Q6/7 not examined
in 2011 Assessment Task 1.

Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2010 Year 11Half Yearly

Mathematics
Extension 1

General Instructions

- Working time – 100 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Ireland
- O Mr Fletcher
- O Mr Lam
- O Mr Rezcallah
- O Mr Trenwith
- O Mr Weiss

Student Name:_____

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	Total %
Mark	___/12	___/12	___/14	___/12	___/10	___/19	___/6	___/15	___/100

Question 1 (12 marks) Marks

a) Evaluate $\sqrt[3]{\frac{8.5+15.2}{\pi-2.15}}$ correct to 3 significant figures. 2

b) Express $0.35\dot{2}$ as a fraction in simplest rational form. Show working. 2

c) Factorise completely:

(i) $m^2 - mn + 6m - 6n$ 2

(ii) $x^5 - 32$ 2

d) If $x = 3 - \sqrt{2}$, simplify fully $(x-2)^2$ 2

e) Solve: $|2x + 1| = 5$ 2

Question 2 (12 marks) Start a new page.

a) Express $\frac{23\pi}{15}$ in degrees. 1

b) Evaluate *cosec* 0.8 correct to two decimal places. 1

c) Find, in simplest surd form, the value of $\frac{1+\cos 120°}{\tan 240°}$ 3

d) If $\cos x = -\frac{2}{3}$, find the exact value(s) of $\tan x$. 2

e) In triangle ABC, AC = 8 cm, BC= 10 cm and ∠ABC = 50°.

Find the possible values of ∠BAC to the nearest degree 3

f) A triangular airfield is bordered by three straight lines of length 4 km, 5 km and 6 km. If the largest angle in the triangle is 83°, find the area of the airfield. 2

Question 3 (14 marks) Start a new page.

a) Sketch the following relations on separate number planes, showing all important features (asymptotes, x and y intercepts):

(i) $y = \sqrt{25 - x^2}$ 2

(ii) $y = |3 + 2x|$ 2

(iii) $y = \frac{-3}{x+2}$ 2

(iv) $y = 2^{-x} - 1$ 2

b) (i) Sketch a graph of $f(x) = 4x - x^2$ showing all main features. 2

Use this graph to answer the following questions:

(ii) Find the largest possible domain and range of $y = 4x - x^2$ 2

(iii) Solve $4x - x^2 \geq 0$ 1

(iv) What is the natural domain of the function $H(x) = \frac{3}{\sqrt{4x - x^2}}$ 1

Question 4 (12 marks) Start a new page.

a) Solve the following equations:

(i) $\frac{x+3}{2} - \frac{3x-1}{7} = 2$ 2

(ii) $5x^2 - 9x = 2$ 2

(iii) $|3x + 1| = 4x - 2$ 3

b) Simplify fully the following:

(i) $\frac{a^2-9}{a^4-27a} \div \frac{a+3}{a^2+3a+9}$ 2

(ii) $\frac{x+2}{x-1} - \frac{2x-1}{x+1} - \frac{6x}{x^2-1}$ 3

Question 5 (10 marks) Start a new page.

a) Solve the following inequations:

i) $|5 - 2x| < 7$ 2

ii) $\frac{4x-1}{x-1} \geq 3$ 3

b) Expand and simplify $(2x - 1)^4$ 2

c) Find rational numbers a and b such that $a + b\sqrt{2} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ 3

Question 6 (19 marks) Start a new page.

a) Solve the following equations for $0 \leq x \leq 360°$:

i) $2\sin 2x = 1$ 2

ii) $\tan x = \cot x$ 3

b) Prove that $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. 3

c) Simplify $\tan(90° - \theta)\ \sin(360° - \theta)$ 2

d) i) State the amplitude, period and the range of $y = 2\sin 4x$. 3

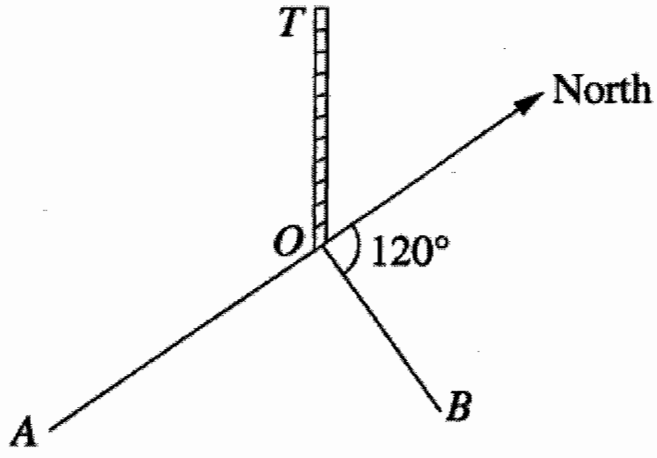
ii) Sketch the graph of $y = 2\sin 4x, 0 \leq x \leq 180°$. 2

e) A is 3 km North and 4.3 km east of P. The bearing of B from A is 145°.

