NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2023 Year 11 Course Assessment Task 2

Wednesday, 21st June 2023

General instructions

- Working time – 60 minutes.
(plus 5 minutes reading time)
- Answer each question in a separate writing booklet. Extra writing booklets are available.
- Write using a blue or a black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.
- A NESA Reference Sheet is provided.

Class (please ✔)

- ◯ 11MAX.1 – Ms Ham
- ◯ 11MAX.2 – Miss Chai
- ◯ 11MAX.3 – Miss J. Kim / Mr Lam
- ◯ 11MAX.4 – Mr Ho
- ◯ 11MAX.5 – Mr Sekaran
- ◯ 11MAX.6 – Miss Lee

NESA STUDENT # ....

Marker's use only.

QUESTION	1	2	3	4	5	6	**Total**	%
MARKS	$\overline{7}$	$\overline{6}$	$\overline{5}$	$\overline{16}$	$\overline{9}$	$\overline{11}$	$\overline{54}$	

Question 1 (7 Marks) Commence a NEW writing booklet **Marks**

(a) Find the exact value of $\tan 600°$. **2**

(b) Solve for θ, if $\sin\theta = -\dfrac{\sqrt{3}}{2}$ and $0° \leq \theta \leq 360°$. **2**

(c) Solve for x, given $2\cos^2 x - \cos x = 0$ and $0° \leq x \leq 180°$. **3**

Question 2 (6 Marks) Commence a NEW writing booklet **Marks**

Let $f(x) = \sqrt{5-2x}$.

(a) Find the domain and range of $f(x)$. **2**

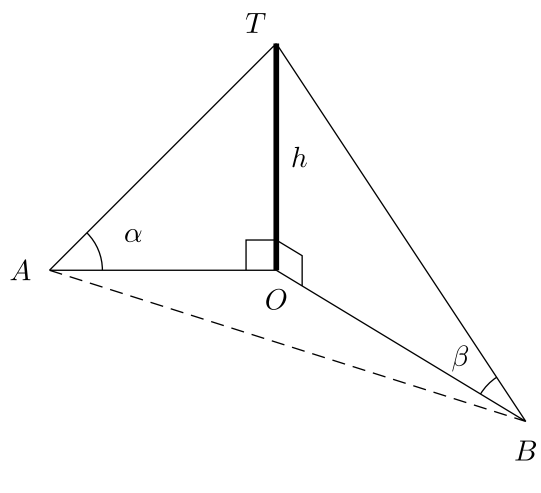
(b) Find the equation of the inverse function, $f^{-1}(x)$, and state its domain. **3**

(c) Briefly explain why $f\left(f^{-1}(x)\right) \neq x$ where $x < 0$. **1**

Question 3 (5 Marks) Commence a NEW writing booklet **Marks**

A vertical flagpole OT of height h metres stands with its base O on horizontal ground. Additionally, A and B are points on the ground.

From A and B, the angles of elevation to the top T of the flagpole are α and β respectively, where $\tan\alpha = \dfrac{5}{3}$ and $\tan\beta = \dfrac{5}{4}$.



(a) Find OA and OB in terms of h. **2**

(b) Given that $\angle AOB = 120°$, find AB in terms of h as an exact value. **3**

Examination continues overleaf...

Question 4 (16 Marks) Commence a NEW writing booklet **Marks**

(a) Use differentiation by first principles to find $f'(x)$ given $f(x) = -2x^2$. **2**

(b) Differentiate the following with respect to x. Leave your answers in the simplest form.

i. $x(2x+1)^4$ **2**

ii. $\dfrac{2x^2-1}{x+1}$ **2**

(c) Consider the curve

$$f(x) = 2x^3 - 7x^2 + 4x + 6$$

i. Find the coordinates of the turning points and determine their nature. **5**

ii. Find the coordinates of any point(s) of inflection. **2**

iii. Hence sketch the graph of $y = f(x)$, showing the stationary points, the point(s) of inflection and the y-intercept. **3**
(Do NOT find the x-intercept.)

