NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2019 Year 11 Course Assessment Task 2
Wednesday, 29 May 2019

General instructions

- Working time – 1 hour (plus 5 minutes reading time).
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt all questions.
- All necessary working should be shown in every question.
- Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: ..... **# BOOKLETS USED:** .....

Class: (please ✓)

- ○ 11MAT.1 – Mr Lam
- ○ 11MAT.2 – Mr Sekaran
- ○ 11MAT.3 – Ms Park
- ○ 11MAT.4 – Ms Ham
- ○ 11MAT.5 – Ms Gan

Marker's use only

QUESTION	1	2	3	4	Total	%
MARKS	/22	/11	/14	/13	/60	

Question 1 (22 marks) **Marks**

(a) Is the function $f(x) = x^3 - x$ odd, even or neither? Justify your answer. **2**

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(b) Find the domain and range of the following in interval notation.

i. $y = 3x^2 - 1$ **2**

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ii. $y = \frac{1}{x-2}$ **2**

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iii. $y = -\sqrt{9 - x^2}$ **2**

.....

.....

(c) Given $f(x) = x^2 + 1$ and $g(x) = 3 - x$, find:

i. $f(g(2))$. **1**

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ii. $g\big(f(x)\big)$ in simplest form. **1**

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iii. The value(s) of x such that $g\big(f(x)\big) = f(x)$. **2**

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(d) Solve $|7 - 3x| = 1$ **2**

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(e) Sketch the following graphs showing all important features, including x and y intercepts.

i. $y = (x - 1)(x + 3)(4 - x)$ **2**

ii. $y = -x^2 - 4x + 5$ **2**

iii. $y = \sqrt{x+3}$ **2**

iv. $y = |x - 2| + 1$ **2**

Examination continues overleaf

Question 2 (11 marks) **Marks**

(a) Solve the equation $\log_2 x + \log_2(x+7) = 3$. **3**

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(b) Fully simplify $\sqrt{\frac{3^{5k+2}}{27^k}}$ **2**

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(c) Sketch the graph of $y = \log_4(3 - x)$, showing all important features. **2**

(d) The number N of individuals in a population at time t years is given by

$$N = 100 - 60e^{-0.1t} \quad \text{where } e = 2.71828 \ldots$$

i. Find the initial population N at $t = 0$ and what N tends towards as $t \to \infty$. **2**

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ii. Find the exact time taken for the population to double its initial size. **2**

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Question 3 (14 marks) **Marks**

(a) If $\sin\theta = \frac{1}{2}$ and $\cos\theta < 0$, what is the exact value of $\tan\theta$? **2**

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(b) Solve $\cos^2 x + 2\sin x = 0$, for $0° \leq x \leq 360°$. **2**

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(c) Solve $\tan 2x = -\sqrt{3}$, for $-180° \leq x \leq 180°$. **2**

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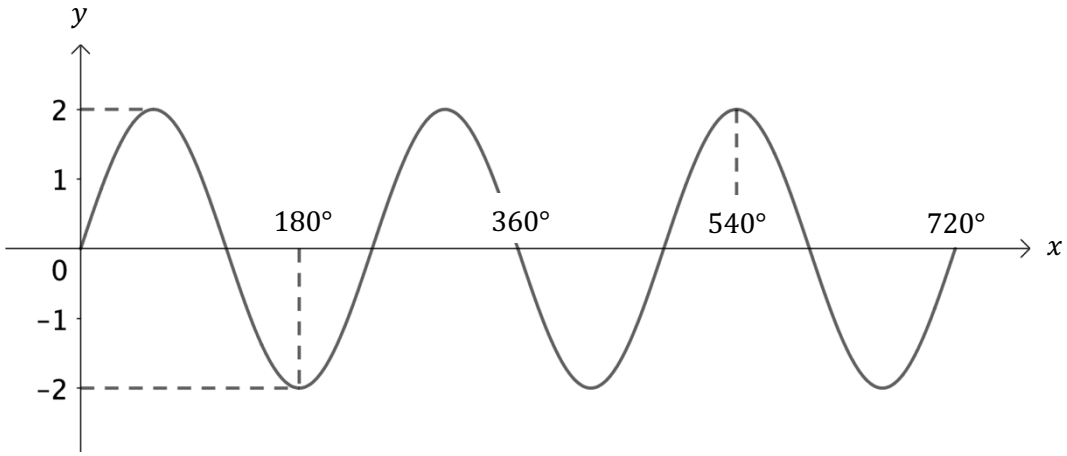
(d) Find the equation of the trigonometric graph below. **2**



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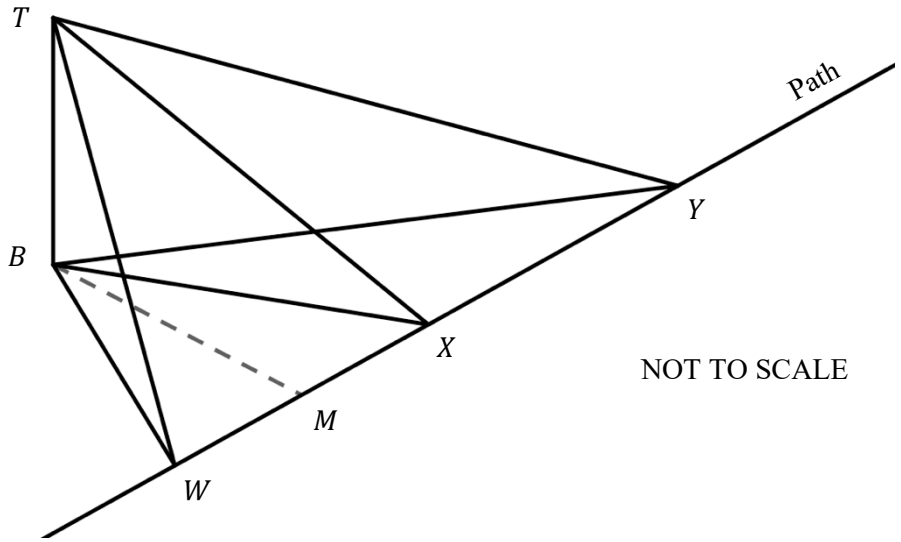
Examination continues overleaf

(e) Three points W, X and Y lie on a straight path on the ground, where $WX = XY = 40$ m. The base, B, of a flagpole, TB, is on the same horizontal plane as the path. M is the midpoint of WX. The angles of elevation to the top of the flagpole from the points W, X and Y are α, α, and β respectively. The height of the flagpole is h.



i. Prove that ΔBWX is isosceles. **2**

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ii. Find the length of BM in terms of h and α. **1**

