NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2024 Year 11 Course Assessment Task 2

Friday, 28th June 2024

General instructions

- Working time – 60 minutes. (plus 5 minutes reading time)
- Answer each question in a separate writing booklet. Extra writing booklets are available.
- Write using a blue or a black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.
- A NESA Reference Sheet is provided.

Class (please ✔)

- ◯ 11MAX.1 – Miss J. Kim
- ◯ 11MAX.2 – Miss Chaussivert
- ◯ 11MAX.3 – Miss Lee
- ◯ 11MAX.4 – Mr Tran
- ◯ 11MAX.5 – Ms Ham
- ◯ 11MAX.6 – Mr Ho

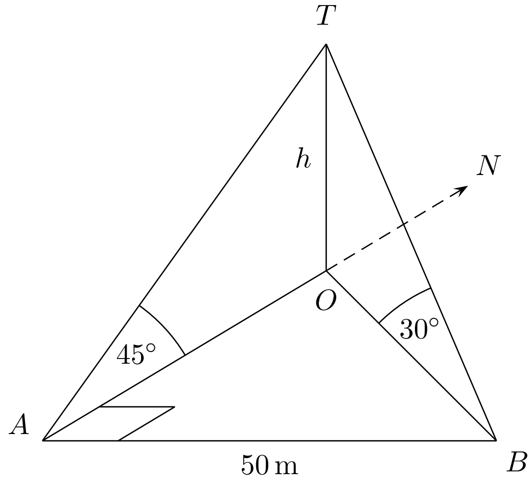
NESA STUDENT # ....

Marker's use only.

QUESTION	1	2	3	4	5	6	7	**Total**	%
MARKS	$\overline{6}$	$\overline{9}$	$\overline{3}$	$\overline{8}$	$\overline{6}$	$\overline{3}$	$\overline{12}$	$\overline{47}$	

Question 1 (6 Marks) Commence a NEW writing booklet **Marks**

A person is standing at point A, which is due south of a tower OT that is h metres tall. The person walks 50 metres east to point B. The angle of elevation at points A and B to the top of the tower are $45°$ and $30°$ respectively.



(a) Express OA and OB in terms of h. **2**

(b) Show that $h = 25\sqrt{2}$ metres. **2**

(c) Find the bearing of B from the base of the tower O, correct to the nearest minute. **2**

Question 2 (9 Marks) Commence a NEW writing booklet **Marks**

(a) Differentiate the following with respect to x:

i. $x^2\sqrt{x}$ **2**

ii. $\left(2x^2+1\right)^6$ **2**

iii. $\dfrac{1+7x}{x^3}$ **2**

(b) If $f(x) = \dfrac{1}{x-1}$, show that $f'(x) = -\dfrac{1}{(x-1)^2}$ by differentiation from first principles. **3**

Question 3 (3 Marks) Commence a NEW writing booklet **Marks**

Solve for x: **3**

$$2\log_3(x) = \log_3(6-x)$$

Question 4 (8 Marks) Commence a NEW writing booklet **Marks**

(a) For $f(x) = x^2 - 6x + 11$

i. Find the range of $f(x)$. **1**

ii. State the largest domain of $f(x)$ containing $x = 4$ such that it is monotonic increasing. **1**

iii. Find an expression for the inverse function, $f^{-1}(x)$ by using the restricted domain from part (ii). **3**

(b) Find the Cartesian equation of the curve defined by the parametric equations: **3**

$$\begin{cases} x = 3\sin\theta - 2 \\ y = 4\cos\theta + 1 \end{cases}$$

Question 5 (6 Marks) Commence a NEW writing booklet **Marks**

Consider the curve

$$f(x) = -3x^5 + 5x^3 + 1$$

(a) Find the coordinates of the stationary points and determine their nature. **4**

(b) Hence sketch the graph of $y = f(x)$, showing the stationary points, and the y-intercept. (Do NOT find the x-intercept(s).) **2**

Examination continues overleaf...

