NORTH SYDNEY BOYS' HIGH SCHOOL
2009 Year 11 Assessment Task 3

MATHEMATICS
(Extension 1)

General instructions

- Working time – 60 minutes.
- Write on your own A4 paper. **Each question is to commence on a new page.**
- Write using blue or black pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question (Insufficient/illegible working may cause a deduction of marks).
- Attempt **all** questions.

Class teacher (please ✔)

- ◯ Mr Barrett
- ◯ Mr Lowe
- ◯ Mr Rezcallah
- ◯ Mr Trenwith
- ◯ Mr Ireland
- ◯ Mr Weiss
- ◯ Mr Lam

NAME: .... **PAGES USED**: ....

Marker's use only.

QUESTION	1	2	3	4	5	Total	%
MARKS	$\overline{12}$	$\overline{9}$	$\overline{13}$	$\overline{9}$	$\overline{11}$	$\overline{54}$	

Question 1 (12 Marks) Commence a NEW page. **Marks**

(a) Differentiate with respect to x, simplifying the answer fully.

i. $f(x) = 3x^3 + x^2 - 12x + 4.$ **2**

ii. $y = \left(5x^2 - 4\right)^2.$ **2**

iii. $f(x) = \dfrac{3x}{x^2 - 4}.$ **2**

iv. $y = 5x^3\sqrt{x+2}.$ **3**

(b) Find the value(s) of x for which the tangent to the curve $y = \frac{5}{3}x^3 - 10x^2 + 6$ is parallel to the x-axis. **3**

Question 2 (9 Marks) Commence a NEW page. **Marks**

(a) Shade in the region defined by **3**

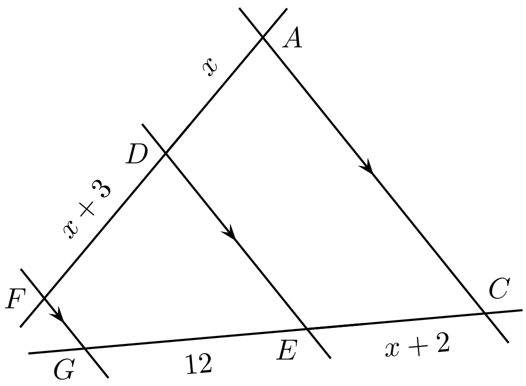
$$y \leq \sqrt{4 - x^2} \quad \text{and} \quad y > x - 2$$

(b) AB is divided by Q externally in the ratio $2:3$. **3**
Find the co-ordinates of B if A is $(3, -5)$ and Q is $(-4, 7)$.

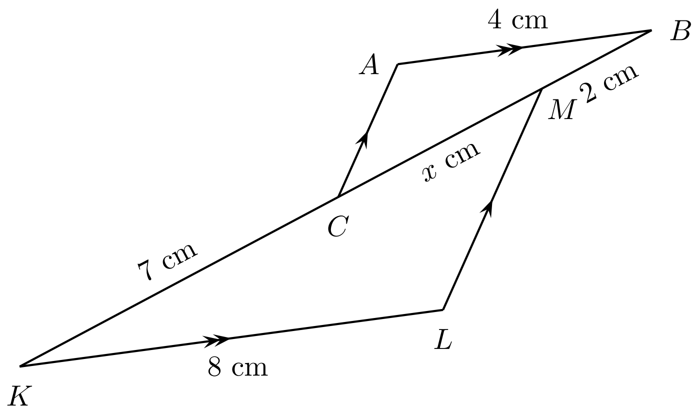
(c) Find the equation of the line through the point of intersection of the two lines given by the equations $x + 2y + 1 = 0$ and $2x - 3y + 5 = 0$ and through the point $(1, -3)$ **3**

Question 3 (13 Marks) Commence a NEW page. **Marks**

(a) In the diagram, $DE \parallel AC$. Find the value(s) of x, showing full working. **3**



