

Question 1 (11 Marks)**Marks**

(a) Simplify $\frac{(2^3)^n (4^{2n-1})}{(8^n)^{\frac{1}{3}}}$. 2

(b) Convert $0.\overline{485}$ to a fraction in its simplest terms. 2

(c) Simplify $\frac{1}{x-y} + \frac{2x-y}{x^2-y^2}$. 2

- (d) Factorise fully
- (i) $2x^3 + 54$. 2
- (ii) $a^2 - b^2 - 6b - 9$. 3

Question 2 (9 Marks)

(a) Rearrange the formula $\frac{1}{f} + \frac{3}{g} = \frac{7}{h}$ so that g is the subject. 2

(b) Expand and simplify $(1 - \sqrt{2})^3$ 2

(c) Simplify the following by rationalising the denominator.

$$\frac{\sqrt{10} - \sqrt{6}}{\sqrt{10} + \sqrt{6}}$$
 2

(d) If $2x - y + \sqrt{x - y} = 2\sqrt{3}$ find x and y . 3

Question 3 (15 Marks) **START A NEW PAGE** **Marks**

- (a) Solve $|x+1| \leq 5$. **2**
- (b) State the domain of the function $f(x) = \sqrt{x^2 - 9}$. **2**
- (c) Sketch each of the following graphs on separate axes. Label carefully.
- (i) $x^2 + (y-2)^2 = 4$. **2**
- (ii) $f(x) = \sqrt{1-x}$. **2**
- (d) Consider $f(x) = \frac{2x}{1-x^2}$.
- (i) Determine the equation of the vertical and horizontal asymptotes. **2**
- (ii) Show that $f(x)$ is odd. **2**
- (iii) Show that $f(x)$ passes through the origin. **1**
- (iv) Hence sketch the curve. **2**

Question 4 (10 Marks) **START A NEW PAGE**

- (a) Solve $\frac{x}{x+5} \leq 2$. **3**
- (b) Shade the region where $y \leq \sqrt{2-x^2}$ and $y > x$ hold simultaneously. Carefully label curves and show points of intersection. **3**
- (c) Sketch $y = |2x-1|$ and $y = x+1$ on the same axes. Find any points of intersection. Hence use your graph to solve $|2x-1| \leq x+1$. **4**

END OF PAPER

Question 1 [11 Marks]

$$\begin{aligned}
 (a) \quad & \frac{(2^3)^n (4^{2n-1})}{(8^n)^{1/3}} \\
 &= \frac{2^{3n} \cdot 2^{4n-2}}{2^n} \checkmark \\
 &= 2^{6n-2} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & \text{let } p = 0.485 \\
 & \therefore 100p = 48.585 \checkmark \\
 & \therefore 99p = 48.1 \\
 & p = \frac{481}{990} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad & \frac{1}{x-y} + \frac{2x-y}{(x-y)(x+y)} \\
 &= \frac{x+y + 2x-y}{(x-y)(x+y)} \checkmark \\
 &= \frac{3x}{(x-y)(x+y)} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad (i) \quad & 2x^3 + 54 \\
 &= 2(x^3 + 27) \checkmark \\
 &= 2(x+3)(x^2 - 3x + 9) \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & a^2 - b^2 - 6b - 9 \\
 &= a^2 - (b^2 + 6b + 9) \checkmark \\
 &= a^2 - (b+3)^2 \checkmark \\
 &= (a-b-3)(a+b+3) \checkmark
 \end{aligned}$$

Question 2 [9 Marks]

(a) $\frac{1}{f} + \frac{3}{g} = \frac{7}{h}$ find g

$$\frac{3}{g} = \frac{7}{h} - \frac{1}{f}$$

$$\frac{3}{g} = \frac{7f-h}{hf}$$

$$\frac{g}{3} = \frac{hf}{7f-h}$$

$$g = \frac{3hf}{7f-h}$$

(b) $(1-\sqrt{2})^3 = 1 - 3(\sqrt{2}) + 3(\sqrt{2})^2 - (\sqrt{2})^3$ ✓
 $= 1 - 3\sqrt{2} + 6 - 2\sqrt{2}$
 $= 7 - 5\sqrt{2}$ ✓

(c) $\frac{\sqrt{10}-\sqrt{6}}{\sqrt{10}+\sqrt{6}} \times \frac{\sqrt{10}-\sqrt{6}}{\sqrt{10}-\sqrt{6}}$

$$= \frac{(\sqrt{10}-\sqrt{6})^2}{10-6}$$

$$= \frac{10 - 2\sqrt{60} + 6}{4}$$

$$= \frac{16 - 2\sqrt{60}}{4}$$

$$= \frac{16 - 4\sqrt{15}}{4}$$

$$= 4 - \sqrt{15}$$

(d) $2x - y + \sqrt{x-y} = 2\sqrt{3}$

$$\therefore (2x-y) + \sqrt{x-y} = \sqrt{12}$$

$$\therefore 2x-y = 0 \quad \dots (i)$$

$$x-y = 12 \quad \dots (ii)$$

$$\therefore x = -12 \quad [(i)-(ii)]$$

$$\therefore y = -24$$

Question 3 [15 Marks]