Find the distance PB if the distance AB = 14 km. 4

Question 7 (6 marks) Start a new page.

From a point A due south of a tower, the angle of elevation of the top of the tower T, is 28°. From another point B, on a bearing of 120° from the tower, the angle of elevation of T is 33°.
The distance AB is 200 metres.



i) Copy or trace the diagram into your writing booklet, adding the given information to your diagram. 1

ii) If $OT = h$, express the distances OA and OB in terms of h. 2

iii) Hence find the height of the tower to the nearest metre. 3

Question 8 (15 marks) Start a new page.

a)

$$f(x) = \begin{cases} \frac{1}{x}, & x \leq 1 \quad x \neq 0 \\ 2x^3 - 1, & x > 1 \end{cases}$$

(i) Find $f(2)$ 1

(ii) Sketch the function $f(x)$. 2

(iii) State the range of $f(x)$. 1

b) If $f(n) = n(n+1)(n+2)$ express $f(n) + f(n+1)$ in factored form. 2

c) If $y = f(x) = \frac{x^2+1}{x^2-1} = 1 + \frac{2}{x^2-1}$

i) State the equations of the asymptotes. 2

ii) Show that $y = f(x)$ is an even function. 2

iii) Sketch the graph showing intercepts and asymptotes. 3

iv) Hence, or otherwise, solve $\frac{x^2+1}{x^2-1} < 1$ 1

d) Find $g(x)$ if the function $f(x)$ below is an odd function.

$$f(x) = \begin{cases} x^4, \ for \ x > 0 \\ g(x), \ for \ \ x < 0 \end{cases}$$ 1

End of Exam

2010 Yr 11 Ext 1 Maths Half Yearly

Question 1

a) $2.880525 = 2.88$.

✓ ✓ for sig. figures 2.8

b) let $x = 0.3522$

$10x = 3.5222$.

$\therefore \quad 9x = 3.17$

$x = \frac{3.17}{9} = \frac{317}{900}$.

✓ for correct method

✓ for $\frac{317}{900}$.

c) (i) $m^2 - mn + 6m - 6n$

$= m(m-n) + 6(m-n)$

$= (m-n)(m+6)$.

✓

✓

(ii) $x^5 - 32 = x^5 - 2^5$

$= (x-2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$

✓ for $(x-2)$

✓ for correct factoring.

d) $x = 3 - \sqrt{2}$.

$x - 2 = 1 - \sqrt{2}$.

$(x-2)^2 = (1-\sqrt{2})^2$

$= 1 - 2\sqrt{2} + 2 \quad = 3 - 2\sqrt{2}$.

✓

✓ for $3 - 2\sqrt{2}$.

e) $2x + 1 = 5$ $\qquad$ $2x + 1 = -5$

$2x = 4$ $\qquad$ $2x = -6$

$x = 2$ $\qquad$ $x = -3$.

✓ for $x = 2$

✓ for $x = -3$

[or Award 1 for correct working wrong answer].

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Solution of Yr 11 2010 Ext1 Half Yearly.

Question 2:

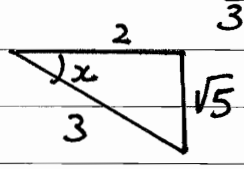
a) $\frac{23\pi}{15} = \frac{23 \times 180}{15} = 276$ ✓

b) $\operatorname{cosec} 0.8 = \frac{1}{\sin 0.8} = 1.39$ (2 dec. places) ✓

c) $\frac{1+\cos 120^\circ}{\tan 240} = \frac{1+(-\frac{1}{2})}{\sqrt{3}} = \frac{1}{2\sqrt{3}}$ ✓ for correct ratios.

$= \frac{1}{2}\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{6}$. ✓ for $\frac{\sqrt{3}}{6}$.

d) $\cos x = -\frac{2}{3}$. $\therefore$ x is in 2ND or 3rd quad.