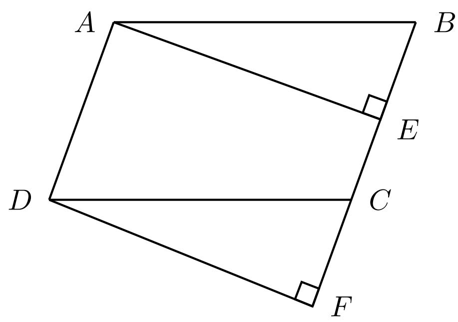
(b) In the diagram AB is parallel to KL and AC is parallel to ML. $AB = 4$ cm, $MB = 2$ cm and $KC = 7$ cm.



i. Prove $\triangle ABC$ is similar to $\triangle KLM$. **3**

ii. Find the value of x. **2**

(c) $ABCD$ is a parallelogram. $\angle AEB = \angle DFC = 90°$.



i. Prove $\triangle ABE \equiv \triangle DCF$. **3**

ii. Show that $AEFD$ is a rectangle. **2**

Question 4 (9 Marks) Commence a NEW page. **Marks**

(a) Given $\log_a 9 = 1.585$ and $\log_a 10 = 1.661$, find the following values correct to 3 decimal places.

i. $\log_a 90$. **1**

ii. $\log_a 0.9$. **1**

(b) Find the value of $\log_9 10$, correct to 2 decimal places. **2**

(c) Solve for x. **2**

$$0.4^x \leq 5$$

(d) Solve the equation $2\log_2(x-1) = \log_2(3x-3)$. **3**

Question 5 (11 Marks) Commence a NEW page. **Marks**

(a) Evaluate the following limits, expressing your answer in simplest form:

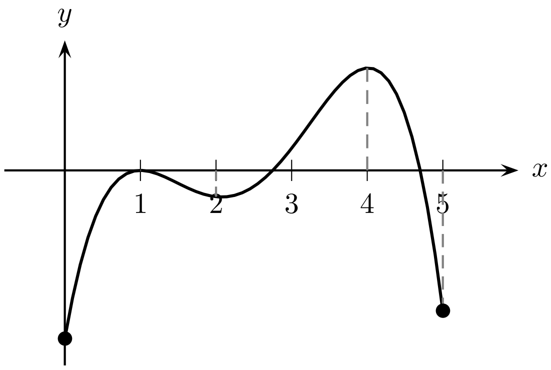
i. $\lim\limits_{x \to 3} \left(-x^2 - 2x + 3\right)$. **2**

ii. $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x + 2}$. **2**

iii. $\lim\limits_{x \to \infty} \dfrac{2x^2 - 3x + 7}{5x^2 - 4x + 3}$. **2**

(b) Differentiate $f(x) = x^2 - 2x$ from first principles. **3**

(c) The figure below shows the graph of $f(x)$ for $0 \leq x \leq 5$. Sketch the graph of its derivative $f'(x)$. **2**



End of paper.

Suggested Solutions

Question 1

(a) i. (2 marks)

✓ [-1] for each mistake.

$$f(x) = 3x^3 + x^2 - 12x + 4$$
$$f'(x) = 9x^2 + 2x - 12$$

ii. (2 marks)

✓ [1] for attempting to apply the chain rule.

✓ [1] for correct final answer. If student gave the final correct answer via expansion, award full marks.

$$\begin{aligned} y &= \left(5x^2 - 4\right)^2 \\ \frac{dy}{dx} &= 2 \times 10x\left(5x^2 - 4\right) \\ &= 20x\left(5x^2 - 4\right) \end{aligned}$$

iii. (2 marks)

✓ [1] for attempting to apply the quotient rule.

