NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2020 Year 11 Course Assessment Task 3

Tuesday, 30 June 2020

General instructions

- Working time – 60 minutes.
  (plus 5 minutes reading time)
- Commence each new question on a new Writing Booklet. Write on both sides of the paper.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.
- At the conclusion of the examination, arrange the booklets used in the correct order after this paper and hand to examination supervisors.
- A NESA Reference Sheet is provided.

Class (please ✔)

◯ 11MAX.1 – Ms Madden

◯ 11MAX.2 – Mr Siu

◯ 11MAX.3 – Ms Ham

◯ 11MAX.4 – Mr Sekaran

◯ 11MAX.5 – Mr Ho

NESA STUDENT #: .... **# BOOKLETS USED**: .....

Marker's use only.

QUESTION	1	2	3	4	5	Total	%
MARKS	$\overline{7}$	$\overline{10}$	$\overline{9}$	$\overline{14}$	$\overline{10}$	$\overline{50}$	

Question 1 (7 Marks) Commence a NEW writing booklet **Marks**

(a) Rationalise the denominator $\dfrac{4+3\sqrt{5}}{2-\sqrt{6}}$, leaving your answer in the simplest form. **2**

(b) Sketch the graph of $y = |x+4| - 3$, showing all important features. **2**

(c) Consider the piecewise defined function

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{for } x \le -1 \\ 2^x + 2 & \text{for } x > -1 \end{cases}$$

i. Draw a neat sketch of the graph of $y = f(x)$ on one-third of a page in your writing booklet. **2**

ii. Hence state the range of the function, using interval notation. **1**

Question 2 (10 Marks) Commence a NEW writing booklet **Marks**

(a) Differentiate $y = \dfrac{11}{x^5}$, writing your answer without a negative index. **2**

(b) Find the equation of the normal to the curve $y = 6 + 8x - 3x^4$ at $x = 1$. **3**

(c) Evaluate: $\displaystyle\lim_{x\to\infty} \frac{3 - x - 2x^2}{5x^2 - 7x + 4}$. **2**

(d) Differentiate $f(x) = 6 - 9x^2$ from first principles. **3**

Question 3 (9 Marks) Commence a NEW writing booklet **Marks**

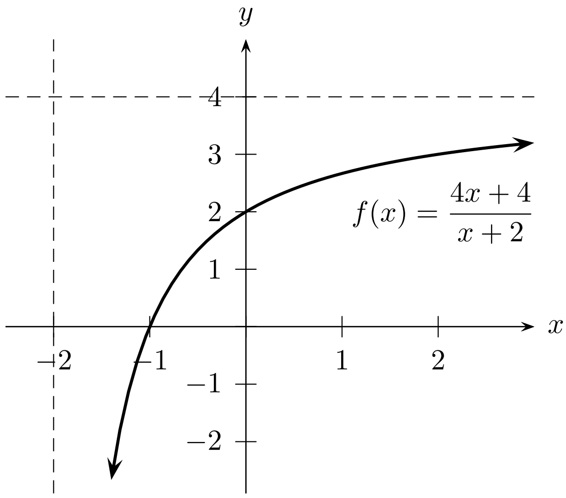
(a) Consider the functions $f(x) = x - 3$ and $g(x) = 2x^2 + 1$.

i. Find $h(x)$, where $h(x) = g\Big(f(x)\Big)$. **1**

ii. Find the largest domain containing only positive values of x, for which $h(x)$ has an inverse function. **1**

iii. Hence find the inverse function of $h(x)$. **2**

(b) The diagram below shows the graph of the curve $y = f(x)$ where $f(x) = \dfrac{4x+4}{x+2}$ for $x > -2$.



On separate diagrams, sketch the graphs of the following curves, showing clearly the intercepts with the axes and equations of any asymptotes.

It is recommended you draw or trace a copy of $y = f(x)$ on to your writing booklet to use as guidance to demonstrate your understanding.

i. $y = |f(x)|$. **1**

ii. $y^2 = f(x)$. **2**

iii. $y = \dfrac{1}{f(x)}$. **2**

Question 4 (14 Marks) Commence a NEW writing booklet **Marks**

(a) Fully simplify: $9\cos^2 x + (2 - 3\sin x)^2$. **2**

(b) Find the exact value for $\tan(\alpha + \beta)$ given that $\cos\alpha = \frac{\sqrt{3}}{4}$ for $-\frac{\pi}{2} < \alpha < 0$ and $\sin\beta = \frac{\sqrt{2}}{2}$ for $0 < \beta < \frac{\pi}{2}$. **2**