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iii. Hence, or otherwise, show that the height of the flagpole is given by **3**

$$h = \frac{40\sqrt{2}}{\sqrt{\cot^2 \beta - \cot^2 \alpha}}$$

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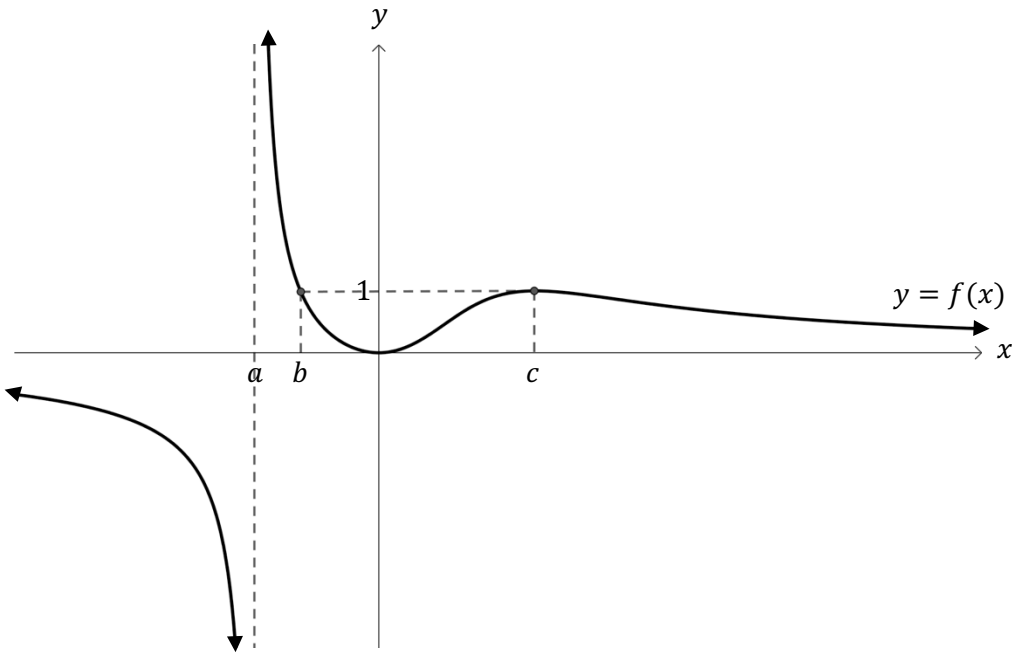
Examination continues overleaf

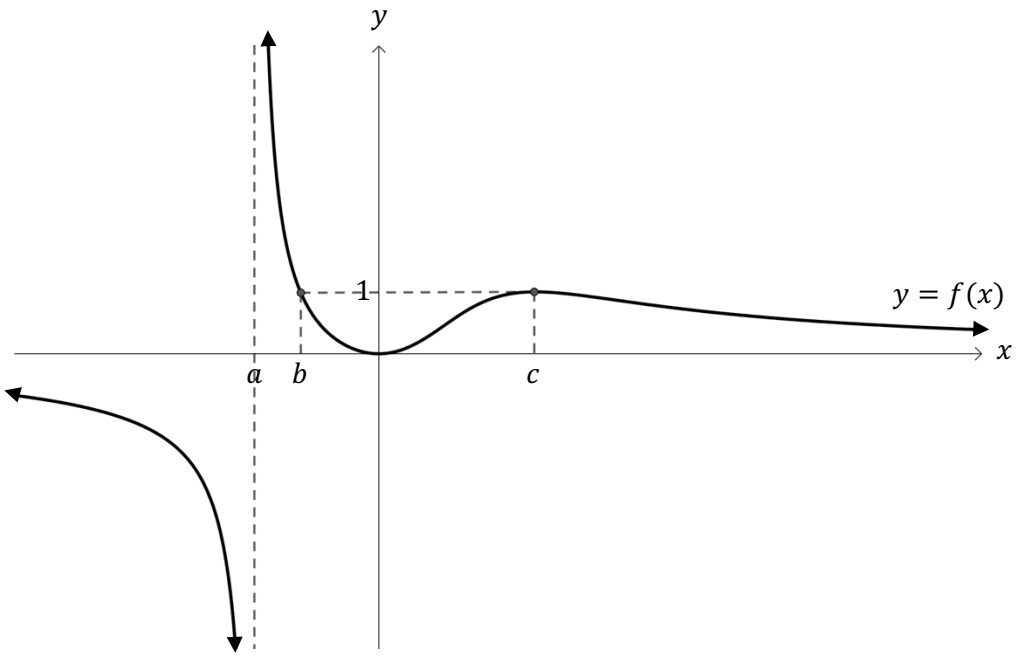
Question 4 (13 marks) **Marks**

(a) Given the graph of $y = f(x)$, sketch the following on the same set of axes:

