NORTH SYDNEY BOYS HIGH SCHOOL

2008 YEAR 11 EXTENSION 1
ASSESSMENT TASK 3

Mathematics

General Instructions

- Working time – **50 minutes**
- Write on one side of the paper using blue or black pen
- This examination paper contains 5 sections
- All necessary working should be shown in every question
- Each new **section** is to be started on a **new page.**
- Attempt all questions

Total Marks (50)

Class Teacher:
(Please tick or highlight)

- O Mr Barrett
- O Mr Weiss
- O Mr Lowe
- O Mr Rezcallah
- O Mr Taylor
- O Ms Wei

Student Name: ....
No of working pages used: ____ (fill this at the end of the exam)

Section	Intro Calc	Plane Geom	Co-ord Geom	Logs and Indices	3D Trig	Total	Total
Mark	$\frac{\quad}{11}$	$\frac{\quad}{11}$	$\frac{\quad}{13}$	$\frac{\quad}{10}$	$\frac{\quad}{5}$	$\frac{\quad}{50}$	$\frac{\quad}{100}$

Question 1 Introductory Calculus

a) Differentiate using appropriate techniques: (7 marks)

(i) $y = \frac{1}{3}x^3 + x - 1.5$

(ii) $f(x) = -\sqrt[4]{x^3} + \frac{1}{x}$

(iii) $y = x(3x-2)^4$

(iv) $f(x) = \frac{2x^2}{3x-1}$

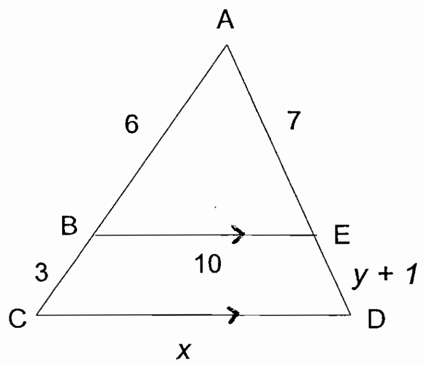
b) Evaluate : $\lim_{x \to \infty} \frac{2x}{x-1}$ (2 marks)

c) Find the derivative of $f(x) = 3x - 1$ from first principles. (2 marks)

Question 2 Plane Geometry

a) Find the number of sides of a regular polygon has if each interior angle is 144°. (2 marks)

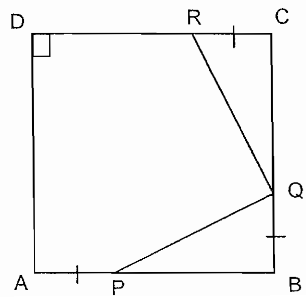
b) Find the values of *x* and *y*. (No reasons required) (4 marks)



c) In this square ABCD, R, Q and P are points chosen such AP = BQ = RC. (Show all necessary working and reasoning) (5 marks)

(i) Prove that $\Delta PBQ \equiv \Delta QCR$

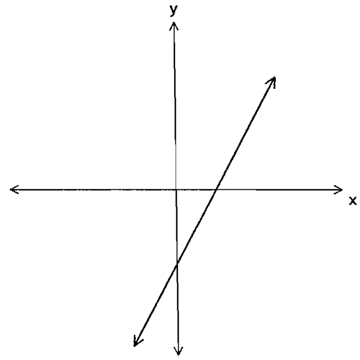
(ii) Prove that $\angle PQR$ is a right angle.



Question 3 Coordinate Geometry

a) Find the coordinates of the point that divides AB, A(3, -1) and B(9, 7), *externally* in the ratio of 3 : 2. (3 marks)

b) In the diagram, The line L_1 passes through the point (2,0) on the x-axis, and (0, -6) on the y-axis.



(i) Show that the equation of L_1 is $3x - y - 6 = 0$. (2 marks)

(ii) The line L_2 has equation $2x + y - 14 = 0$. By using the 'k method' or otherwise, find the equation of a line that passes through the intersection of L_1 and L_2 and the point (1, 4). (3 marks)

(iii) Calculate the perpendicular distance from the point (1, 4) to L_2. (2 marks)

c) Sketch the region satisfying: $y < \sqrt{64 - x^2}$ and $y \geq -4$ (3 marks)

Question 4 Logarithm and Indices

a) Calculate the value of $\log_3 15$, correct to 2 decimal places. (1 mark)

b) If $\log_{64} a = \frac{1}{3}$, find the value of a. (1 mark)

c) Give the exact value of $(\log_6 4 + \log_6 9)^2$ (2 marks)

d) Simplify $b^{2\log_b 7}$ (1 mark)

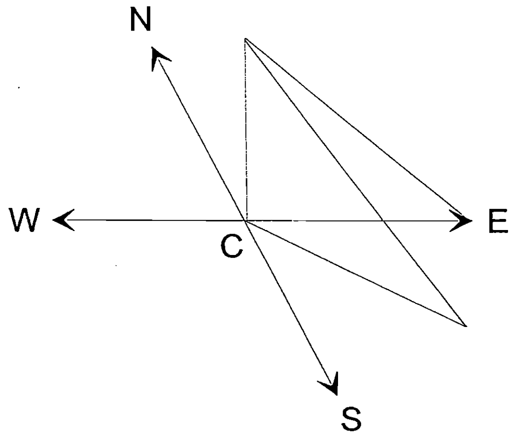
e) Solve for x: $\log_3(x + 3) - \log_3(x - 3) = 1$ (3 marks)

f) If $2^{x+1} < 12$, find the values of x to 2 decimal places. (2 marks)

Question 5   3D Trigonometry

Point A is located east of a hill and point B has a true bearing of 135° from the hill. The angle of elevation to the top of the hill from A is 43 °, from B 62°. A and B lie on level ground and are 600m apart.