Question 6 (3 Marks) Commence a NEW writing booklet **Marks**

Prove $(1+\sin\theta+\cos\theta)^2 = 2\,(1+\sin\theta)\,(1+\cos\theta)$. **3**

Question 7 (12 Marks) Commence a NEW writing booklet **Marks**

(a) Given the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{b} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{c} = \underset{\sim}{a} - \underset{\sim}{b}$.

i. Find the value of $\left|\underset{\sim}{c}\right|$. **1**

ii. Hence, find a vector in the same direction as $\underset{\sim}{c}$ that has a magnitude of 3 units. **2**

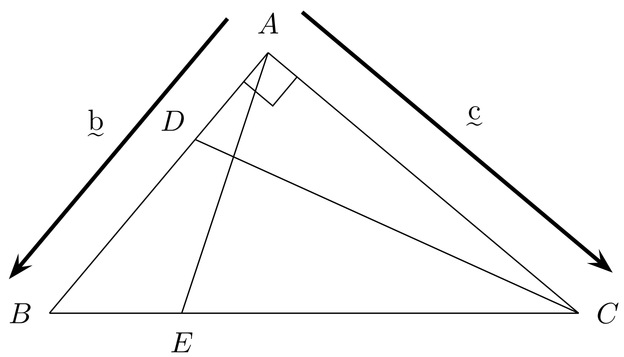
(b) The projection of $\underset{\sim}{u} = \begin{pmatrix} 3 \\ \lambda \end{pmatrix}$ onto $\underset{\sim}{v} = \begin{pmatrix} 8 \\ -15 \end{pmatrix}$ has length $\dfrac{159}{17}$.

i. Show that $|24 - 15\lambda| = 159$. **2**

ii. It is given that $A(3,\lambda)$ and $B(8,-15)$. Hence, find the shortest distance from $A(3,\lambda)$ to the line OB where $\lambda < 0$. **3**

(c) In the right-angled triangle ABC, point D divides the interval AB so that $AD : DB = 1 : 2$, and point E divides the interval BC so that $BE : EC = 1 : 3$. **4**

Let $\underset{\sim}{b} = \overrightarrow{AB}$ and $\underset{\sim}{c} = \overrightarrow{AC}$.



Show that $\overrightarrow{AE}\cdot\overrightarrow{CD} = \dfrac{1}{4}\left(\left|\overrightarrow{AB}\right|^2 - \left|\overrightarrow{AC}\right|^2\right)$.

End of paper.

Question 1 (6 marks) (LEE)

(a) (2 marks)

✓ [1] for finding OA
✓ [1] for finding OB

$$\begin{aligned}
\tan 45^\circ &= \frac{h}{OA}\\
OA &= \frac{h}{\tan 45^\circ}\\
&= h \qquad \checkmark\\
\tan 30^\circ &= \frac{h}{OB}\\
\frac{1}{\sqrt{3}} &= \frac{h}{OB}\\
OB &= \sqrt{3}h \qquad \checkmark
\end{aligned}$$

(b) (2 marks)

✓ [1] for applying Pythagoras' Theorem
✓ [1] for final answer

$$\begin{aligned}
OA^2 + 50^2 &= OB^2\\
h^2 + 2500 &= \left(\sqrt{3}h\right)^2 \qquad \checkmark\\
2h^2 &= 2500\\
h &= \pm\sqrt{1250}\\
&= 25\sqrt{2} \ \ (h > 0) \qquad \checkmark
\end{aligned}$$

(c) (2 marks)

✓ [1] for finding $\angle AOB$
✓ [1] for finding the bearing, correct to the nearest minute

$$\begin{aligned}
\tan\angle AOB &= \frac{50}{OA}\\
&= \frac{50}{25\sqrt{2}}\\
\angle AOB &= \tan^{-1}\left(\frac{2}{\sqrt{2}}\right) \qquad \checkmark\\
\text{Bearing of B} &= 180^\circ - \angle AOB\\
&= 180^\circ - \tan^{-1}\left(\frac{2}{\sqrt{2}}\right)\\
&= 125^\circ 16' \qquad \checkmark
\end{aligned}$$

Question 2 (9 marks) (Ham)

(a) i. (2 marks)

✓ [1] for applying the appropriate differentiation rule

✓ [1] for final answer

$$\begin{aligned}\frac{d}{dx}\left(x^2\sqrt{x}\right) &= \frac{d}{dx}\left(x^{\frac{5}{2}}\right)\\ &= \frac{5}{2}x^{\frac{3}{2}} \qquad \checkmark\\ &= \frac{5}{2}x\sqrt{x} \qquad \checkmark\end{aligned}$$

ii. (2 marks)