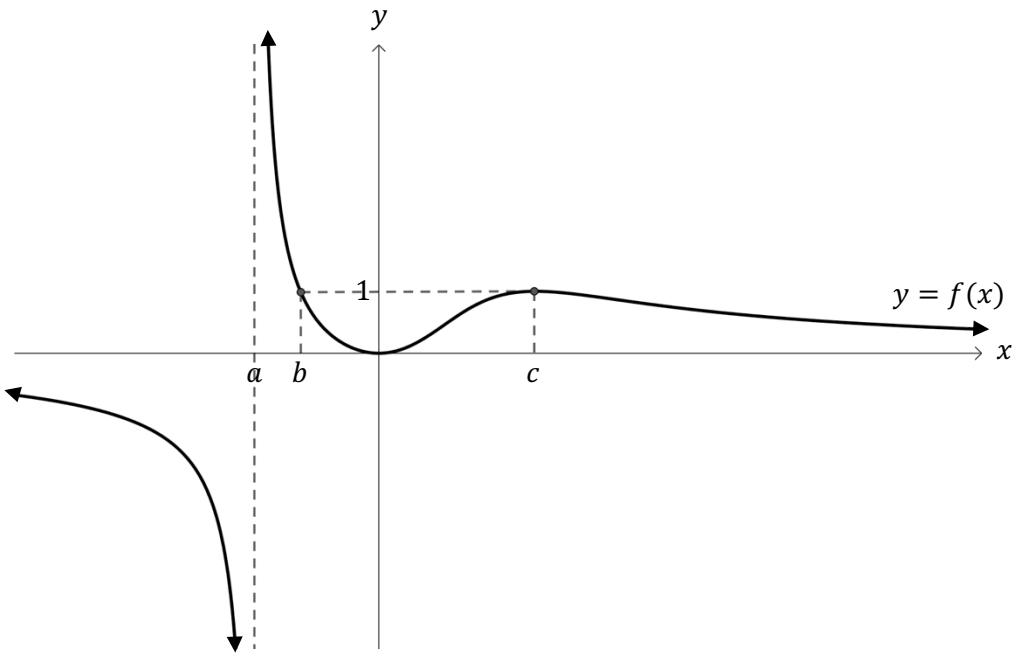
i. $y = f(-x)$ **2**



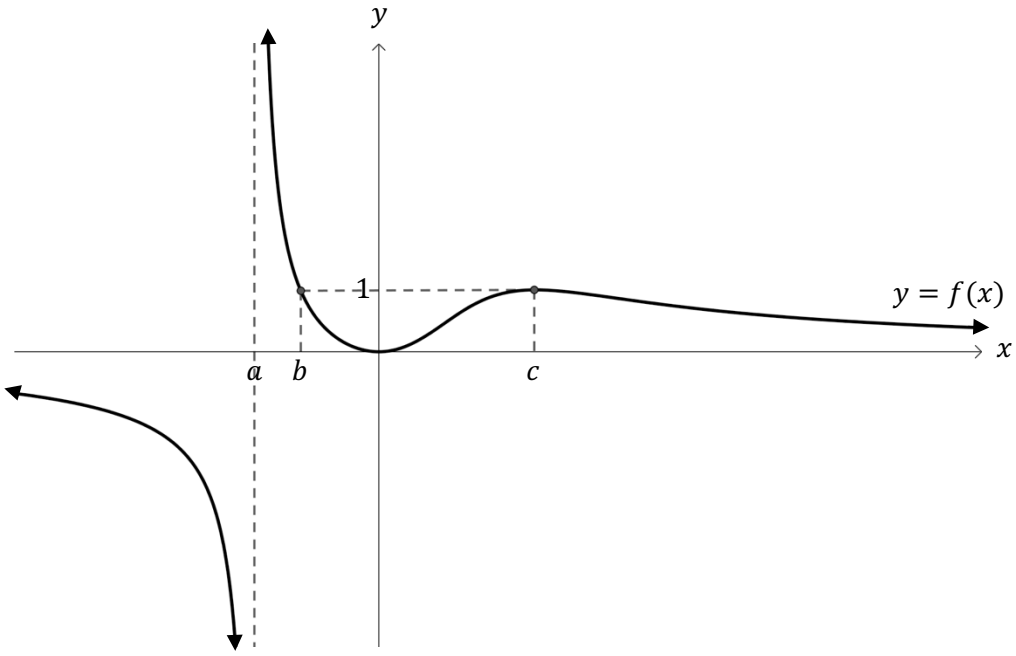
ii. $y = \frac{1}{f(x)}$ **2**



iii. $y^2 = f(x)$ **2**



iv. $y = f(|x|)$ **2**



(b) For the function $f(x) = x^2 - 4x + 4$.

i. State the largest domain containing positive values for which $f(x)$ has an inverse. **1**

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ii. Find the equation of the inverse function. **2**

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(c) i. Sketch the graph of $y = x$ and $y = 2^{-x}$ on the same axes. **1**

ii. Hence, sketch the graph of $y = x + 2^{-x}$ by addition of ordinates, showing all important features. **1**

End of Examination

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QUESTION	1	2	3	4	Total	%
MARKS	___/22	___/11	___/14	___/13	___/60	

Question 1 (22 marks) **Marks**

(a) Is the function $f(x) = x^3 - x$ odd, even or neither? Justify your answer. **2**

$f(-x) = (-x)^3 - (-x)$ ✓

$= -x^3 + x$

$= -(x^3 - x)$

$= -f(x)$

$\therefore$ odd function ✓

(b) Find the domain and range of the following in interval notation.

i. $y = 3x^2 - 1$ **2**

$D: (-\infty, \infty)$ ✓

$R: [-1, \infty)$ ✓

ii. $y = \frac{1}{x-2}$ **2**

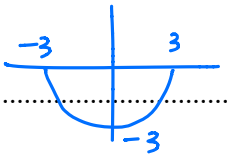
$D: (-\infty, 2) \cup (2, \infty)$ ✓

$R: (-\infty, 0) \cup (0, \infty)$ ✓

iii. $y = -\sqrt{9 - x^2}$ **2**

$D: [-3, 3]$ ✓

$R: [-3, 0]$ ✓



deduct 1 mark overall if not in interval notation

(c) Given $f(x) = x^2 + 1$ and $g(x) = 3 - x$, find:

i. $f(g(2))$. **1**

$g(2) = 3 - 2 = 1$

$f(g(2)) = 1^2 + 1 = 2$ ✓

ii. $g(f(x))$ in simplest form. **1**

$g(f(x)) = 3 - (x^2 + 1)$

$= 2 - x^2$ ✓

iii. The value(s) of x such that $g(f(x)) = f(x)$. **2**

$2-x^2 = x^2+1$ ✓

$0 = 2x^2 - 1$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$ ✓

(d) Solve $|7-3x| = 1$ **2**

$7-3x = 1$, $7-3x = -1$

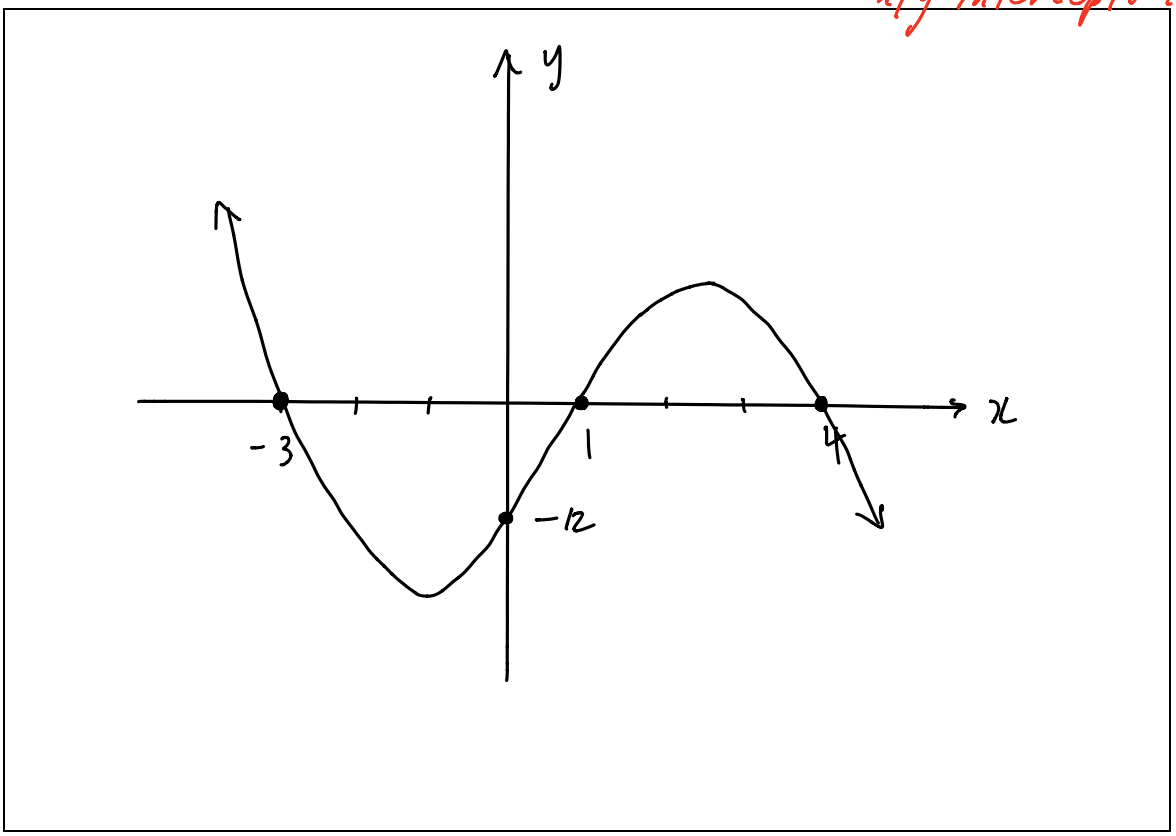
$6 = 3x$ $8 = 3x$

$x = 2$ ✓ $x = \frac{8}{3}$ ✓

(e) Sketch the following graphs showing all important features, including x and y intercepts.

i. $y = (x-1)(x+3)(4-x)$ **2**

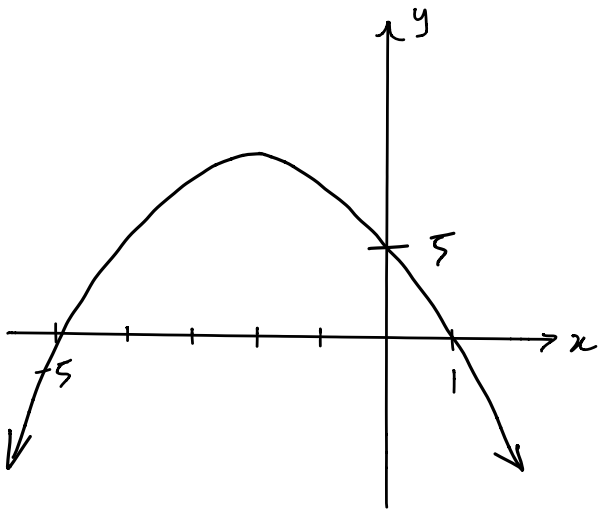
✓ shape ✓ critical point/ x/y intercepts labelled