(You do not need to express your answer with a rational denominator)

(c) Consider the trigonometric function: **3**

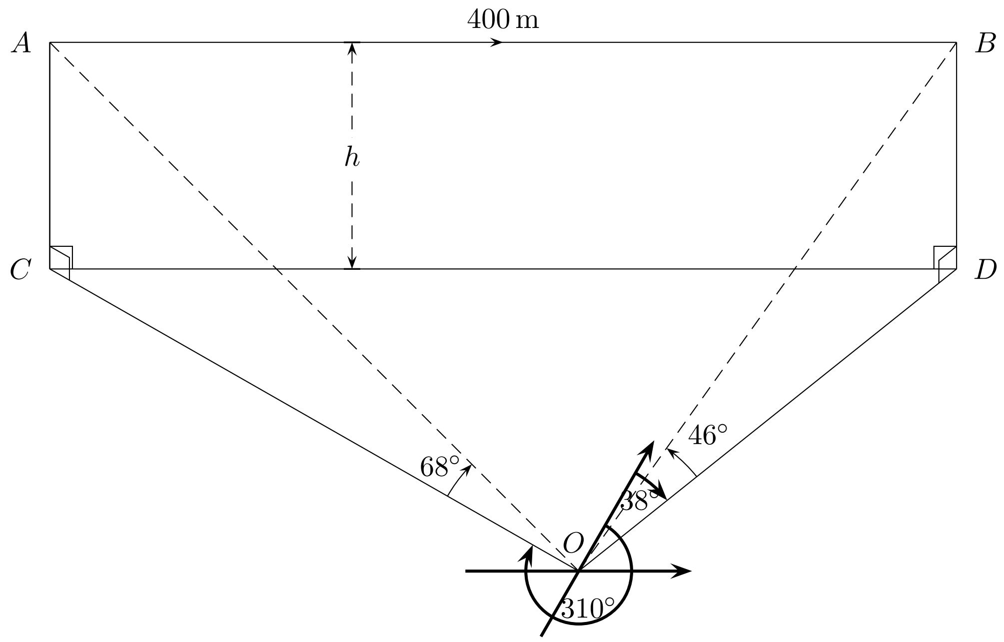
$$y = \sin(3x - \pi) + 1$$

State the necessary transformations, in the correct order, that are need to be applied to $y = \sin x$ to graph the above trigonometric function.

(d) Prove the following trigonometric identity: **2**

$$\frac{\sin 2x + \sin x}{1 + \cos 2x + \cos x} = \tan x$$

(e) From point O on the ground, Mannan observes a drone flying horizontally from point A at a bearing of 310°T and angle of elevation is 68°. After flying 400 m, the drone reaches point B and the bearing changes to 038°T with an angle of elevation of 46°.



i. Show that $OC = h\tan 22°$. **1**

ii. Show that the height h of the drone is given by **3**

$$h = \frac{400}{\sqrt{\tan^2 22° + \tan^2 44° - 2\tan 22°\tan 44°\cos 88°}}$$

iii. Hence calculate the height of the drone correct to 2 decimal places. **1**

Question 5 (10 Marks) Commence a NEW writing booklet **Marks**

(a) Differentiate with respect to x : $f(x) = \dfrac{3x^2 + 2}{x + 1}$ **2**

(b) Find the coordinates of the stationary point of the function below, and classify its nature with suitable working out. **3**

$$y = x^2 - \frac{2}{x}$$

(c) A water tank is being filled at a variable rate. The height of water h cm at any time t minutes, is given by

$$h(x) = 2t^2 - 4t + 50$$

i. Find the initial height of water in the tank. **1**

ii. Find the instantaneous rate of change of the height of water when $t = 4$ minutes. **2**

iii. Find the average rate at which the height has changed over the first 4 minutes. **2**

End of paper.