✓ [1] for correct final answer. If student gave the final correct answer via product + chain rules, award full marks.

$$\begin{aligned} f(x) &= \frac{3x}{x^2 - 4} \\ u = 3x &\quad v = x^2 - 4 \\ u' = 3 &\quad v' = 2x \\ f'(x) &= \frac{vu' - uv'}{v^2} \\ &= \frac{3(x^2 - 4) - 6x^2}{(x^2 - 4)^2} \\ &= \frac{3x^2 - 12x - 6x^2}{(x^2 - 4)^2} \\ &= \frac{-3x^2 - 12}{(x^2 - 4)^2} \end{aligned}$$

iv. (3 marks)

✓ [1] for attempting to apply the product rule.

✓ [1] for obtaining $\frac{d}{dx}(x+2)^{\frac{1}{2}} = \frac{1}{2}(x+2)^{-\frac{1}{2}}$.

✓ [1] for correct final answer.

$$\begin{aligned} y &= 5x^3 (x+2)^{\frac{1}{2}} \\ u = 5x^3 &\quad v = (x+2)^{\frac{1}{2}} \\ u' = 15x^2 &\quad v' = \frac{1}{2}(x+2)^{-\frac{1}{2}} \\ \frac{dy}{dx} &= uv' + vu' \\ &= \frac{1}{2} \cdot 5x^3(x+2)^{-\frac{1}{2}} + 15x^2(x+2)^{\frac{1}{2}} \\ &= \frac{5x^3}{2\sqrt{x+2}} + 15x^2\sqrt{x+2} \end{aligned}$$

(b) (3 marks)

✓ [1] for correct differentiation.

✓ [1] for identifying $\frac{dy}{dx} = 0$ for the tangent to be parallel to the x axis.

✓ [1] for $x = 0, 4$.

$$\begin{aligned} y &= \frac{5}{3}x^3 - 10x^2 + 6 \\ \frac{dy}{dx} &= 5x^2 - 20x \end{aligned}$$

The gradient function is $= 0$ when the tangent to the curve is parallel to the x axis.

$$\begin{aligned} \therefore 5x^2 - 20x &= 0 \\ 5x(x - 4) &= 0 \\ \therefore x &= 0, 4 \end{aligned}$$

Question 2

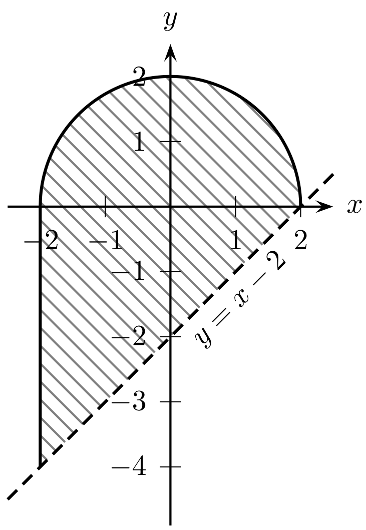
(a) (3 marks)

✓ [1] for correctly sketching semi circle.

✓ [1] for correctly sketching $y = x - 2$ with dashed line.

✓ [1] for correct shading.

$$\left\{(x, y) : y \le \sqrt{4 - x^2}\right\} \cap \{(x, y) : y > x - 2\}$$

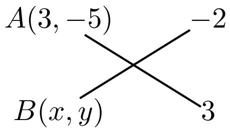


(b) (3 marks)

✓ [1] for obtaining

$$(-4,7) = (9-2x, -15-2y)$$

✓ [1] each for correct value of x and y obtained from expression.



$$\begin{aligned}(-4,7) &= \left(\frac{3(3)-2x}{1}, \frac{3(-5)-2y}{1}\right)\\ &= (9-2x, -15-2y)\end{aligned}$$

$$\begin{aligned} 9-2x &= -4 \\ 2x &= 13 \\ x &= \frac{13}{2}\end{aligned} \qquad \begin{aligned} -15-2y &= 7 \\ 2y &= -22 \\ y &= -11\end{aligned}$$

$$\therefore Q\left(\frac{13}{2}, -11\right)$$

(c) (3 marks)

$$\begin{cases} x+2y+1=0 \\ 2x-3y+5=0\end{cases}$$

$$x+2y+1+k(2x-3y+5) = 0 \qquad (2.1)$$

$$x(1+2k)+y(2-3k)+5k+1 = 0$$

Substitute $x = 1$ and $y = -3$ to obtain the value of k for the line passing through $(1, -3)$:

$$\begin{aligned} 1+2k-3(2-3k)+5k+1 &= 0 \\ 16k &= 4 \\ k &= \frac{1}{4} \qquad (2.2)\end{aligned}$$