(a) $|x+1| \leq 5$

or $-5 \leq x+1 \leq 5$ ✓

$x+1 \leq 5$ and $x+1 \geq -5$

$-6 \leq x \leq 4$ ✓

$\therefore -6 \leq x \leq 4$

(b) $f(x) = \sqrt{x^2 - 9}$

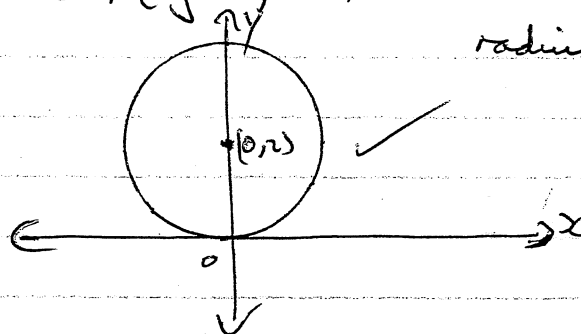
Domain: $x^2 - 9 \geq 0$

$(x-3)(x+3) \geq 0$ ✓

$\therefore x \geq 3$ or $x \leq -3$ ✓

(c) (i) $x^2 + (y-2)^2 = 4$ centre $(0, 2)$ ✓

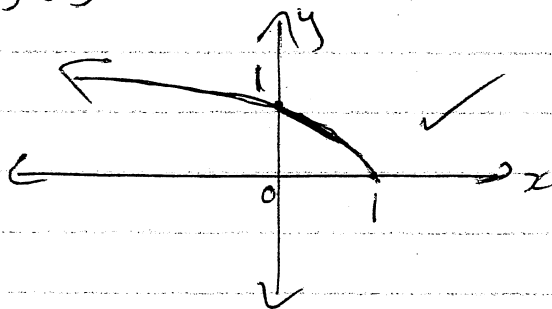
radius 2



(ii) $f(x) = \sqrt{1-x}$

Domain: $1-x \geq 0$ ✓

$x \leq 1$ ✓



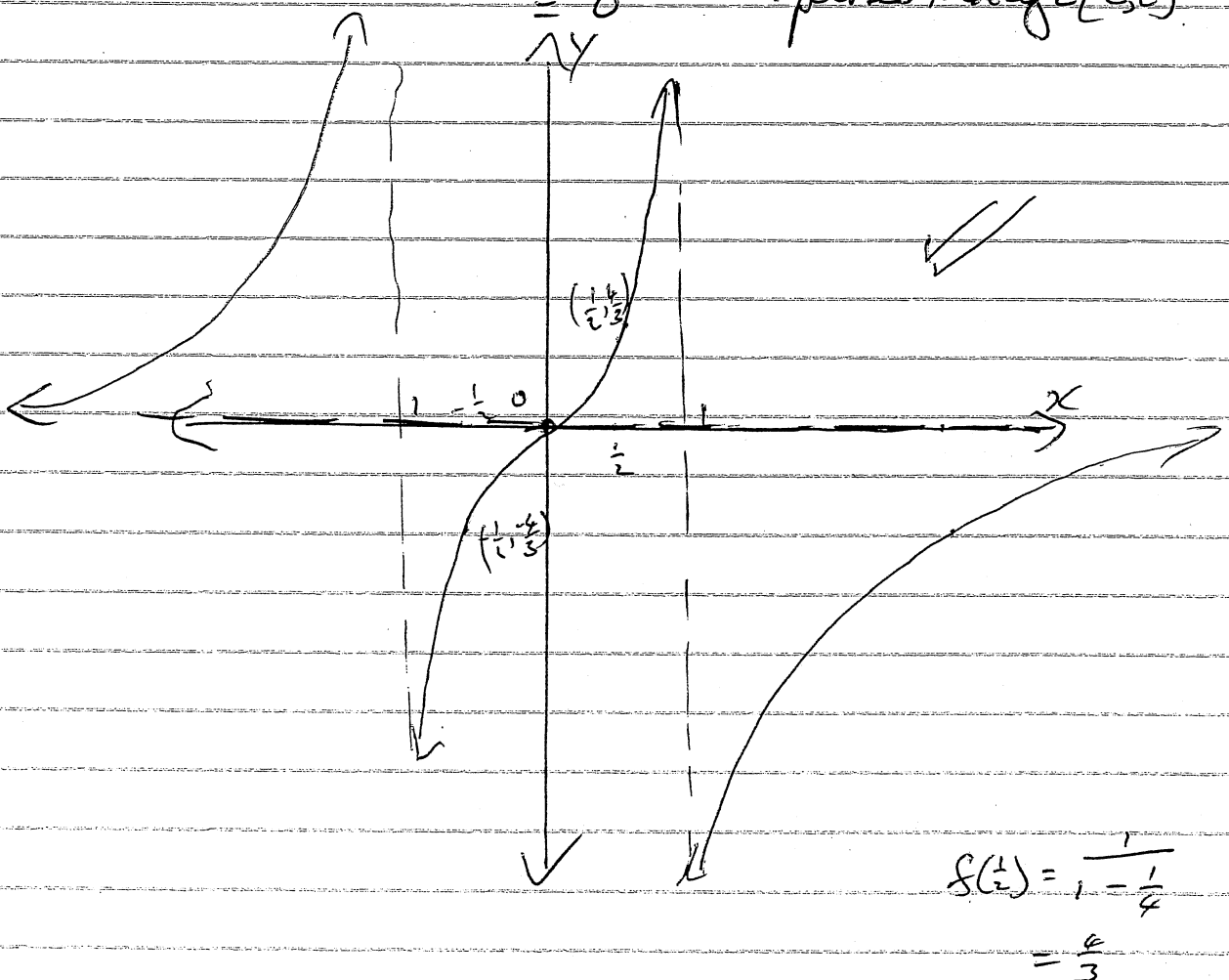
Question 3 (continued)

