of Multiple Choice

2021

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Year 11 Assessment 1

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Section II Answer Booklet

33 marks

Attempt Questions 4–14

Allow about 50 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 4 (2 marks)

Solve $6c^2 > 2 - c$

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Question 5 (3 marks)

Solve $\frac{3}{|x - 4|} \geq 2$

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Question 6 (3 marks)

Solve $\frac{2x}{x-1} \geq 1$

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Question 7 (4 marks)

Nick and Megan are hosting a dinner party. The two hosts and their eight guests will sit around a circular table, which has ten evenly-spaced seats.

- (i) How many seating arrangements are possible?

1

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- (ii) How many seating arrangements are possible where Nick and Megan sit together? **1**

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Question 7 continues on page 5

Question 7 (continued)

- (iii) Due to a global pandemic, dinner parties are restricted to a maximum of 5 people, and Nick and Megan must now choose just three of their friends to invite.

Furthermore, all attendees (including hosts) must be separated by at least one empty seat.

How many seating arrangements are now possible, given that Nick and Megan can sit anywhere, as long as they comply with the restrictions? **2**

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Question 8 (4 marks)

How many arrangements of the word PATTERN are there if:

- (i) there are no restrictions? **1**

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- (ii) there must be exactly two letters separating the Ts? **2**

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- (iii) the letters are not in alphabetical order? **1**

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Question 9 (2 marks)

Simplify $\frac{n! + (n-1)!}{(n+1)!}$ **2**

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Question 10 (2 marks)

Find the 6th term in the expansion of $(1 - 2\sqrt{x})^9$ **2**

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Question 11 (3 marks)

Find the term independent of x in $\left(x^2 - \frac{1}{x}\right)^6$ **3**

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Question 12 (2 marks)

Find the coefficient of x^5 in $(x - 1)(x + 1)^7$ **2**

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Question 13 (5 marks)

In 2021, a school offers three Humanities subjects {English, History, Geography} and four Science subjects {Mathematics, Physics, Chemistry and Biology}. Students must choose exactly five subjects.

- (i) How many different selections are possible if English is mandatory? **1**

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- (ii) In 2022, the school changes its policy so that English is no longer mandatory. However, students must now choose at least two Humanities subjects. How many different selections are possible in 2022? **2**

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Question 13 continues on page 8

Question 13 (continued)

- (iii) In 2023, the school decides that students are free to choose any five subjects.

If there are 90 students enrolled in 2023, explain why there will be five or more students doing the same combination of subjects. 2

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Question 14 (3 marks)

A binary sequence is any list of symbols, where each symbol is a 0 or a 1. For example, [11010] is a binary sequence of length 5.
Consider a binary sequence of length n .

- (i) Explain why there are 2^n possible binary sequences of length n . 1

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- (ii) Hence explain why $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$ by
considering the possible number of 1s in a binary sequence of length n . 2

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End of Paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

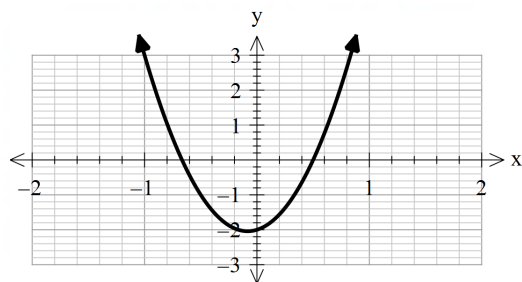
Section II extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

1	Since $\leq$ it is between -11 and 11 $-11 = 2x - 3 \leq 11$ $-8 \leq 2x \leq 14$ $-4 \leq x \leq 7$ $\therefore B$	B
2	With repeats allowed so $6^3 = 216 \text{ in total ,}$ Odd if ends in 3, 5, 7 or 9 (4 options) > 600 if starts with 6, 7, 9 (3 options) Middle number any of the 6 numbers $\therefore = 3 \times 6 \times 4 = 72$ $P(> 600) = \frac{72}{216} = \frac{1}{3}$ $\therefore B$	B
3	${}^n\mathbf{P}_2 = \frac{n!}{(n-2)!}$ $= \frac{n(n-1)(n-2)!}{(n-2)!}$ $= n(n-1)$ ${}^n\mathbf{C}_2 \times 2! = \frac{n!}{2!(n-2)!} \times 2!$ $= n(n-2)$ ${}^n\mathbf{P}_n - 2 = \frac{n!}{n - (n-2)!}$ $= \frac{n!}{2!} \text{ incorrect}$ ${}^n\mathbf{C}_n - 2 \times 2 \neq \frac{n!}{(n-2)!(n - (n-2)!)} \times 2!$ $= \frac{n!}{(n-2)!2!} \times 2!$ $n(n-1)$ $\therefore C$ is incorrect	C

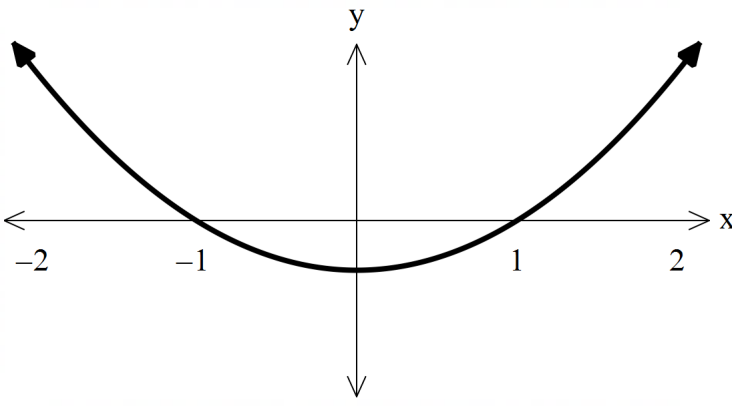
Q	Solution	Marking guidelines
4	$6c^2 + c - 2 = 0$ $6c^2 + 4c - 3c - 2 = 0$ $2c(3c + 2) - (3c + 2) = 0$ $(2c - 1)(3c + 2) = 0$ $c = -\frac{2}{3} \text{ or } \frac{1}{2}$	2 marks for correct solution 1 mark for factorising equation correctly 1 mark for correct conclusion given incorrect factorising (must include sketch of graph or testing points)