Question 5 (9 Marks) Commence a NEW writing booklet **Marks**

(a) Consider the curve defined by

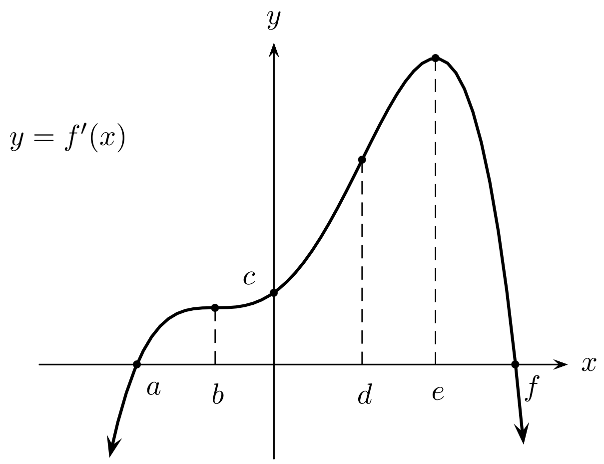
$$x = t^2 + 2t \quad \text{and} \quad y = t^{-3} - 1$$

where $t > 0$.

i. Find the expression for $\dfrac{dy}{dx}$, in terms of t. **2**

ii. Hence, find the equation of the tangent to the curve where $t = 1$. **3**

(b) The diagram below shows a sketch of $y = f'(x)$ with x-intercepts at $x = a$ and $x = f$, y-intercept at $y = c$, stationary points at $x = b$ and $x = e$, and a point of inflection at $x = d$.



i. State the value(s) of x for which the original function $y = f(x)$ has a stationary point. **2**

ii. Sketch the graph of $y = f''(x)$, showing all important features. **2**

Examination continues overleaf...

Question 6 (11 Marks) Commence a NEW writing booklet **Marks**

(a) Fully simplify $3\log_a \sqrt{a^5} + \log_a \dfrac{1}{a^2}$. **2**

(b) Given that $7^{-\frac{x}{2}} = \dfrac{1}{3}$ and $7^{\frac{y}{4}} = 2$, evaluate $\dfrac{1}{7^{y-x}}$. **3**

(c) i. Prove that $\log_{\frac{1}{x}} \dfrac{1}{y} = \log_x y$ where x and y are both positive numbers and $x \neq 1$. **2**

ii. Hence solve the equation $4\log_x 3 + \log_3 27 = \log_{\frac{1}{3}} \dfrac{1}{x}$. **4**

End of paper.

Question 1 (7 marks) (LEE)

(a) (2 marks)

✓ [1] for correct sign
✓ [1] for exact value

$$\begin{aligned} \tan 600^\circ &= \tan 240^\circ \\ &= \tan 60^\circ \\ &= \sqrt{3} \qquad \checkmark\checkmark \end{aligned}$$

(b) (2 marks)

✓ [1] for related reference angle
✓ [1] for final answers

$$\begin{aligned} \sin\theta &= \frac{-\sqrt{3}}{2} \\ \text{ref } \angle &= 60^\circ \qquad \checkmark \\ x &= 240^\circ, 300^\circ \qquad \checkmark \end{aligned}$$

(c) (3 marks)

✓ [1] for factorising and/or obtaining two trigonometric equations
✓ [1] for solution to $\cos x = 0$
✓ [1] for solution to $\cos x = \frac{1}{2}$

$$\begin{aligned} \cos x(2\cos x - 1) &= 0 \qquad \checkmark \\ \cos x = 0 \qquad &\text{or} \cos x = \frac{1}{2} \\ \therefore x = 60^\circ, 90^\circ \qquad &\checkmark\checkmark \end{aligned}$$

Question 2 (6 marks) (SEKARAN)

(a) (2 mark)

✓ [1] for domain
✓ [1] for range

$$\text{Domain : all real } x \le \frac{5}{2} \quad \text{or} \quad \left[-\infty, \frac{5}{2}\right) \qquad \checkmark$$

$$\text{Range : all real } y \ge 0 \quad \text{or} \quad [0, \infty) \qquad \checkmark$$

(b) (3 marks)