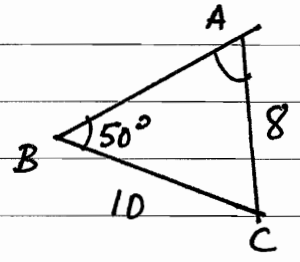


$\tan x = \pm \frac{\sqrt{5}}{2}$.

✓ for correct working ($\sqrt{5}$).

✓ for $\pm \frac{\sqrt{5}}{2}$.

e)



$\frac{\sin A}{10} = \frac{\sin 50}{8}$.

$\sin A = \frac{10 \sin 50}{8} = 0.95755$. ✓ for correct sine rule

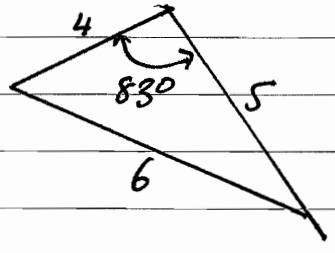
$\therefore \angle A = 73^\circ$, $180 - 73 = 107^\circ$. ✓ for 73°.

↓ works since $107 + 50 < 180^\circ$.

$\therefore \angle BAC = 73^\circ, 107^\circ$. ✓ for obtuse answer.

(Accept: $73^\circ 15'$, $106^\circ 45'$)

f)



$A = \frac{1}{2} \times 4 \times 5 \sin 83^\circ$.

$= 10 \times \sin 83^\circ = 9.925 \text{ km}^2$

$= 9.9 \text{ km}^2$.

✓ for correct area rule

✓ for correct answer.

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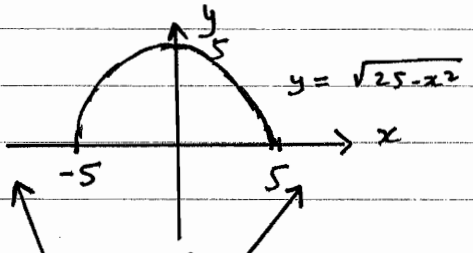
Solution of Yr 11 Ext 1 Half Yearly 2010 -

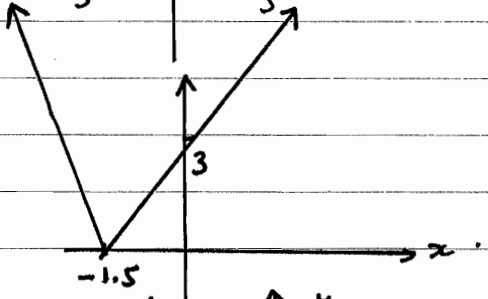
Question 3.

a) (i)



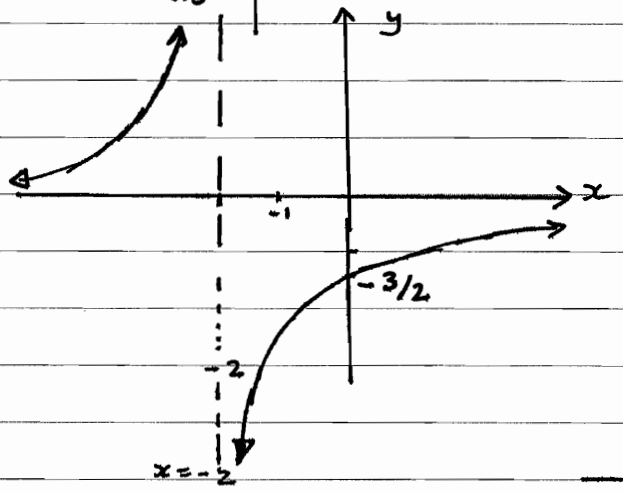
(i)
✓ for shape.
✓ for intercepts.

(ii)



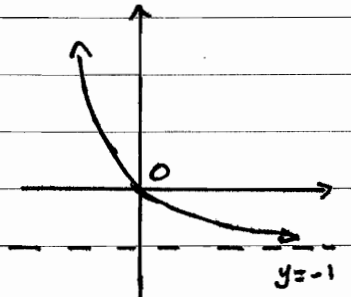
(ii)
✓ for shape.
✓ for both intercepts

(iii)



(iii)
✓ for shape.
✓ for asymptote + intercept.