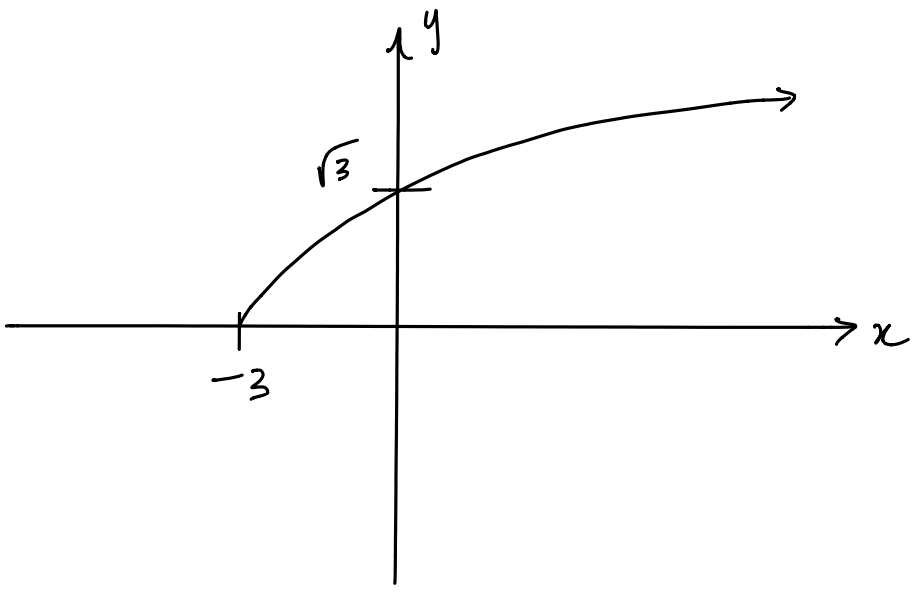
ii. $y = -x^2 - 4x + 5$ **2**

$= -(x^2+4x-5)$

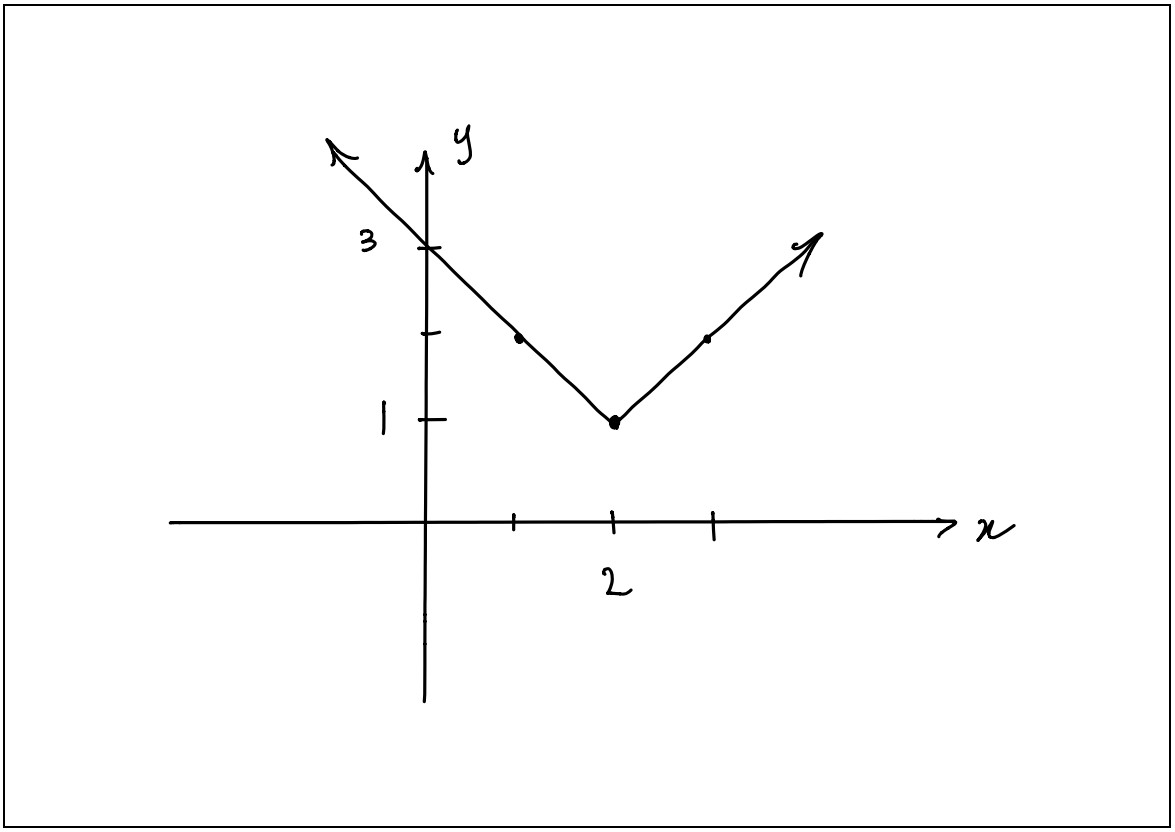
$= -(x-1)(x+5)$



iii. $y = \sqrt{x+3}$ **2**



iv. $y = |x - 2| + 1$ **2**



Examination continues overleaf

Question 2 (11 marks) **Marks**

(a) Solve the equation $\log_2 x + \log_2(x+7) = 3$. **3**

$\log_2 (x(x+7)) = 3$ ✓

$2^3 = x^2 + 7x$

$0 = x^2 + 7x - 8$

$= (x-1)(x+8)$

$x = 1, -8$ ✓

$= 1$ ✓ $(x+7 > 0 \therefore x > -7)$

(b) Fully simplify $\sqrt{\frac{3^{5k+2}}{27^k}}$ **2**

$= \left(\frac{3^{5k+2}}{3^{3k}}\right)^{\frac{1}{2}}$ ✓

$= (3^{5k+2-3k})^{\frac{1}{2}}$

$= (3^{2k+2})^{\frac{1}{2}}$

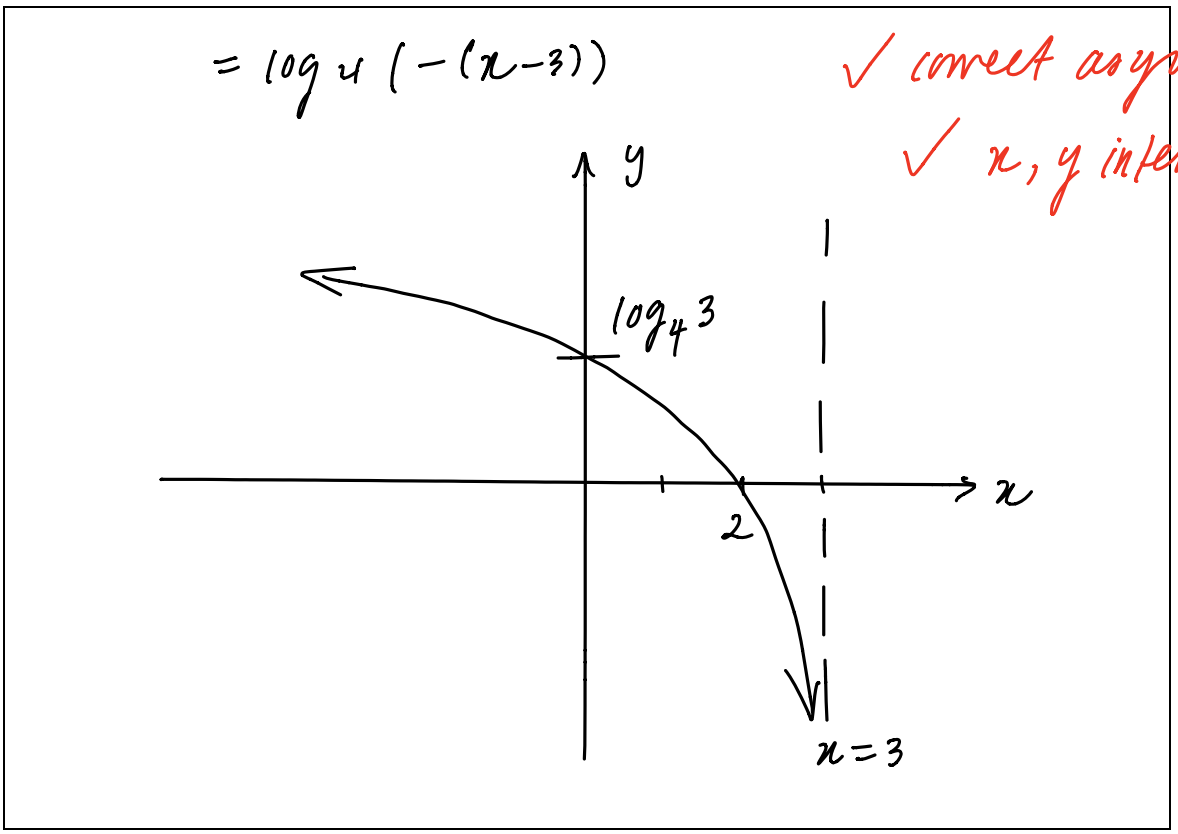
$= 3^{k+1}$ ✓

(c) Sketch the graph of $y = \log_4(3 - x)$, showing all important features. **2**

$= \log_4(-(x-3))$

✓ correct asymptote

✓ x, y intercepts



(d) The number N of individuals in a population at time t years is given by

$$N = 100 - 60e^{-0.1t} \quad \text{where } e = 2.71828\ldots$$

i. Find the initial population N at $t = 0$ and what N tends towards as $t \to \infty$. **2**

$N = 100 - 60e^{0}$

$= 40$ ✓

as $t \to \infty$, $N \to 100$ ✓ (since $e^{-0.1t} \to 0$)

ii. Find the exact time taken for the population to double its initial size. **2**

$80 = 100 - 60e^{-0.1t}$

$-20 = -60e^{-0.1t}$

$\frac{1}{3} = e^{-0.1t}$ ✓

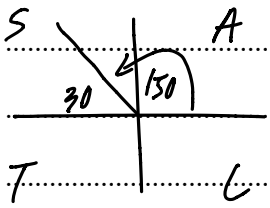
$\log_e \frac{1}{3} = -0.1t$

$t = \frac{1}{-0.1} \log_e \frac{1}{3}$ ✓

$= 10 \log_e 3$

Question 3 (14 marks) **Marks**

(a) If $\sin\theta = \frac{1}{2}$ and $\cos\theta < 0$, what is the exact value of $\tan\theta$? **2**



$\tan\theta = -\frac{1}{\sqrt{3}}$ ✓ ✓

(b) Solve $\cos^2 x + 2\sin x = 0$, for $0° \le x \le 360°$. **2**

$1 - \sin^2 x + 2\sin x = 0$

$\sin^2 x - 2\sin x - 1 = 0$ ✓

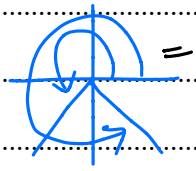
$u^2 - 2u - 1 = 0$ (using quad. formula)

$u = 1 \pm \sqrt{2}$

$\sin x = 1 \pm \sqrt{2}$

$= 1 - \sqrt{2}$ only since $-1 \le \sin x \le 1$

$= -0.41\ldots$



$= 204°28', 335°32'$ ✓

correct to the nearest minute

accept correct to the nearest degree also.