- ✓ [1] for procedure of swapping x and y
- ✓ [1] for $f^{-1}(x)$
- ✓ [1] for domain of $f^{-1}(x)$

$$\begin{aligned}
\text{Let } y &= \sqrt{5-2x}, \text{ where } x \leq \frac{5}{2} \\
\text{Then inverse has equation } x &= \sqrt{5-2y}, \text{ where } y \leq \frac{5}{2} \qquad \checkmark \\
y &= \frac{5-x^2}{2} \\
\therefore \quad f^{-1}(x) &= \frac{5-x^2}{2} \qquad \checkmark \\
\text{Domain : all real } x \geq 0 \quad &\text{or} \quad [0,\infty) \qquad \checkmark
\end{aligned}$$

(c) (1 mark)

- ✓ [1] for appropriate reasoning

$$\text{Since the domain of } f^{-1}(x) \text{ is all real } x \geq 0, \text{ when } x < 0, f\left(f^{-1}(x)\right) \neq x \qquad \checkmark$$

Question 3 (5 marks) (HO)

(a) (2 marks)

- ✓ [1] for OA in terms of h
- ✓ [1] for OB in terms of h

$$\begin{aligned}
\tan\alpha &= \frac{h}{OA} \\
\frac{5}{3} &= \frac{h}{OA} \\
\therefore \quad OA &= \frac{3h}{5} \qquad \checkmark
\end{aligned}$$

$$\begin{aligned}
\tan\beta &= \frac{h}{OB} \\
\frac{5}{4} &= \frac{h}{OB} \\
\therefore \quad OB &= \frac{4h}{5} \qquad \checkmark
\end{aligned}$$

(b) (3 marks)

✓ [1] for applying cosine rule
✓ [1] for simplification of trigonometric functions
✓ [1] for final answer

Using Cosine rule,

$$\begin{aligned} AB^2 &= OA^2 + OB^2 - 2(OA)(OB)\cos 120^\circ \quad \checkmark \\ AB^2 &= \left(\frac{3h}{5}\right)^2 + \left(\frac{4h}{5}\right)^2 - 2\left(\frac{3h}{5}\right)\left(\frac{4h}{5}\right)\cos 120^\circ \\ AB^2 &= \frac{9h^2}{25} + \frac{16h^2}{25} + \frac{12h^2}{25} \quad \checkmark \\ AB^2 &= \frac{37h^2}{25} \\ \therefore AB &= \frac{\sqrt{37}h}{5} \quad \checkmark \end{aligned}$$

Question 4 (16 marks) (J. KIM and LAM)

(a) (2 mark) (J. KIM)

✓ [1] for applying first principles and/or difference quotient
✓ [1] for final answer

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{-2(x+h)^2 - (-2x^2)}{h} \\ &= \lim_{h\to 0} \frac{-2x^2 - 4xh - 2h^2 + 2x^2}{h} \quad \checkmark \\ &= \lim_{h\to 0} -4x - 2h \\ &= -4x \quad \checkmark \end{aligned}$$

(b) i. (2 marks) (J. KIM)

✓ [1] for applying product rule
✓ [1] for final answer

$$\begin{aligned} \frac{d}{dx}\left(x(2x+1)^4\right) &= (2x+1)^4(1) + x\left(4(2x+1)^3 \times 2\right) \quad \checkmark \\ &= (2x+1)^3(10x+1) \quad \checkmark \end{aligned}$$

ii. (2 marks) (J. KIM)

✓ [1] for applying quotient rule

✓ [1] for final answer

$$\begin{aligned}\frac{d}{dx}\left(\frac{2x^2-1}{x+1}\right) &= \frac{(x+1)(4x)-(2x^2-1)(1)}{(x+1)^2} \qquad \checkmark\\ &= \frac{2x^2+4x+1}{(x+1)^2} \qquad \checkmark\end{aligned}$$

(c) i. (5 marks) (LAM)

✓ [1] for the function of $f'(x)$

✓ [2] for coordinates of stationary points

✓ [2] for nature of stationary points

$$\begin{aligned}f'(x) &= 6x^2-14x+4 \qquad \checkmark\\ f''(x) &= 12x-14\end{aligned}$$