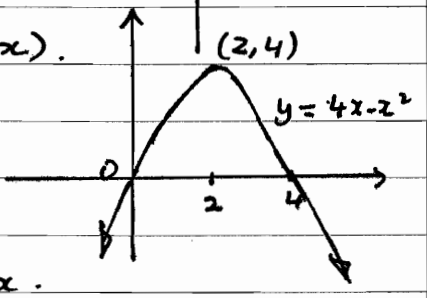
(iv)



(iv)
✓ for correct shape.
✓ for showing $y=-1$

b) (i) $f(x) = 4x - x^2$. $= x(4-x)$.

vertex : $x = 2$, $y = 4$.



b)(i)
✓ for shape.
✓ for intercepts and/or vertex.

(ii) Domain: All real x.

R: $y \leq 4$

(ii)
✓
✓

(iii) $4x - x^2 \geqslant 0$ $\therefore$ $0 \leq x \leq 4$.

iii)
✓

(iv) Domain: $0 < x < 4$.

(iv)
✓ (No equal signs)

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Solution of Yr 11 2010 Ext 1 Half Yearly.

Question 4:

a) (i) $\frac{x+3}{2} - \frac{3x-1}{7} = 2. \quad \times 14$

$7(x+3) - 2(3x-1) = 28.$

$7x + 21 - 6x + 2 = 28.$

$x + 23 = 28 \quad \therefore x = 5.$

(ii) $5x^2 - 9x = 2 \quad \therefore 5x^2 - 9x - 2 = 0.$

$(5x + 1)(x - 2) = 0.$

$x = -\frac{1}{5}, 2.$

(iii) $|3x+1| = 4x - 2.$

$3x + 1 = 4x - 2. \qquad 3x + 1 = -4x + 2.$

$3 = x \qquad 7x = 1.$

$\therefore x = \frac{1}{7}.$

test $x=3$: $10 = 12 - 2$ ✓ $\qquad 1\frac{3}{7} \neq \frac{4}{7} - 2$

$\therefore x = 3$ is only solution.

b) (i) $\frac{(a-3)(a+3)}{a(a^3-27)} \times \frac{a^2+3a+9}{(a+3)}$

$\frac{(a-3)}{a(a-3)(a^2+3a+9)} \times \frac{(a^2+3a+9)}{1}$

$= \frac{1}{a}.$

(ii) $\frac{x+2}{x-1} - \frac{2x-1}{x+1} - \frac{6x}{x^2-1 \to (x+1)(x-1)}$

$= \frac{(x+2)(x+1) - (2x-1)(x-1) - 6x.}{(x+1)(x-1)}$

$= \frac{x^2+3x+2 - (2x^2-3x+1) - 6x}{x^2-1}$

$= \frac{-x^2+6x+1-6x}{x^2-1} = -1\frac{(x^2-1)}{x^2-1} = -1$

(a)

(i)

✓ for correct expandi

✓ for $x = 5$.

(ii)

✓ for correct method.

✓ for correct answers.

(iii)

✓ for getting $x=3$.

✓ for getting $x=\frac{1}{7}$.

[or ✓✓ for correct graphical method].

✓ for testing to get $x=3$. only.

b) (i)

✓✓ for $\frac{1}{a}$.

(Take away 1 for each error).

(ii)

✓ for correct start.

✓

✓ for -1.

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Solution of Yr 11 Ext 1 Half Yearly 2010

Question 5

a) (i) $-7 < 5-2x < 7$

$-12 < -2x < 2 \quad \div -2.$

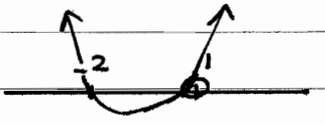
$6 > x > -1 \quad \therefore -1 < x < 6.$

(ii) $\frac{4x-1}{x-1} - 3 \geq 0.$

$\frac{4x-1-3x+3}{x-1} \geq 0.$

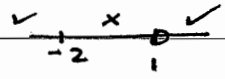
$(x+2)(x-1) \geqslant 0.$

$x > 1$ or $x \leq -2.$



OR: $4x-1 = 3(x-1)$

$x = -2.$



$x > 1$ or $x \leq -2.$

b) $(2x-1)^4$

$= (2x)^4 + 4(2x)^3(-1) + 6(2x)^2(-1)^2 + 4(2x)(-1)^3 + (-1)^4$

$= 16x^4 - 32x^3 + 24x^2 - 8x + 1.$

c) $\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2+1+2\sqrt{2}}{2-1} = 3+2\sqrt{2}.$

$a = 3. \quad b = 2.$

a) (i)

✓ for correct start.

✓ $-1 < x < 6.$

(ii)

✓✓ for a correct method.

✓ for $x > 1$ or $x \leq -$

[Students lose 1 mark for $x \geq 1$]

b)

✓

✓

c)

✓ for correct rationalising.