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NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\dfrac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

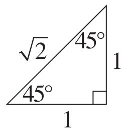
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

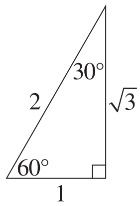
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

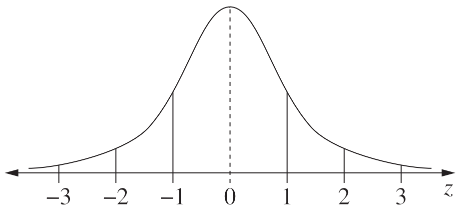
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\Big\{f(a) + f(b) + 2\big[f(x_1) + \cdots + f(x_{n-1})\big]\Big\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

Solutions to Year 11 Extension 1 Task 3

Question 1:

(a)

$$\frac{4+3\sqrt{5}}{2-\sqrt{6}} \times \frac{2+\sqrt{6}}{2+\sqrt{6}}$$

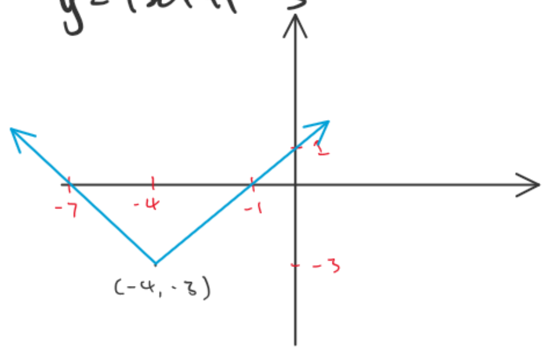
$$= \frac{(4+3\sqrt{5})(2+\sqrt{6})}{4-6}$$ ① Mark for rationalising correctly

$$= \frac{8+4\sqrt{6}+6\sqrt{5}+3\sqrt{30}}{-2}$$

$$= -\frac{8+4\sqrt{6}+6\sqrt{5}+3\sqrt{30}}{2}$$ ① Mark for simplest form

(b)

$y = |x+4| - 3$



① Mark for correct vertex

① Mark for correct graph.

→ Find the x-intercept by letting $y=0$

$0 = |x+4| - 3$

$|x+4| = 3$

$x+4 = 3$ $\qquad\qquad$ $x+4 = -3$

$x = -1$ $\qquad\qquad$ $x = -7$

→ Find the y-intercept by letting $x=0$

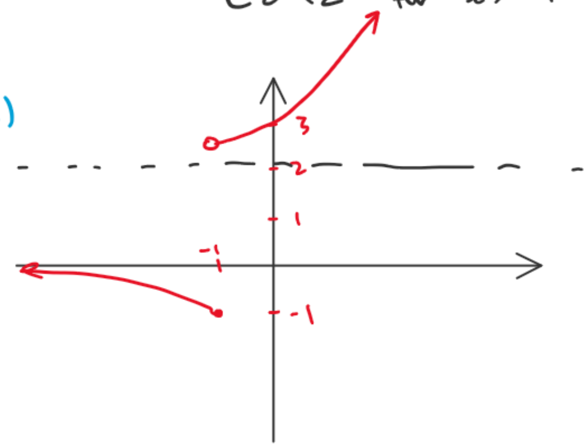
$y = |4| - 3$

$y = 1$

(c)

$$f(x) = \begin{cases} \frac{1}{x} & \text{for } x \le -1 \\ 2^x + 2 & \text{for } x > -1 \end{cases}$$

(i)



① Mark for indicating correct domain restrictions

① Mark for correct graph and labelling of intercepts

(ii) $R: y \in [-1, 0) \cup (2.5, \infty)$

Question 2.

(a)

$y = 11x^{-5}$

$\frac{dy}{dx} = -55x^{-6}$

① Mark for correct derivative

$= -\frac{55}{x^6}$

① Mark for answer without negative index.

(b)

$y = 6 + 8x - 3x^4$

$\frac{dy}{dx} = 8 - 12x^3$

① Mark for correct derivative.

at $x = 1$:

$m = 8 - 12(1)^3$

$m_1 = -4$

$m_2 = \frac{1}{4}$

① Mark for correct gradient

substitute $x = 1$ into

y for y-coordinate: $y = 11$

$\therefore\ y - y_1 = m(x - x_1)$

$y - 11 = \frac{1}{4}(x - 1)$

$y - 11 = \frac{1}{4}x - \frac{1}{4}$

$y = \frac{1}{4}x + \frac{43}{4}$

① Mark for correct equation

(c)

$$\lim_{x \to \infty} \frac{3 - x - 2x^2}{5x^2 - 7x + 4} \times \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x \to \infty} \frac{\frac{3}{x^2} - \frac{1}{x} - 2}{5 - \frac{7}{x} + \frac{4}{x^2}}$$

① Correct simplification

$$= \frac{0 - 0 - 2}{5 - 0 + 0}$$

$$= -\frac{2}{5}$$

① Correct answer.