Let the base of the hill be point C, and its perpendicular height be h.



Copy the diagram in your answer booklet.

(i) Show that BC has a distance of $\dfrac{h}{tan62°}$ (1 mark)

(ii) Derive a similar expression for AC. (1 mark)

(iii) Hence, find the value of h to the nearest metre. (3 marks)

Year 11 Extension 1 2008

Q1 Introductory Calculus

a i) $y' = x^2 + 1$ 1

ii) $f(x) = -x^{\frac{3}{4}} + x^{-1}$

$f'(x) = -\frac{3}{4}x^{-\frac{1}{4}} - x^{-2}$ 2

1 1 each

iii) $u = x \quad v = (3x-2)^4$

$u' = 1 \quad v' = 4(3x-2)^3 \times 3$ 1

$= 12(3x-2)^3$

$y' = (3x-2)^4 + 12x(3x-2)^3$ 1

$(3x-2)^3\left[3x-2+12x\right] = (3x-2)^3(15x-2)$

iv) $u = 2x^2 \quad v = 3x-1$

$u' = 4x \quad v' = 3$

$f'(x) = \dfrac{4x(3x-1) - 6x^2}{(3x-1)^2}$ 1

$= \dfrac{12x^2 - 4x - 6x^2}{(3x-1)^2}$

$= \dfrac{6x^2 - 4x}{(3x-1)^2}$

Q1 – cont.

b) $\lim\limits_{x \to \infty} \dfrac{2x}{x-1}$

$= \lim\limits_{x \to \infty} \dfrac{\frac{2x}{x}}{\frac{x}{x} - \frac{1}{x}}$ 1

$= \lim\limits_{x \to \infty} \dfrac{2}{1 - \frac{1}{x}}$

answer only?

$= \dfrac{2}{1-0}$

$= 2$ 1 ✓✓

c) $f(x) = 3x - 1$

$f(x+h) = 3(x+h) - 1$

$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{3(x+h) - 1 - (3x-1)}{h}$ 1

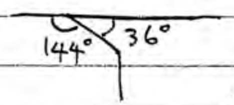
$= \lim\limits_{h \to 0} \dfrac{\cancel{3x} + 3\cancel{h} \cancel{-1} \cancel{-3x} \cancel{+1}}{\cancel{h}}$

$= \lim\limits_{h \to 0} 3$

$= 3$ 1

Q2. Plane Geometry

a)



ext $\angle = 36°$ 1

$\therefore$ no. of sides $= 360° \div 36°$

$= 10$ 1

b) $\frac{6}{3} = \frac{7}{y+1}$ 1

$\frac{1}{2} = \frac{y+1}{7}$

$\frac{7}{2} = y+1$

$y = \frac{7}{2} - 1$

$= \frac{5}{2}$ or $2\frac{1}{2}$ or 2.5 1

$\frac{6}{6+3} = \frac{10}{x}$ 1

$\frac{3}{2} = \frac{x}{10}$

$x = 15$ 1

c) (i) In Δs PBQ and QCR

- $BQ = RC$ (given)
- $\angle PBQ = \angle QCR = 90°$ ($\angle$s in a square) } 1
- since $AB = BC$ (= sides of square)

$\therefore AB - AP = BC - BQ$

$\therefore PB = QC$ 1

$\therefore \Delta PBQ \equiv \Delta QCR$ (SAS) 1

Q2 cont.

(ii) construct a line TQ, // to AB and CD

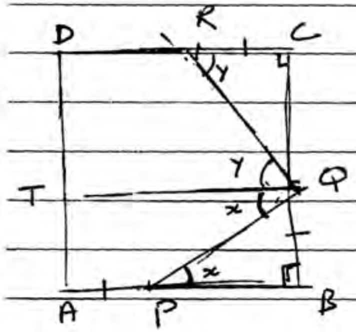
$\angle TQP = \angle BPQ$ (alt $\angle$s, $TQ \parallel PB$) $= x$ } 1

$\angle CRQ = \angle RQT$ (alt $\angle$s, $DC \parallel TQ$)

$= \angle BQP$ (corresp. $\angle$s in $\equiv \Delta$s) } 1

$\therefore \angle TQP + \angle RQT = \underbrace{\angle BPQ}_{x} + \underbrace{\angle BQP}_{+y}$

$= 90°$ ($\angle$ sum, right $\angle$ed Δ)



Q3 Coordinate Geometry

a) $A(3, -1)$ $B(9, 7)$

$3 : -2$ (or $-3 : 2$)