Substitute (2.2) to (2.1)

$$\underbrace{x+2y+1}_{\times 4} + \underbrace{\frac{1}{4}(2x-3y+5)}_{\times 4} = 0$$

$$\begin{aligned} 4x+8y+4+2x-3y+5 &= 0 \\ 6x+5y+9 &= 0\end{aligned}$$

Question 3

(a) (3 marks)

✓ [1] For attempting to apply the ratio of intercepts.

✓ [1] For each correct final answer. If not fully correct, deduct [1] for each incorrect step.

By the ratio of intercepts,

$$\frac{12}{x+2} = \frac{x+3}{x}$$

Cross multiplying,

$$\begin{aligned} 12x &= (x+2)(x+3) \\ \underset{-12x}{12x} &= x^2 + \underset{-12x}{5x} + 6 \\ x^2 - 7x + 6 &= 0 \\ (x-6)(x-1) &= 0 \\ \therefore x &= 1, 6\end{aligned}$$

(b) i. (3 marks)

✓ [1] for each reason, up to 2 marks max (subtract [1] for each incorrect reason).

✓ [1] for concluding that the triangles are similar as they are equiangular.

In $\triangle ACB$ & $\triangle LMK$,

- $\angle ACB = \angle KML$ (alternate $\angle$ on $\parallel$ lines AC & ML.)
- $\angle ABC = \angle MKL$ (alternate $\angle$ on $\parallel$ lines AB & KL.)
- $\angle CAB = \angle KLM$ (remaining $\angle$)

$\therefore \triangle ACB \,|||\, \triangle LMK$ (equiangular).

ii. (2 marks)

$$\frac{BC}{KM} = \frac{AB}{KL} \qquad \Rightarrow \frac{2+x}{x+7} = \frac{1}{2}$$
$$4 + 2x = x + 7$$
$$CM = x = 3$$

(c) i. (3 marks)

- ✓ [1] for each reason, up to 2 marks max (subtract [1] for each incorrect reason).
- ✓ [1] for concluding that the triangles are congruent (AAS).

In $\triangle ABE$ and $\triangle DCF$,

- $\angle DFC = \angle AEB$ (data)
- $DC = AB$ (opposite sides of a parallelogram)
- $\angle ABE = \angle DCF$ (corresponding $\angle$ on $\parallel$ lines AB & CD)

$\therefore \triangle ABE \equiv \triangle DCF$ (AAS)

ii. (2 marks)

- ✓ [1] for each correct reason (subtract [1] for each incorrect reason), up to 2 marks max.

Since $\triangle ABE \equiv \triangle DCF$, $\therefore$ all corresponding sides & angles are equal.

- $AE = DF$ (corresponding sides in $\equiv \triangle$)
- $\angle AEF = \angle EFD = 90°$ (supplementary $\angle$)
- $AE \parallel DF$ as $\angle AEF = \angle EFD$ (corresponding $\angle$ on $\parallel$ lines)

$\therefore AEFD$ is a rectangle – 2 adj. $\angle$ are right angles and 2 opposite sides equal & parallel.