(d)

$$f(x) = 6 - 9x^2$$

$$f(x+h) = 6 - 9(x+h)^2$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{6 - 9(x+h)^2 - (6 - 9x^2)}{h}$$ ① Mark for correct substitution

$$\lim_{h \to 0} \frac{6 - 9(x^2 + 2xh + h^2) - 6 + 9x^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{6} - \cancel{9x^2} - 18xh - 9h^2 - \cancel{6} + \cancel{9x^2}}{h}$$ ① Mark for correct simplifying

$$\lim_{h \to 0} \frac{-18xh - 9h^2}{h}$$

$$\lim_{h \to 0} \frac{-9\cancel{h}(2x + h)}{\cancel{h}}$$

$$= -18x$$ ① Mark for correct answer.

Question 3

(a)

(i) $f(x) = x - 3$

$g(x) = 2x^2 + 1$

$g(f(x)) = 2(x-3)^2 + 1$

$g(f(x)) = 2(x^2 - 6x + 9) + 1$

$g(f(x)) = 2x^2 - 12x + 18 + 1$

$g(f(x)) = 2x^2 - 12x + 19$

$\therefore\ h(x) = 2x^2 - 12x + 19$ ✓ (1) Mark for correct function.

(ii)

$x = -\frac{b}{2a}$

$x = \frac{12}{4}$

$x = 3$

→ "Largest positive domain"

$\therefore\ x \geq 3$ ✓ (1) Mark for correct domain

(iii)

$h(x) = 2(x-3)^2 + 1$

$x = 2(y-3)^2 + 1$ ✓ (1) Mark for correct change in "x" and "y"

$2(y-3)^2 = x - 1$

$(y-3)^2 = \frac{x-1}{2}$

$y - 3 = \sqrt{\frac{x-1}{2}}\ (y \geq 3)$

$$y = 3 + \sqrt{\frac{x-1}{2}}$$

$$\therefore h^{-1}(x) = \sqrt{\frac{x-1}{2}} + 3$$ ✓ ① Mark for correct inverse function

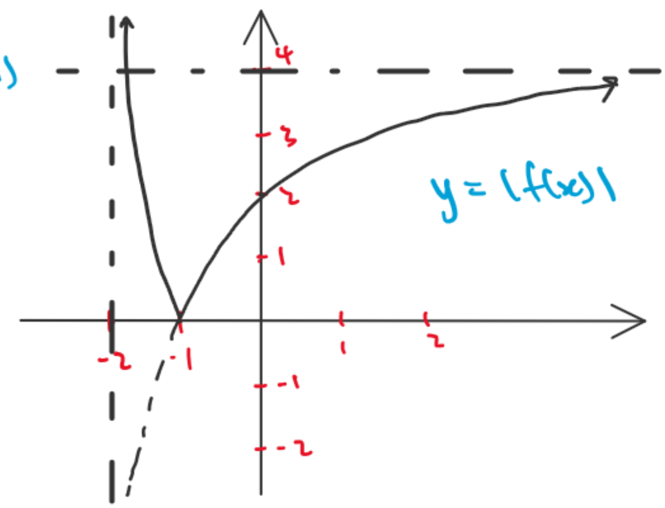
(b)

$$y = f(x)$$

$$f(x) = \frac{4x+4}{x+2}$$

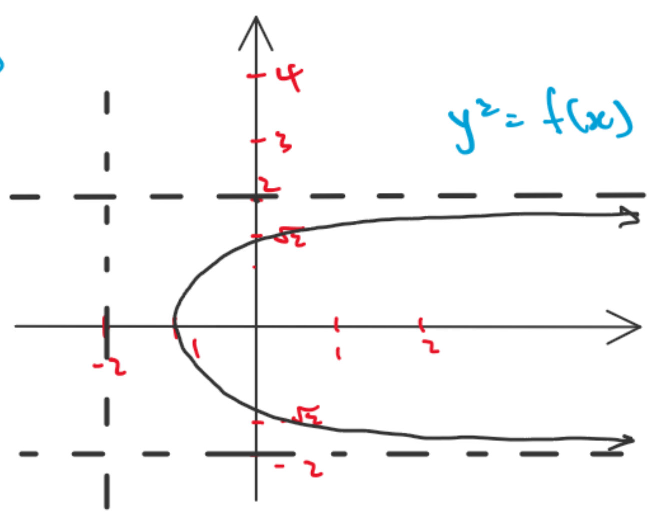
for $x > -2$

(i)