$x = \dfrac{3 \times 9 + (-2) \times 3}{3 + (-2)}$ 1 for method

1 for correct x

$= 27 - 6$ 1 for correct y

$= 21$

$y = \dfrac{3 \times 7 + (-2)(-1)}{3 + (-2)}$

$= 21 + 2$

$= 23$

b) i) gradient of $L_1 = \dfrac{-6-0}{0-2} = 3$ 1 for gradient

use either $(2,0)$ or $(0,-6)$

eg: $y - y_1 = m(x - x_1)$

$y - 0 = 3(x - 2)$

$y = 3x - 6$

$3x - y - 6 = 0$ 1 for correct answer

ii) $(3x - y - 6) + k(2x + y - 14) = 0$ 1

sub in $(1, 4)$

$(3 - 4 - 6) + k(2 + 4 - 14) = 0$

$-7 - 8k = 0$

$k = \dfrac{-7}{8}$ 1 for k value

$\therefore (3x - y - 6) - \frac{7}{8}(2x + y - 14) = 0$

$24x - 8y - 48 - 14x - 7y + 98 = 0$

$10x - 15y + 50 = 0$

$2x - 3y + 10 = 0$ 1 for correct equation

(or $y = \frac{2}{3}x + \frac{10}{3}$)

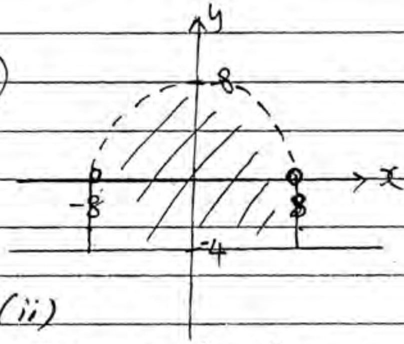
iii) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \dfrac{|2 \times 1 + 4 - 14|}{\sqrt{2^2 + 1^2}}$ 1 for working

$= \dfrac{8}{\sqrt{5}}$ units 1 for correct answer

$\left(= \dfrac{8\sqrt{5}}{5}\right)$

b)



1 for dotted semi circle

1 for filled vertical lines at ±8

1 for filled line $y = -4$ and correct region shaded

Q3(ii)

or: $3x - y - 6 = 0$

$2x + y - 14 = 0$

$5x = 20$

$x = 4$

$y = 3x - 6 = 12 - 6 = 6.$

$(4, 6)$ and $(1, 4)$ 1

$m = \dfrac{6-4}{4-1} = \dfrac{2}{3}$

$y - 4 = \frac{2}{3}(x - 1)$ 1

$3y - 12 = 2x - 2 \longrightarrow 2x - 3y + 10 = 0$ 1

Q4 Logarithm and Indices

a) $\log_3 15 = 2.46$ (2d.p.) 1

b) $64^{\frac{1}{3}} = a$

$\therefore a = 4$ 1

c) $(\log_6 4 \times 9)^2$

$= (\log_6 36)^2$ 1

$= 2^2$

$= 4$ 1

d) $b^{\log_b 7^2}$

$= 7^2$

$= 49$ 1

e) $\log_3 \frac{x+3}{x-3} = \log_3 3$ 1 1

$\frac{x+3}{x-3} = 3$ /3

$x + 3 = 3x - 9$

$2x = 12$

$x = 6$ 1

f) $\log_2 2^{x+1} < \log_2 12$ 1

$x + 1 < \frac{\log 12}{\log 2}$

$x < 3.58 - 1$ (2d.p.) 1

2.58

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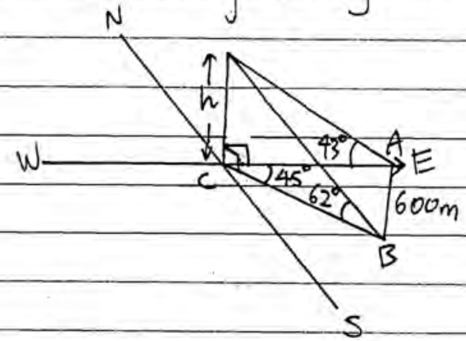
Q5 3D Trigonometry



(i) $\frac{h}{BC} = \tan 62°$

$\therefore BC = \frac{h}{\tan 62°}$ 1

(ii) $\frac{h}{CA} = \tan 43°$

$\therefore CA = \frac{h}{\tan 43°}$ 1

(iii) using cosine rule:

$AB^2 = \left(\frac{h}{\tan 62°}\right)^2 + \left(\frac{h}{\tan 43°}\right)^2$ 2

(−1 for each wrong working)

$- 2 \times \frac{h}{\tan 62°} \times \frac{h}{\tan 43°} \cos 45°$

$600^2 = h^2\left(\frac{1}{(\tan 62°)^2} + \frac{1}{(\tan 43°)^2}\right) - \frac{2h^2 \cos 45°}{\tan 62° \tan 43°}$

0.2827149

$= 1.43h^2 - 0.81h^2$

$360000 = 0.62h^2$

$h^2 = 580645.16$

$h = \cancel{762}$ m → not accurate 1

758.14