✓ [1] for applying the chain rule

✓ [1] for final answer

$$\begin{aligned}\frac{d}{dx}\left((2x^2+1)^6\right) &= 6(2x^2+1)^5(4x) \qquad \checkmark\\ &= 24x(2x^2+1)^5 \qquad \checkmark\end{aligned}$$

iii. (2 marks)

✓ [1] for applying appropriate differentiation rule

✓ [1] for final answer

$$\begin{aligned}\frac{d}{dx}\left(\frac{1+7x}{x^3}\right) &= \frac{d}{dx}\left(x^{-3}+7x^{-2}\right)\\ &= -3x^{-4}+7(-2)x^{-3} \qquad \checkmark\\ &= -\frac{3}{x^4}-\frac{14}{x^3} \qquad \checkmark\end{aligned}$$

(b) (3 marks)

✓ [1] for applying differentiation by first principles

✓ [1] for obtaining $\dfrac{-1}{(x-1)(x+h-1)}$

✓ [1] for final answer

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\dfrac{1}{x+h-1} - \dfrac{1}{x-1}}{h} \quad \checkmark \\
&= \lim_{h\to 0} \frac{\dfrac{(x-1)-(x+h-1)}{(x-1)(x+h-1)}}{h} \\
&= \lim_{h\to 0} \frac{-h}{h(x-1)(x+h-1)} \\
&= \lim_{h\to 0} -\frac{1}{(x-1)(x+h-1)} \quad \checkmark \\
&= -\frac{1}{(x-1)^2} \quad \checkmark
\end{aligned}$$

Question 3 (3 marks) (Lam)

✓[1] for applying logarithm laws to obtain quadratic equation
✓[1] for factorising the quadratic equation
✓[1] for final answer

$$\begin{aligned}
2\log_3(x) &= \log_3(6-x) \\
\log_3(x^2) &= \log_3(6-x) \\
x^2 &= 6-x \quad \checkmark \\
x^2+x-6 &= 0 \\
(x+3)(x-2) &= 0 \quad \checkmark \\
x &= -3 \text{ or } 2 \\
\text{Since } 0<x<6:\ x &= 2 \quad \checkmark
\end{aligned}$$

Question 4 (8 marks) (Chaussivert)

(a) i. (1 mark)
✓ [1] for final answer

$$f(x) \geq 2 \text{ or } f(x) \in [2,\infty) \quad \checkmark$$

ii. (1 mark)
✓ [1] for final answer

$$x \geq 3 \text{ or } x \in [3,\infty) \quad \checkmark$$

iii. (3 marks)

✓ [1] for using difference of 2 squares
✓ [1] for switching the positions of x and y
✓ [1] for final answer

$$\begin{aligned} y &= x^2 - 6x + 11 \\ &= (x-3)^2 + 2 && \checkmark \\ x &= (y-3)^2 + 2 && \checkmark \\ y - 3 &= \pm\sqrt{x-2} \\ y &= 3 \pm \sqrt{x-2} \\ \text{Since } D_{f(x)} &: x \geq 3 \\ \text{Then, } R_{f^{-1}(x)} &: y \geq 3 \\ y &= 3 + \sqrt{x-2} && \checkmark \end{aligned}$$

(b) (3 mark)

✓ [1] for making $\sin\theta$ the subject
✓ [1] for making $\cos\theta$ the subject
✓ [1] for final answer

$$\begin{aligned} \sin\theta &= \frac{x+2}{3} && \checkmark \\ \cos\theta &= \frac{y-1}{4} && \checkmark \\ \text{Since } \sin^2\theta + \cos^2\theta &= 1 \\ \left(\frac{x+2}{3}\right)^2 + \left(\frac{y-1}{4}\right)^2 &= 1 && \checkmark \end{aligned}$$

Question 5 (6 marks) (J.Kim)

(a) (4 marks)