① Mark for correct graph

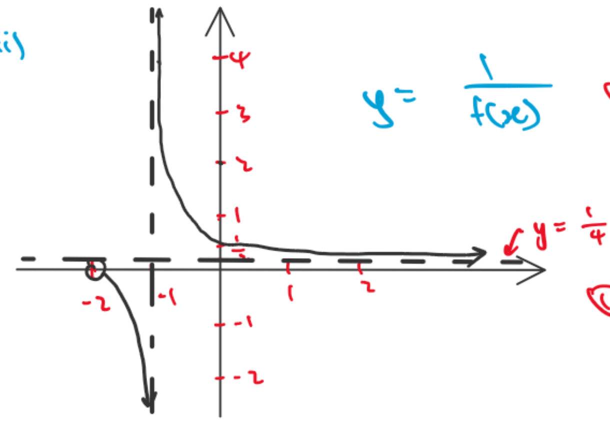
ii)



① Mark for correct intercepts

① Mark for correct graph

(iii)



$y = \frac{1}{f(x)}$

① Mark for correct intercepts

① Mark for correct graph.

Question 4.

(a)

$$9\cos^2 x + (2 - 3\sin x)^2$$
$$= 9\cos^2 x + (4 - 12\sin x + 9\sin^2 x)$$
$$= 9\cos^2 x + 9\sin^2 x + 4 - 12\sin x$$
$$= 9(\cos^2 x + \sin^2 x) + 4 - 12\sin x$$
$$= 13 - 12\sin x$$

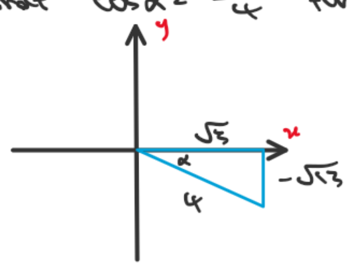
① Correct expansion

① Correct answer.

(b)

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

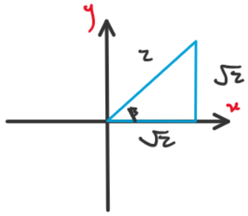
Given that $\cos\alpha = \frac{\sqrt{3}}{4}$ for $-\frac{\pi}{2} < \alpha < 0$



$\tan\alpha = -\frac{\sqrt{13}}{\sqrt{3}}$

① Mark for the correct $\tan\alpha$ and $\tan\beta$ solutions

Given that $\sin\beta = \frac{\sqrt{3}}{2}$ for $0 < \beta < \frac{\pi}{}$



$$\tan\beta = \frac{\sqrt{2}}{\sqrt{2}}$$
$$= 1$$

① Mark for the correct answer.

$$\therefore \tan(\alpha+\beta) = \frac{-\frac{\sqrt{13}}{\sqrt{3}}+1}{(1+\frac{\sqrt{13}}{\sqrt{3}}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{-\sqrt{13}+\sqrt{3}}{\sqrt{3}+\sqrt{13}}$$

(c)

$$y = \sin(3x - \pi) + 1$$

1. Factorise out 3:

$$y = \sin[3(x - \frac{\pi}{3})] + 1$$

$$\text{period} = \frac{2\pi}{n}$$

$$= \frac{2\pi}{3}$$ = horizontal stretch by a factor of $\frac{1}{3}$

① Mark for correct transformation terminology.

2. Consider the phase shift:

The function moves $\frac{\pi}{3}$ units to the right.

① Mark for correct phase shift

3. Consider the vertical upwards shift of 1.

$\therefore$ 1. Horizontal stretch by a factor of $\frac{\zeta}{3}$.

2. Phase shift $= \frac{\pi}{3}$

3. Vertical shift $= 1$ up. ① Mark for correct vertical shift.

(d)

$$\frac{\sin 2x + \sin x}{1 + \cos 2x + \cos x} = \tan x$$

★ $\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

L.H.S:

$$= \frac{2\sin x \cos x + \sin x}{1 + 2\cos^2 x - 1 + \cos x}$$

① Mark for changing $\sin 2x$ and $\cos 2x$

$$= \frac{\sin x(\cancel{2\cos x + 1})}{\cos x(\cancel{2\cos x + 1})}$$

$$= \frac{\sin x}{\cos x}$$

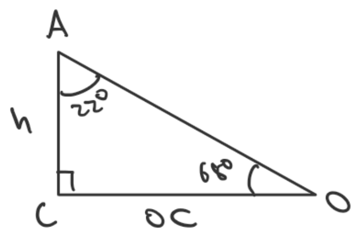
① Mark for correct simplification

$$= \tan x$$

$= R.H.S$

(e)

(i)



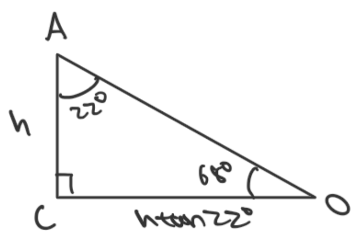
$\tan 22° = \dfrac{OC}{h}$

① Mark for correct trigonometric ratio.