(c) Solve $\tan 2x = -\sqrt{3}$, for $-180° \le x \le 180°$. **2**

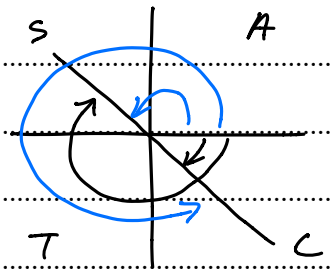
$-360° \le 2x \le 360°$

$2x = \tan^{-1}(-\sqrt{3})$

$= -60°, -240°,$

$120°, 300°$

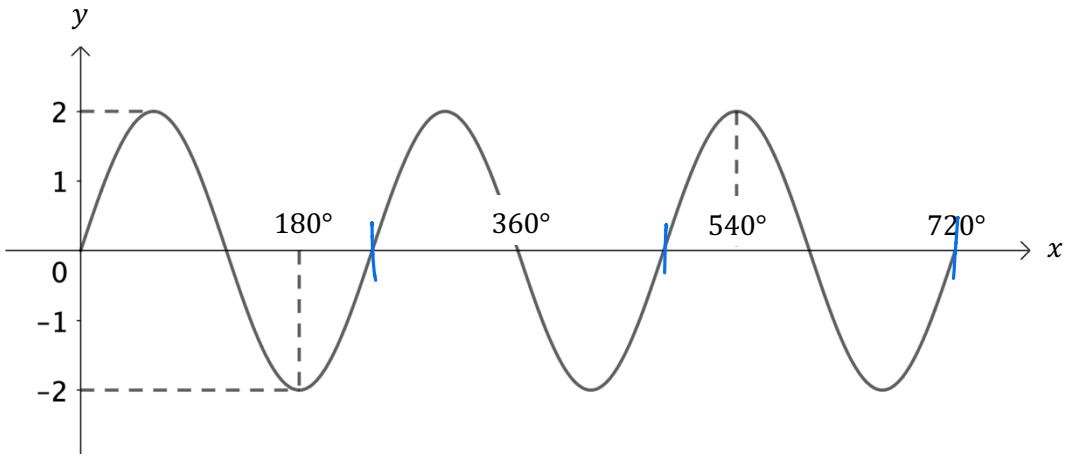
$x = -30°, -120°, 60°, 150°$



✓ for $-30°, 60°$

✓ for $-120°, 150°$

(d) Find the equation of the trigonometric graph below. **2**



$y = 2\sin(nx)$, $(180°, -2) \rightarrow -2 = 2\sin(n \times 180°)$

$-1 = \sin(n \times 180°)$

$n \times 180° = \sin^{-1}(-1)$

$= 270°$

$n = \frac{3}{2}$ ✓

$\therefore y = 2\sin(\frac{3}{2}x)$ ✓

Alternatively,

$3 \times \text{period} = 720°$

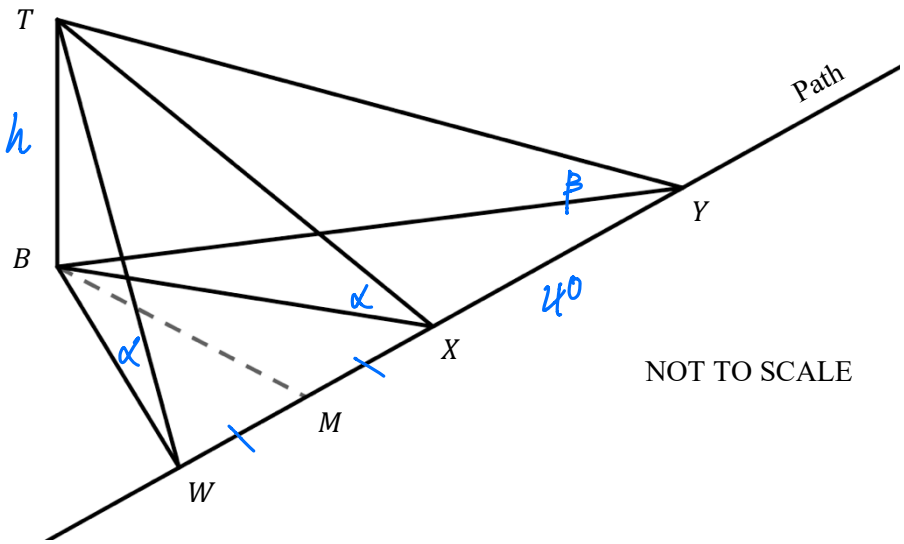
$\text{period} = 240°$

$\frac{2 \times 180°}{n} = 240°$

$\frac{3}{2} = n$

Examination continues overleaf

(e) Three points W, X and Y lie on a straight path on the ground, where $WX = XY = 40$ m. The base, B, of a flagpole, TB, is on the same horizontal plane as the path. M is the midpoint of WX. The angles of elevation to the top of the flagpole from the points W, X and Y are α, α, and β respectively. The height of the flagpole is h.



i. Prove that ΔBWX is isosceles. **2**

In $\triangle TBW$, $\tan\alpha = \frac{h}{BW} \rightarrow BW = h\cot\alpha$ ✓

In $\triangle TBX$, $\tan\alpha = \frac{h}{BX} \rightarrow BX = h\cot\alpha$

$\therefore BW = BX$ ✓

$\therefore \Delta BWX$ isosceles

ii. Find the length of BM in terms of h and α. **1**

In $\triangle TBW$, $\tan\alpha = \frac{h}{BW}$

$BW = h\cot\alpha$

In $\triangle BWM$, $BM = \sqrt{BW^2 - 20^2}$

$= \sqrt{h^2\cot^2\alpha - 400}$ ✓ or $\sqrt{\frac{h^2}{\tan^2\alpha} - 400}$

iii. Hence, or otherwise, show that the height of the flagpole is given by **3**

$$h = \frac{40\sqrt{2}}{\sqrt{\cot^2\beta - \cot^2\alpha}}$$

In $\triangle TBY$, $\tan\beta = \frac{h}{BY}$

$BY = h\cot\beta$ ✓

In $\triangle BMY$, $\angle BMY = 90°$ ($\triangle BWX$ isosceles)

$\therefore BY^2 = BM^2 + MY^2$ (pythagoras' theorem)