✓ [1] for correct derivative
✓ [1] for finding stationary points
✓ [1] for nature of stationary points
✓ [1] for coordinates with correct nature

$$\begin{aligned} f'(x) &= -15x^4 + 15x^2 \quad \checkmark \\ &= 15x^2(-x^2+1) \\ &= 15x^2(1-x)(1+x) \end{aligned}$$

$$\text{Stationary Points occur when } f'(x) = 0:$$

$$\begin{aligned} x = 0,\ & f'(0) = 1 \Rightarrow (0,1) \\ x = 1,\ & f'(1) = 3 \Rightarrow (1,3) \\ x = -1,\ & f'(-1) = -1 \Rightarrow (-1,-1) \quad \checkmark \end{aligned}$$

x	-2	-1	$\frac{-1}{2}$	0	$\frac{1}{2}$	1	2
$f'(x)$	-180	0	2.81	0	2.81	0	-180
$f(x)$	↘	–	↗	–	↗	–	↘

✓

$$\begin{aligned} \therefore (0,1) &\quad \text{is a HPOI} \\ (1,3) &\quad \text{is a local max} \\ (-1,-1) &\quad \text{is a local min} \quad \checkmark \end{aligned}$$

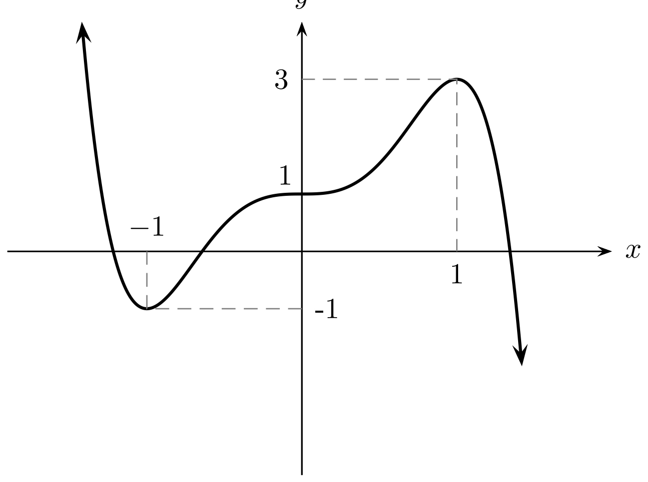
(b) (2 marks)

✓ [1] for shape and y-intercept
✓ [1] for stationary points

$$f(0) = 1$$

$$\therefore\ y-\text{intercept at } (0,1)$$

$$f(x) = -3x^5 + 5x^3 + 1$$



✓✓

Question 6 (3 marks) (J.Kim)

✓[1] for correct expansion
✓[1] for correct substitution
✓[1] for correct factorisation to get RHS

$$\begin{aligned}
\text{LHS} &= (1+\sin\theta+\cos\theta)^2 \\
&= 1+(\sin\theta+\cos\theta)^2+2(\sin\theta+\cos\theta) \qquad \checkmark \\
&= 1+(\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta)+2\sin\theta+2\cos\theta \\
&= 2+2\sin\theta\cos\theta+2\sin\theta+2\cos\theta \qquad \checkmark \\
&= 2\left(1+\sin\theta\cos\theta+\sin\theta+\cos\theta\right) \\
&= 2\left[1+\sin\theta+\cos\theta\left(1+\sin\theta\right)\right] \qquad \checkmark \\
&= 2(1+\sin\theta)(1+\cos\theta) \\
&= \text{RHS}
\end{aligned}$$

Question 7 (12 marks) (Ho and Tran)

(a) i. (1 mark) (Ho)

✓ [1] for correct answer

$$\begin{aligned}
\left|\underset{\sim}{c}\right| &= \left|-2\underset{\sim}{i}+3\underset{\sim}{j}\right| \\
&= \sqrt{(-2)^2+3^2} \\
&= \sqrt{13} \qquad \checkmark
\end{aligned}$$

ii. (2 marks) (Ho)

✓ [1] for finding a unit vector in the same direction as $\underset{\sim}{c}$
✓ [1] for final answer

$$\begin{aligned}
\hat{\underset{\sim}{c}} &= \frac{1}{\sqrt{13}}\left(-2\underset{\sim}{i}+3\underset{\sim}{j}\right) \qquad \checkmark \\
3\hat{\underset{\sim}{c}} &= \frac{3}{\sqrt{13}}\left(-2\underset{\sim}{i}+3\underset{\sim}{j}\right) \qquad \checkmark
\end{aligned}$$

(b) i. (2 marks) (Ho)