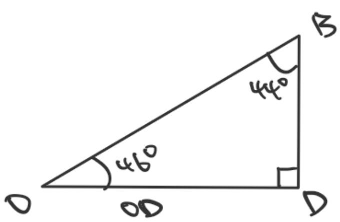
$OC = h\tan 22°$

(ii)

Draw three 2-D triangles:

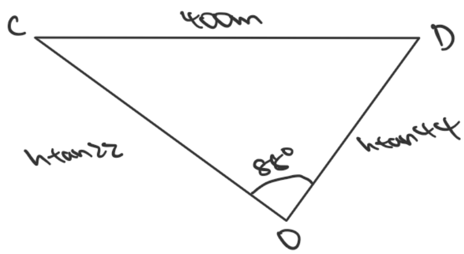


$OC = h\tan 22°$



$\tan 44 = \dfrac{OD}{h}$

$OD = h\tan 44°$ ✓ ① Mark for correct side of OD.



$c^2 = a^2 + b^2 - 2ab\cos C$

$400^2 = (h\tan 22)^2 + (h\tan 44)^2 - (2 \times h\tan 32 \times h\tan 44 \times \cos 88)$ ✓

① Mark for correct substitution

$400^2 = h^2\tan^2 22 + h^2\tan^2 44 - 2h\tan 22\tan 44\cos 88$

$400^2 = h^2(\tan^2 22 + \tan^2 44 - 2\tan 22\tan 44\cos 88)$

$h^2 = \dfrac{400^2}{\tan^2 22 + \tan^2 44 - 2\tan 22\tan 44\cos 88}$ ✓ ① Mark for correct simplification.

$h = \dfrac{400}{\sqrt{\tan^2 22 + \tan^2 44 - 2\tan 22\tan 44\cos 88}}$

(iii)

$h = 386.96\text{m}$. (2 d.p) ✓ ① Mark for correct answer.

Question 5.

(a)

$f(x) = \dfrac{3x^2+2}{x+1}$

$u = 3x^2+2 \qquad v = x+1$

$u' = 6x \qquad v' = 1$

$$f'(x) = \frac{6x^2+6x-(3x^2+2)}{(x+1)^2}$$ ✓ ① Mark for correct use of quotient rule

$$= \frac{6x^2+6x-3x^2-2}{(x+1)^2}$$

$$= \frac{3x^2+6x-2}{(x+1)^2}$$ ✓ ① Mark for correct simplification

(b)

$y = x^2 - 2x^{-1}$

$\frac{dy}{dx} = 2x + 2x^{-2}$

$= 2x + \frac{2}{x^2}$ ✓ ① correct first derivative

Let $\frac{dy}{dx} = 0$ to find stationary point.

$-2x = \frac{2}{x^2}$

$-2x^3 = 2$

$x^3 = -1$

$x = -1$ ← substitute into y for y-coordinate.

$y = 3$

Sub stationary point into second derivative to determine nature:

$$\frac{d^2y}{dx^2} = 2 - 4x^{-3}$$
$$= 2 - \frac{4}{x^3}$$

sub in $x = -1$

$$\frac{d^2y}{dx^2} = 2 - \frac{4}{(-1)^3}$$
$$= 2 + 4$$
$$= 6$$

$\therefore$ minimum ✓ (1) Mark for correct concavity

$\therefore$ $(-1, 3)$ is a minimum ✓ (1) Mark for correct coordinates

(c)

For $h(x) = 2t^2 - 4t + 50$

(i) Let $t = 0$

$h(0) = 50$ cm ✓ (1) Mark for correct answer

(ii) For rate of change, find the gradient function:

$h'(t) = 4t - 4$ ✓ (1) Mark for correct gradient function

$h'(4) = 16 - 4$

$= 12$ cm/min ✓ (1) Mark for correct rate with units

(iii) $h(0) = 50$

$h(4) = 66$ ✓ (1) Mark for $h(4)$

$$\therefore \quad \frac{66-50}{4-0}$$

$$= \frac{16}{4}$$

$\therefore$ 4cm/min for ✓ ① Mark for correct
average rate of change. answer with substitution.