✓ for $3+2\sqrt{2}.$

✓ for $a = 3$ and $b = 2.$

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Solution of Yr 11 Ext1 Half Yrly 2010

Question 6

a) (i) $\sin 2x = \frac{1}{2}$.

$2x = 30°, 150°, 390°, 510°.$

$x = 15°, 75°, 195°, 255°.$

(ii) $\tan x = \frac{1}{\tan x}$. $\therefore \tan^2 x = 1.$

$\tan x = 1$ $\quad$ $\tan x = -1.$

$x = 45°, 180+45°.$ $\quad$ $x = 180-45°, 360-45°.$

$x = 45°, 225°$ $\quad$ $x = 135°, 315°.$

b) LHS $(\tan x + \cot x)^2$

$= \tan^2 x + \cot^2 x + 2$ ✓

$= 1 + \tan^2 x + 1 + \cot^2 x.$ ✓

$= \sec^2 x + \operatorname{cosec}^2 x.$ ✓

$= RHS$

or RHS $\sec^2 x + \operatorname{cosec}^2 x$

$= 1 + \tan^2 x + 1 + \cot^2 x$ ✓

$= \tan^2 x + \cot^2 x + 2$ ✓

$= (\tan x + \cot x)^2.$ ✓

$= LHS.$

c) $\cot\theta \times (-\sin\theta)$

$= \frac{\cos\theta}{\sin\theta} \times (-\sin\theta)$

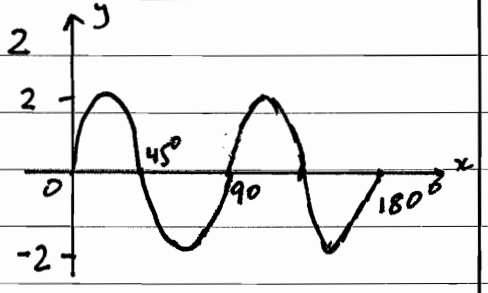
$= -\cos\theta.$

d) i) $a = 2.$

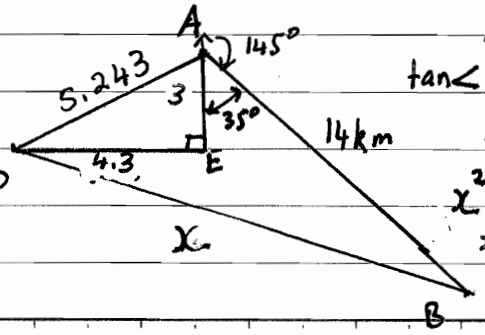
$T = \frac{360}{4} = 90°.$

Range $= -2 \le y \le 2$

ii)



e)



$AP = \sqrt{4.3^2 + 3^2} = 5.243$

$\tan \angle PAE = \frac{4.3}{3} \Rightarrow \angle PAE = 55°5'51''$

$\therefore \angle PAB = 90° 5' 51''$ ✓

$x^2 = 5.243^2 + 14^2 - 2(5.243)(14)\cos(90°5'51'')$ ✓ students have to use cos. rule as $\angle$ is 90.0975°

$x = 14.9579$

14.96 km (2 dec. pl) ✓

a) (i)	
✓ for 2 answers.	
✓ for other 2 answers	
(ii)	
✓ for $\tan x = \pm 1$	
✓ for each 2 correct	
✓ for other 2 correct	
b)	
✓	
✓	
✓	
c)	
✓ for any of the 2 correct.	
✓ for $-\cos\theta$.	
(i)	
✓	
✓	
✓	
(ii)	
✓	
✓	

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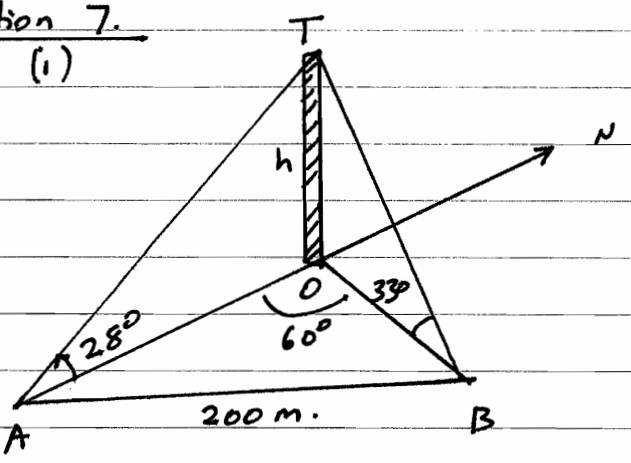
Solution of yr 11 Ext 1 Half yearly 2010.