✓ [1] for finding the $\text{proj}_{\underset{\sim}{v}}\, \underset{\sim}{u}$

✓ [1] for final answer

$$\begin{aligned}
\text{proj}_{\underset{\sim}{v}}\, \underset{\sim}{u} &= \frac{\begin{pmatrix}3\\ \lambda\end{pmatrix}\cdot\begin{pmatrix}8\\ -15\end{pmatrix}}{\left(\sqrt{8^2+(-15)^2}\right)^2}\begin{pmatrix}8\\ -15\end{pmatrix}\\
&= \frac{24-15\lambda}{289}\begin{pmatrix}8\\ -15\end{pmatrix} \qquad ✓\\
\left|\text{proj}_{\underset{\sim}{v}}\, \underset{\sim}{u}\right| &= \frac{|24-15\lambda|}{289}\left|\begin{pmatrix}8\\ -15\end{pmatrix}\right|\\
\frac{|24-15\lambda|}{289}\sqrt{289} &= \frac{159}{17}\\
\frac{|24-15\lambda|}{17} &= \frac{159}{17}\\
|24-15\lambda| &= 159 \qquad ✓
\end{aligned}$$

ii. (3 marks) (Ho)

✓ [1] for finding the value of λ

✓ [1] for a correct equation (or vector equation) to find

✓ [1] for final answer

$$\begin{aligned}
\text{From part (i)}:\quad 24-15\lambda &= \pm 159\\
\lambda &= \frac{24\pm 159}{15}\\
&= -9 \ \text{ or } \ \frac{61}{5}\\
&= -9 \ \ (\lambda<0) \qquad ✓\\
d_{OA} &= \sqrt{3^2+(-9)^2}\\
&= \sqrt{90}\\
d_{AB}^2 &= d_{OA}^2 - \left(\left|\text{proj}_{\underset{\sim}{v}}\, \underset{\sim}{u}\right|\right)^2 \qquad ✓\\
&= \left(\sqrt{90}\right)^2 - \left(\frac{159}{17}\right)^2\\
&= \frac{729}{289}\\
d_{AB} &= \pm\frac{27}{17}\\
&= \frac{27}{17}\ \text{ units } \ (d>0) \qquad ✓
\end{aligned}$$

(c) (4 marks) (Tran)

- ✓ [1] for finding $\overrightarrow{AE}$
- ✓ [1] for finding $\overrightarrow{CD}$
- ✓ [1] for using $\underset{\sim}{b} \cdot \underset{\sim}{c} = 0$
- ✓ [1] for final answer

$$
\begin{aligned}
\overrightarrow{BC} &= \underset{\sim}{c} - \underset{\sim}{b} \\
\overrightarrow{AE} &= \underset{\sim}{b} + \frac{1}{4}\left(\underset{\sim}{c} - \underset{\sim}{b}\right) \\
&= \frac{1}{4}\left(\underset{\sim}{c} + 3\underset{\sim}{b}\right) \qquad \checkmark \\
\overrightarrow{CD} &= \frac{1}{3}\underset{\sim}{b} - \underset{\sim}{c} \\
&= \frac{1}{3}\left(\underset{\sim}{b} - 3\underset{\sim}{c}\right) \qquad \checkmark \\
\overrightarrow{AE} \cdot \overrightarrow{CD} &= \frac{1}{12}\left(\underset{\sim}{c} + 3\underset{\sim}{b}\right) \cdot \left(\underset{\sim}{b} - 3\underset{\sim}{c}\right) \\
&= \frac{1}{12}\left(3\left|\underset{\sim}{b}\right|^2 - 3\left|\underset{\sim}{c}\right|^2 - 8\underset{\sim}{b} \cdot \underset{\sim}{c}\right)
\end{aligned}
$$

Since $\underset{\sim}{b} \cdot \underset{\sim}{c} = 0$ ✓

$$
\begin{aligned}
\overrightarrow{AE} \cdot \overrightarrow{CD} &= \frac{1}{4}\left(\left|\underset{\sim}{b}\right|^2 - \left|\underset{\sim}{c}\right|^2\right) \\
&= \frac{1}{4}\left(\left|\overrightarrow{AB}\right|^2 - \left|\overrightarrow{AC}\right|^2\right) \qquad \checkmark
\end{aligned}
$$