Alternative: Instead of using graph, students can test points

$$\therefore c < -\frac{2}{3} \text{ or } c > \frac{1}{2}$$

5	$ x - 4 \neq 0 \quad \therefore x \neq 4$ $2 x - 4 \leq 3$ $ x - 4 \leq \frac{3}{2}$ $-\frac{3}{2} \leq x - 4 \leq \frac{3}{2}$ $\frac{5}{2} \leq x \leq \frac{11}{2}, x \neq 4$ <i>Marker's comments:</i> 1) Many students forgot to include $x \neq 4$ 2) Many students considered 2 cases but made errors in negative case 3) Considerable number of students multiplied by $(x - 4)^2$ made algebraic errors and failed to consider both cases.	3 marks for correct solution 2 marks for solving absolute inequality correctly (must write answer as one inequality) 2 marks for successfully concluding either $\frac{5}{2} \leq x < 4$ or $4 < x \leq \frac{11}{2}$ 1 mark for recognising x cannot equal to 4
6	$x - 1 \neq 0 \quad \therefore x \neq 1$ $2x = x - 1$ $x = -1$ test $x = -2$; $\frac{-4}{-3} \geq 1$ true test $x = 0$; $0 \geq 1$ false test $x = 2$; $\frac{4}{1} \geq 1$ true $\therefore x \leq -1 \text{ or } x > 1$ Alternative method: (Multiply both sides by $(x - 1)^2$) $2x(x - 1) \geq (x - 1)^2$ $2x^2 - 2x \geq x^2 - 2x + 1$ $x^2 - 1 \geq 0$	3 marks for correct solution 2 marks for solving for x and recognising x cannot equal 1 2 marks for one error in solution 1 mark for solving for critical points 1 mark for recognising x cannot equal to 1

	 since $x \neq 1$ $x \leq -1$ or $x > 1$	
7(i)	$9! = 362880$	1 mark for correct answer
7(ii)	$2 \times 8! = 80640$	1 mark for correct answer
7(iii)	${}^8C_3 \times 4! = 1344$ <i>Marker's comments:</i> <i>Common errors-</i> 1) multiplying by 2 (sitting in first set of 5 chairs, then considering the other 5 chairs, originally used as space/gaps) 2) Not choosing 3 friends first	2 marks for correct solution 1 mark for choosing 3 friends out of 8
8 (i)	2520	See last page
(ii)	480	See last page
(iii)	$2520 - 1 = 2519$	See last page
9	Algebraic solution	see last page
10	See below for full solutions to Qn 10 and 11 $-4032x^2\sqrt{x}$	see last page
11	15	see last page
12	x^5 term given by $x \times x^4$ term + $-1 \times x^5$ term $= x \times {}^7C_3 x^{7-3} 1^4 + (-1) \times {}^7C_2 x^{7-2} 1^2$ $= {}^7C_3 x^5 - 1 \times {}^7C_2 x^5$ $= 35x^5 - 21x^5$ $= 14x^5$ $\therefore$ coefficient is 14	2 marks for full correct solution with working 1 mark for correct 2 nd line of solution or equivalent
13(i)	i) English is mandatory so only need from 6 choose 4 $1 \times {}^6C_4 = 15$	1 mark correct solution

13(ii)	i) At least so 2 humanities and 3 from Maths/Science or 3 humanities and 2 from Maths/Science ${}^3C_2 \times {}^4C_3 + {}^3C_3 \times {}^4C_2$ $= 3 \times 4 + 1 \times 6$ $= 12 + 6$ $= 18$	2 marks correct solution 1 mark for either 2 Humanities and 3 other subjects or 3 Humanities and 2 other subjects
13(iii)	i) Any 5 means from 7 choose 5 so ${}^7C_5 = 21$ Since 90 students using the pigeon hole principle $\frac{90}{21} = 4.7$ $\therefore$ 5 or more will do same combination	2 marks correct solution 1 mark for correct identification of 7 Choose 5 subjects =21
14 (i)	Only 2 choices 0 or 1 each time so with repetition allowed $2 \times 2 \times 2 \dots \times 2$ for sequence of length n $= 2^n$	1 mark correct solution
14(ii)	Considering the sequence to have length n we could have $0 \times 1s, 1 \times 1s, 2 \times 1s, 3 \times 1s \dots n \times 1s$ ie number of 1s is n Choose 0, n Choose 1....n Choose n $\therefore {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots {}^nC_n$ and since from above there are 2^n binary sequences $\therefore \sum_{i=0}^n {}^nC_i = 2^n$ Explanation needs to cover fact that ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots {}^nC_n$ covers all possible sequences of 1s And that this is the same as total number of possible sequences 2^n	2 Marks correct fully explained solution including link to fact that there are 2^n possible sequences of length n 1 mark for explaining that $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$ covers all possible 1s Or 1 mark for Explanation linking to fact that there are 2^n possible sequences of length n