$h^2\cot^2\beta = h^2\cot^2\alpha - 400 + 60^2$ ✓

$h^2(\cot^2\beta - \cot^2\alpha) = 3200$

$h = \sqrt{\frac{3200}{\cot^2\beta - \cot^2\alpha}}$

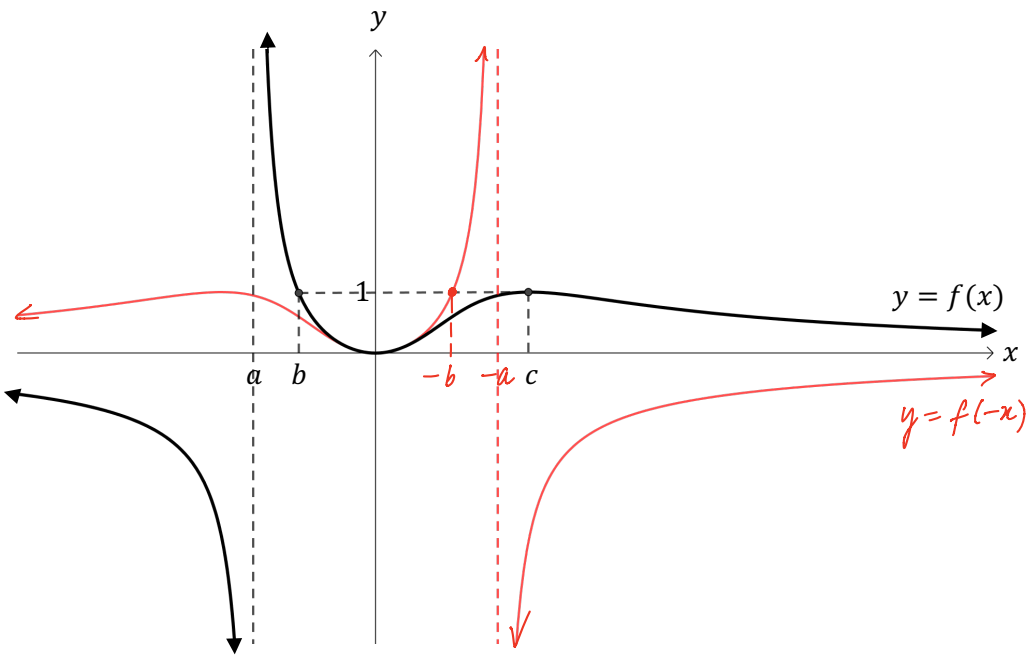
$= \frac{40\sqrt{2}}{\sqrt{\cot^2\beta - \cot^2\alpha}}$ ($h > 0$) ✓

Examination continues overleaf

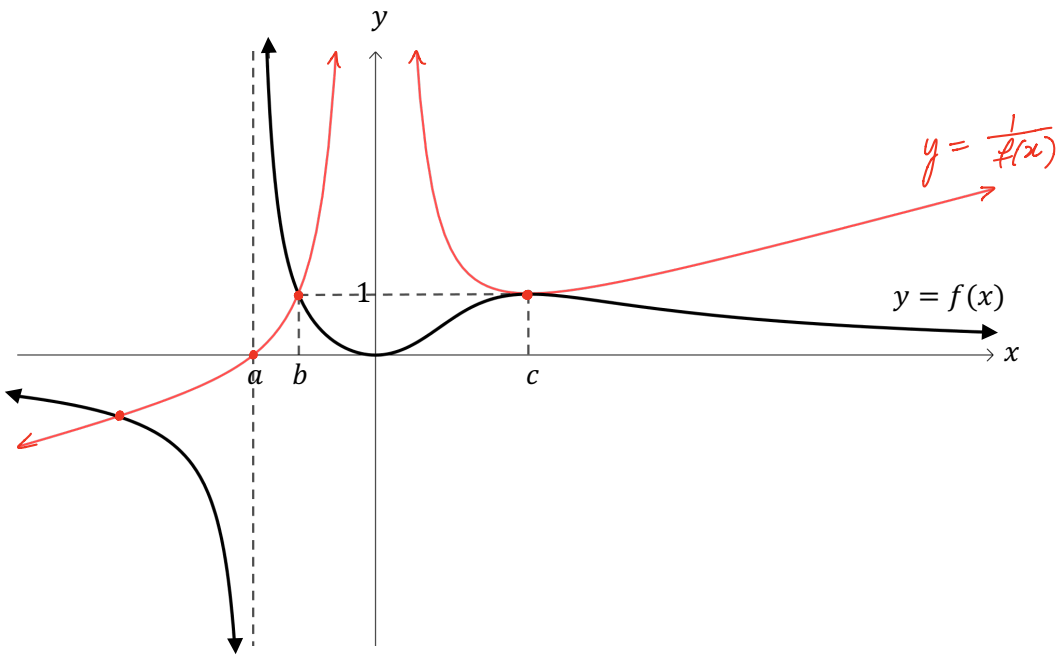
Question 4 (13 marks) **Marks**

(a) Given the graph of $y = f(x)$, sketch the following on the same set of axes:

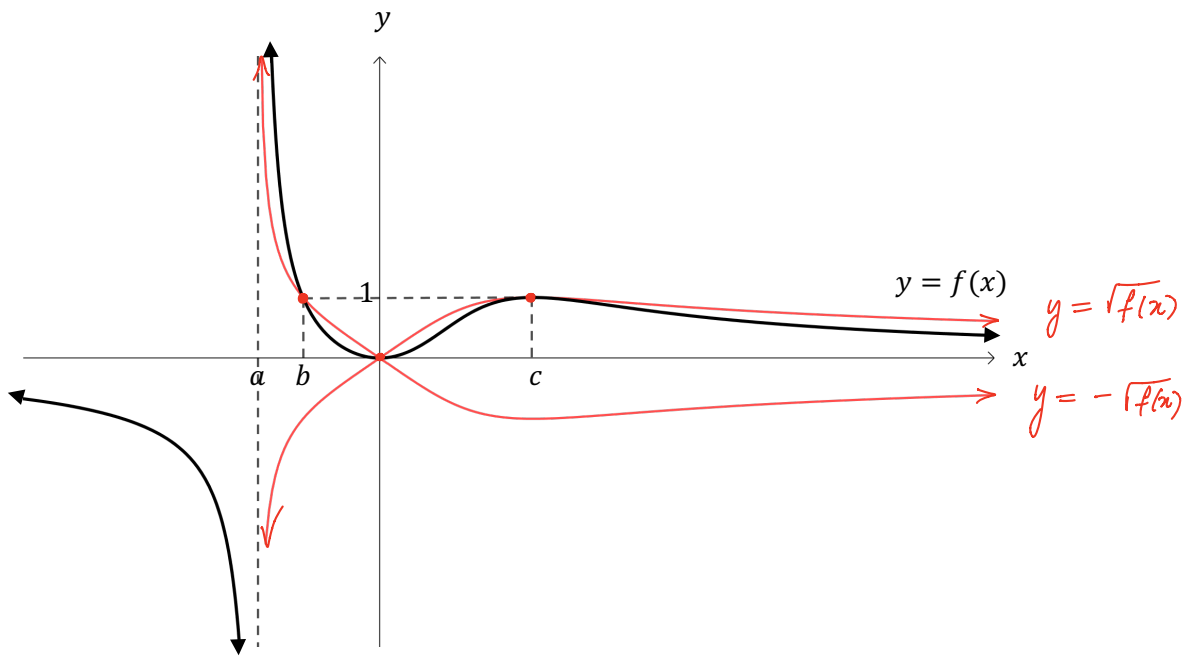
i. $y = f(-x)$ **2**



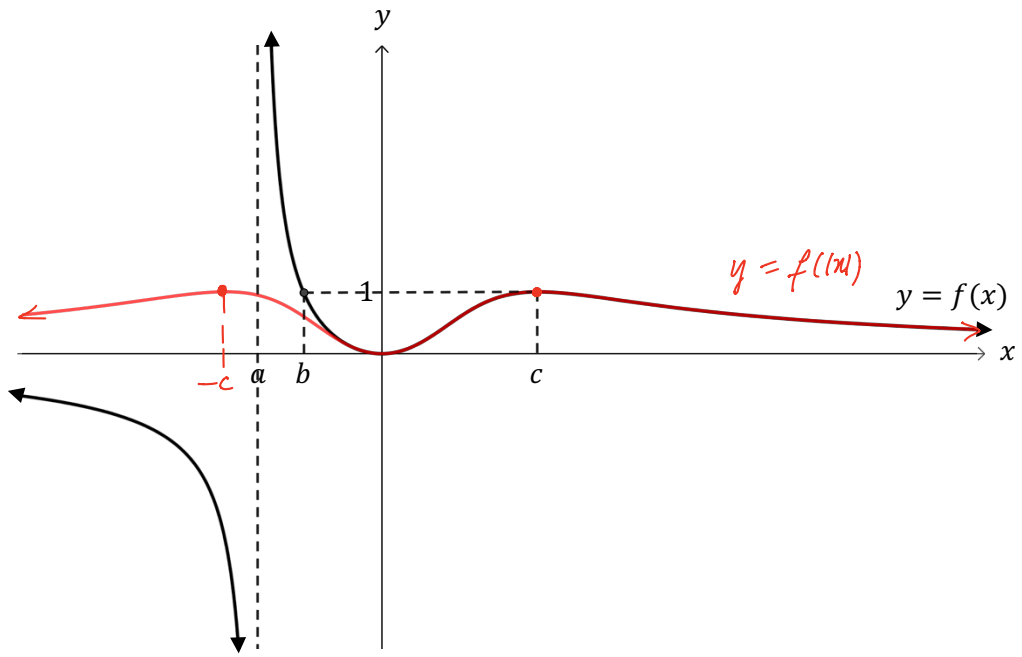
ii. $y = \frac{1}{f(x)}$ **2**



iii. $y^2 = f(x)$ **2**



iv. $y = f(|x|)$ **2**

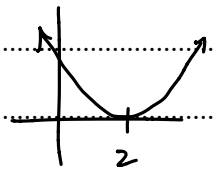


(b) For the function $f(x) = x^2 - 4x + 4$.

i. State the largest domain containing positive values for which $f(x)$ has an inverse. **1**

$f(x) = (x-2)^2$

$D: x \in \mathbb{R}, x \geq 2$ ✓



ii. Find the equation of the inverse function. **2**

$x = (y-2)^2$

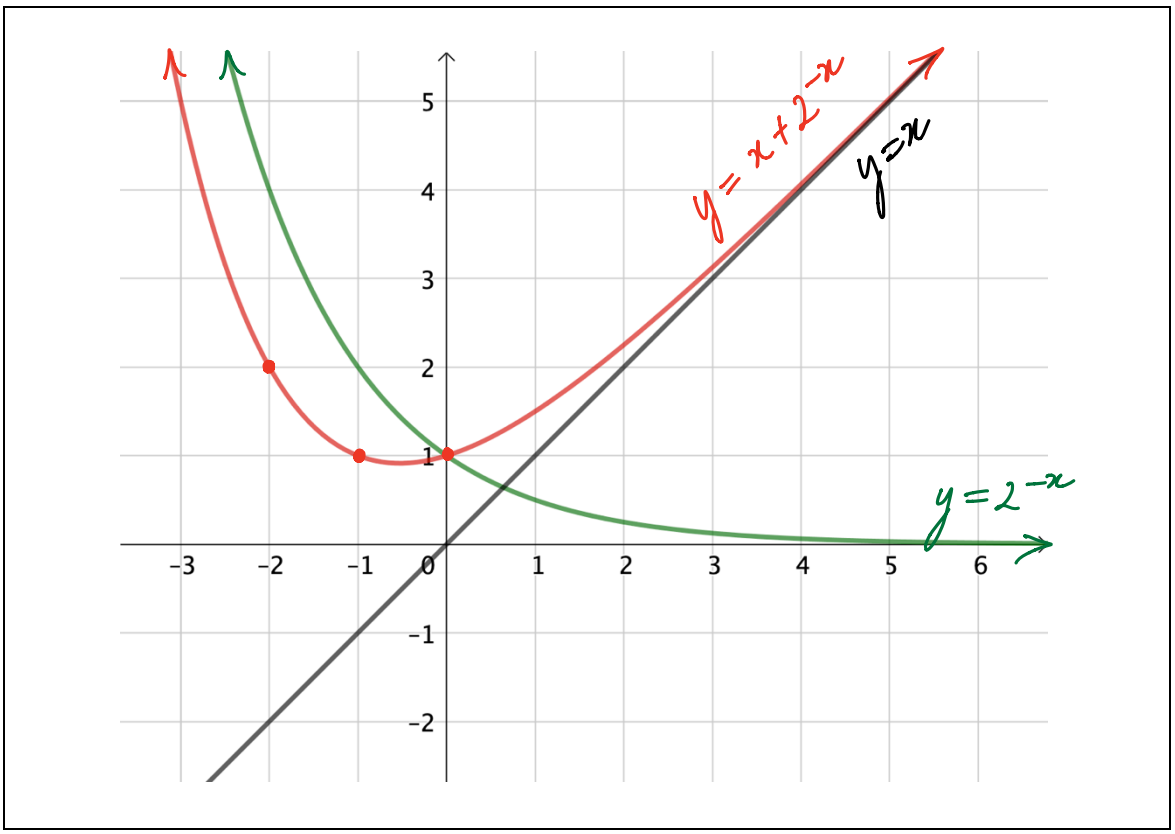
$\pm\sqrt{x} = y-2$

$y = \pm\sqrt{x} + 2$ ✓

$= \sqrt{x} + 2$ ✓ (since $y \geq 2$) since the domain of $f(x)$ becomes range of $f^{-1}(x)$.

(c) i. Sketch the graph of $y = x$ and $y = 2^{-x}$ on the same axes. **1**

ii. Hence, sketch the graph of $y = x + 2^{-x}$ by addition of ordinates, showing all important features. **1**



End of Examination