Stationary points occur when $f'(x)=0$:

$$\begin{aligned}(3x-1)(x-2) &= 0\\ \therefore \quad x &= \frac{1}{3} \quad \text{or} \quad x=2\end{aligned}$$

$$\begin{aligned}f\left(\frac{1}{3}\right) &= \frac{179}{27}\\ f''\left(\frac{1}{3}\right) &= -10 < 0\end{aligned}$$

$\therefore \quad \left(\frac{1}{3}, \frac{179}{27}\right)$ is a local maximum turning point. ✓✓

$$\begin{aligned}f(2) &= 2\\ f''(2) &= 10 > 0\end{aligned}$$

$\therefore \quad (2,2)$ is a local minimum turning point. ✓✓

ii. (2 marks) (LAM)

✓ [1] for x-coordinate of point of inflection

✓ [1] for final answer with confirmation of change in concavity

Points of inflection occur when $f''(x)=0$:

$$\begin{aligned}12x-14 &= 0\\ \therefore \quad x &= \frac{7}{6}\\ f\left(\frac{7}{6}\right) &= \frac{233}{54}\end{aligned}$$

x	1	$\frac{7}{6}$	2
$f''(x)$	-2	0	10

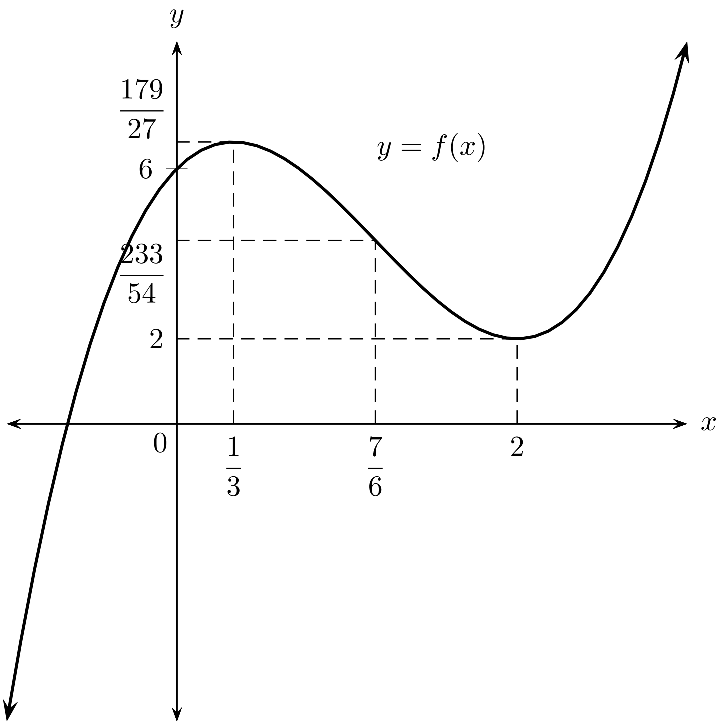
There is a change in concavity. ✓

$$\therefore \left(\frac{7}{6}, \frac{233}{54}\right) \text{ is a point of inflection.} \quad ✓$$

iii. (3 marks) (LAM)

- ✓ [1] for shape and y-intercept
- ✓ [1] for stationary points
- ✓ [1] for point of inflection

$$f(0) = 6$$
$$\therefore \; y - \text{intercept at } (0, 6)$$



Question 5 (9 marks) (HAM)

(a) i. (2 marks)

✓ [1] for $\frac{dy}{dt}$ and $\frac{dx}{dt}$

✓ [1] for final answer

$$\begin{aligned} \frac{dx}{dt} &= 2t+2 \\ \therefore \quad \frac{dt}{dx} &= \frac{1}{2t+2} \\ \frac{dy}{dt} &= -3t^{-4} \\ &= \frac{-3}{t^4} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{-3}{t^4} \times \frac{1}{2t+2} \quad \checkmark \\ \therefore \frac{dy}{dx} &= \frac{-3}{t^4(2t+2)} \quad \checkmark \end{aligned}$$

ii. (3 marks)

✓ [1] for x and y-coordinates when $t = 1$

✓ [1] for gradient of tangent

✓ [1] for final answer

When $t = 1$,

$$\begin{aligned} x &= 3 \\ y &= 0 \quad \checkmark \\ \frac{dy}{dx} &= \frac{-3}{4} \quad \checkmark \end{aligned}$$