Question 7.

(i)



(i) ✓

(ii) $\cot 28° = \frac{OA}{h}$ $\therefore OA = h \cot 28°$

$OB = h \cot 33°$

[smarter: $OA = h \tan 62°$.
$OB = h \tan 57°$]

(ii) ✓ for OA

✓ for OB.

(iii)

$200^2 = OA^2 + OB^2 - 2OA \times OB \cos 60°$.

$40000 = h^2 \tan^2 62 + h^2 \tan^2 57 - 2(h \tan 62)(h \tan 57°) \times \cos 60$

$h^2 = \frac{40000}{\tan^2 62 + \tan^2 57 - 2 \tan 62 \tan 57 \times \frac{1}{2}} =$

$= 13279.10$

or: $h^2 = \frac{40000}{\cot^2 28 + \cot^2 33 - 2 \cot 28 \cot 33 \times \frac{1}{2}}$

$\therefore h = 115$ m.

(iii) ✓ for cos. rule.

✓ for h^2

✓ for 115m.

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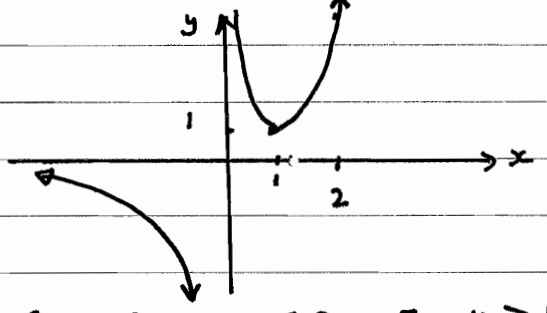
Solution of Yr 11 Ext 1 Maths Half yearly.

Question 8:

(a) (i) $f(2) = 2(8) - 1 = 15$. ✓ (i)

(ii)



✓ (ii)

✓

✓

(iii) Range: $y < 0$ or $y \geq 1$. ✓ (iii)

b) $f(n+1) = (n+1)(n+2)(n+3)$

$f(n) + f(n+1) = n(n+1)(n+2) + (n+1)(n+2)(n+3)$

$= (n+1)(n+2)(n + n + 3)$

$= (n+1)(n+2)(2n+3)$

b) ✓ for correct $f(n+1)$

✓ for final factored answer.

c) (i) Asymptotes: $y = 1$.

$x = 1$, $x = -1$.

c)(i) ✓ for $y=1$

✓ for both $x = \pm 1$

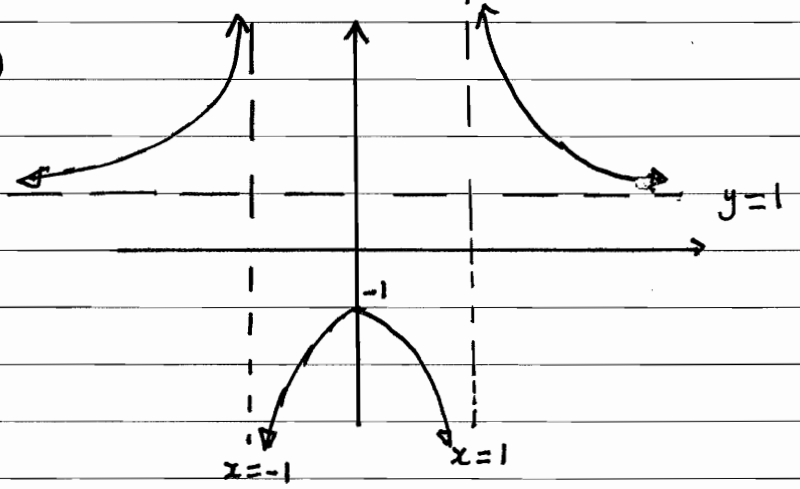
(ii) $f(-x) = 1 + \dfrac{2}{(-x)^2 - 1}$ (ii)

$= 1 + \dfrac{2}{x^2 - 1} = f(x)$.

✓ ✓

$\therefore$ Even function.

(iii)



✓ ✓ ✓

Asymptotes have to be shown clearly eg. $y=1$. The 3 parts of the graph have to be clearly drawn to get the 3 marks.

(iv) $-1 < x < 1$.

✓ only for $-1<x<1$ (question states Hence, or otherwise)

d) $g(x) = -x^4$

✓