(d) $f(x) = \frac{2x}{1-x^2}$

(i) Vertical asymptotes $x = \pm 1$ ✓
Horizontal asymptote $f(x) = 0$ ✓

(ii) $f(-x) = \frac{-2x}{1-(-x)^2}$ ✓
 $= \frac{-2x}{1-x^2}$
 $= -\left(\frac{2x}{1-x^2}\right)$
 $= -f(x) \therefore \text{odd}$ ✓

(iii) let $x = 0 \therefore f(0) = \frac{2(0)}{1-0} = 0$ ✓
 $\therefore \text{passes through } (0,0)$



Question 4

(a) $\frac{x}{x+5} \leq 2, \quad x \neq -5$

$\therefore \frac{x}{x+5} \times \frac{(x+5)^2}{1} \leq 2x(x+5)^2 \quad \checkmark$

$x(x+5) \leq 2(x+5)^2$

$\therefore 2(x+5)^2 - x(x+5) \geq 0 \quad \checkmark$

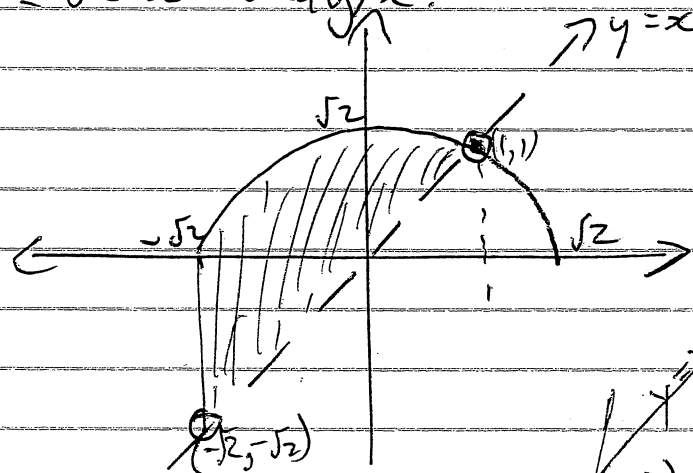
$(x+5)[2(x+5) - x] \geq 0$

$(x+5)(x+10) \geq 0$

$\therefore x \geq -5 \text{ or } x \leq -10 \quad \checkmark$

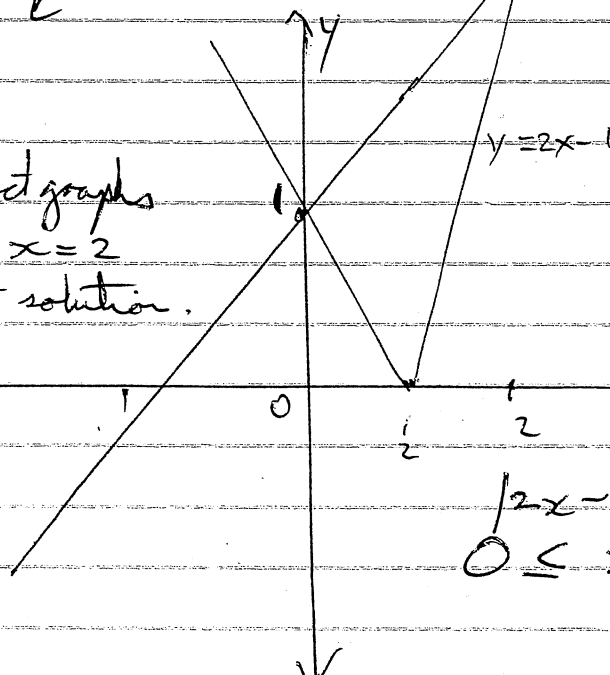
[deduct 1 for ≥ -5]

(b) $y \leq \sqrt{2-x^2}$ and $y > x$



- $\checkmark$ - both graphs
- $\checkmark$ - pts of intersection & dotted line
- $\checkmark$ - correct shading

$\swarrow$



(c)

$\checkmark$ - correct graphs

$\checkmark$ - find $x=2$

$\checkmark$ - correct solution.

let $x = \sqrt{2-x^2}$

$x^2 = 2-x^2 \quad x^2 = 1$

$x = 1$

let $2x-1 = x+1$

$x = 2$

$|2x-1| \leq x+1$
 $0 \leq x \leq 2$