Equation of tangent:

$$\begin{aligned} y - 0 &= -\frac{3}{4}(x-3) \\ \therefore \quad y &= -\frac{3}{4}x + \frac{9}{4} \quad \checkmark \end{aligned}$$

(b) i. (2 marks)

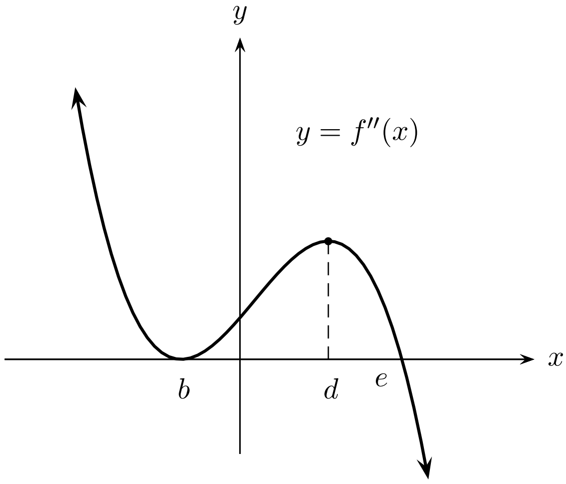
✓ [2] for final answers

At stationary point, $f'(x)$ crosses the x-axis. $\therefore x = a$ and $x = f$ ✓✓

ii. (2 marks)

✓ [1] for correct feature at $x = b$ or between $x = d$ and $x = e$

✓ [1] for final answer



Question 6 (11 marks) (CHAI)

(a) (2 marks)

✓ [1] for applying logarithmic laws

✓ [1] for final answer

$$\begin{aligned} 3\log_a \sqrt{a^5} + \log_a \frac{1}{a^2} &= 3\log_a a^{2.5} + \log_a a^{-2} \quad \checkmark \\ &= 3(2.5) - 2 \\ &= 5.5 \quad \checkmark \end{aligned}$$

(b) (3 marks)

✓ [1] for evaluating 7^x and 7^y

✓ [1] for applying index laws to $\dfrac{1}{7^{y-x}}$

✓ [1] for final answer

$$\begin{aligned} 7^{-\frac{x}{2}} &= \frac{1}{3} \implies 7^x = 9 \\ 7^{\frac{y}{4}} &= 2 \implies 7^y = 16 \quad \checkmark \\ \frac{1}{7^{y-x}} &= 7^{x-y} \\ &= \frac{7^x}{7^y} \quad \checkmark \\ \therefore \quad \frac{1}{7^{y-x}} &= \frac{9}{16} \quad \checkmark \end{aligned}$$

(c) i. (2 mark)

- ✓ [1] for applying logarithmic laws
- ✓ [1] for final answer

$$\begin{aligned}
\text{LHS} &= \log_{\frac{1}{x}} \frac{1}{y} \\
&= \frac{\log_x \frac{1}{y}}{\log_x \frac{1}{x}} \qquad \checkmark \\
&= \frac{-\log_x y}{-1} \qquad \checkmark \\
&= \log_x y \\
&= \text{RHS}
\end{aligned}$$

ii. (4 marks)

- ✓ [1] for applying part (i) results and other logarithm laws
- ✓ [1] for making $\log_3 x$ the subject of a quadratic equation
- ✓ [2] for final answers

$$\begin{aligned}
4\log_x 3 + \log_3 27 &= \log_{\frac{1}{3}} \frac{1}{x} \\
\frac{4}{\log_3 x} + 3 &= \log_3 x \qquad \checkmark \\
4 + 3\log_3 x - (\log_3 x)^2 &= 0 \qquad \checkmark \\
(4 - \log_3 x)(1 + \log_3 x) &= 0 \\
\log_3 x = 4 \qquad &\text{OR} \qquad \log_3 x = -1 \\
x = 81 \qquad &\text{OR} \qquad x = \frac{1}{3} \qquad \checkmark\checkmark
\end{aligned}$$