Question 4

(a) i. (1 mark)

$$\log_a 9 = 1.585 \qquad \log_a 10 = 1.661$$
$$\begin{aligned}\log_a 90 &= \log_a (9 \times 10)\\ &= \log_a 9 + \log_a 10\\ &= 1.585 + 1.661 = 3.246\end{aligned}$$

ii. (1 mark)

$$\begin{aligned}\log_a 0.9 &= \log_a \left(\frac{9}{10}\right)\\ &= \log_a 9 - \log_a 10\\ &= 1.585 - 1.661 = -0.076\end{aligned}$$

(b) (2 marks)

- ✓ [1] for attempting to change base. Penalise if there is no working but only the final answer.
- ✓ [1] for final answer (1.05). Penalise if it was not rounded to 2 d.p.

$$\begin{aligned}\log_9 10 &= \frac{\log_1 010}{\log_1 09}\\ &= \frac{1}{\log_{10} 9}\\ &= 1.05 \text{ (2 d.p.)}\end{aligned}$$

(c) (2 marks)

- ✓ [1] for taking logs correctly to obtain $x \log_{10} 0.4 \leq 5$.
- ✓ [1] for final answer $x \geq -12.56$. No need to penalise for 2 d.p. as long as the sign of the inequality is correct.

$$0.4^x \leq 5$$
$$\underset{\div \log_{10} 0.4}{x \log_{10} 0.4} \leq \underset{\div \log_{10} 0.4}{\log_{10} 5}$$
$$x \geq \frac{\log_{10} 5}{\log_{10} 0.4} = -1.756$$

(d) (3 marks)

- ✓ [1] for each correct solution to the quadratic: $x = 1$, $x = 4$.
- ✓ [1] for reasoning why $x = 1$ cannot be a solution.

$$\begin{aligned}2\log_2(x-1) &= \log_2(3x-3)\\ \log_2(x-1)^2 &= \log_2(3x-3)\\ (x-1)^2 &= 3(x-1)\\ (x-1)^2 - 3(x-1) &= 0\\ (x-1)\Big((x-1)-3\Big) &= 0\\ (x-1)(x-4) &= 0\\ \therefore x &= 1, 4\end{aligned}$$

However, substituting $x = 1$ into the original equation results in $2\log_2 0$. The domain of $\log_2 x$ is $x > 0$. Hence the only possible solution is $x = 4$.

Question 5

(a) i. (2 marks)

✓ [−1] for each incorrect step.

$$\lim_{x\to 3}\left(-x^2-2x+3\right) = -(3^2)-2(3)+3 = -9-6+3 = -12$$

ii. (2 marks)

✓ [1] for realising the need to factorise.

✓ [1] for correct final answer.

$$\lim_{x\to -2}\frac{x^2-x-6}{x+2} = \lim_{x\to -2}\frac{\cancel{(x+2)}(x-3)}{\cancel{x+2}} = -2-3 = -5$$

iii. (2 marks)

✓ [1] for dividing the numerator and denominator by x^2.

✓ [1] for correct final answer.

$$\lim_{x\to\infty}\frac{2x^2-3x+7}{5x^2-4x+3} = \lim_{x\to\infty}\frac{2-\frac{3}{x}+\frac{7}{x^2}}{5-\frac{4}{x}+\frac{3}{x^2}} = \frac{2}{5}$$

(b) (3 marks)

✓ [1] for obtaining $f(x+h)=x^2+2hx+h^2-2x-2h$.

✓ [1] for correctly recalling $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

✓ [1] for final answer.

$$\begin{aligned} f(x) &= x^2-2x \\ f(x+h) &= (x+h)^2-2(x+h) \\ &= x^2+2hx+h^2-2x-2h \end{aligned}$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{\cancel{x^2}+2hx+h^2-\cancel{2x}-2h-\cancel{x^2}+\cancel{2x}}{h} \\ &= \lim_{h\to 0}\frac{\cancel{h}(2x+h-2)}{\cancel{h}} \\ &= 2x-2 \end{aligned}$$

(c) (2 marks)

✓ [1] for correct roots at $x=1,2,4$. Only award if *all* roots are present and correct.

✓ [1] for correct shape.