North Sydney Boys High School

MATHEMATICS (EXTENSION 1)

2011 Preliminary Course Assessment Task 1

General instructions

- Working time – 50 minutes.
- Commence each new question on a new page.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

Class (please ✔)

- ◯ 11M3A – Mr Weiss
- ◯ 11M3B – Mr Ireland
- ◯ 11M3C – Mr Berry
- ◯ 11M3D – Mrs Collins
- ◯ 11M3E – Mr Lam
- ◯ 11M3F – Mr Fletcher/Ms Everingham

NAME .... **# SHEETS USED**: ....

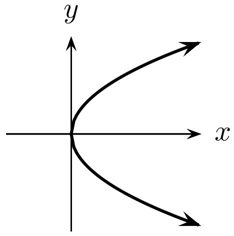
Marker's use only.

QUESTION	1	2	3	4	**Total**	%
MARKS	$\overline{17}$	$\overline{10}$	$\overline{31}$	$\overline{19}$	$\overline{77}$	

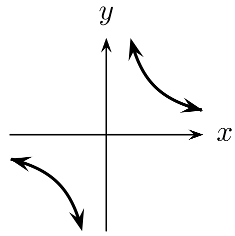
Question 1 (17 Marks) Commence a NEW page. **Marks**

(a) Which of the following are functions? **3**

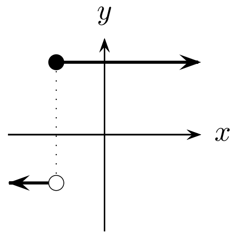
i.



ii.



iii.



(b) For the function $g(x) = \sqrt{x} + 3x$, find

i. $g(4)$ **1**

ii. $g\left(x^2\right)$ **1**

(c) Sketch, showing the essential features.

i. $y = |x| + 2$ **2**

ii. $y = \dfrac{4}{x-2}$ **2**

iii. $y = 4 - x^2$ **2**

iv. $(x-2)^2 + y^2 = 9$ **2**

(d) Consider the function $y = \sqrt{9 - x^2}$

i. State the domain and range. **2**

ii. Sketch the function. **1**

(e) Find the largest possible domain for $f(x) = \sqrt{x-4} + \sqrt{7-x}$. **1**

Question 2 (10 Marks) Commence a NEW page. **Marks**

(a) Express $0.3\dot{7}\dot{8}$ as a simplest fraction. **2**

(b) If $2\sqrt{6} = \sqrt{x}$, find x. **1**

(c) Simplify $\sqrt{18} + \sqrt{8} - \sqrt{2}$ fully. **1**

(d) Expand and simplify:

i. $\left(2\sqrt{3} + \sqrt{5}\right)^2$. **2**

ii. $\left(\sqrt{7} - 3\right)\left(\sqrt{7} + 3\right)$. **1**

(e) Rationalise the denominator and express in simplest form:

i. $\dfrac{1}{2\sqrt{3}}$. **1**

ii. $\dfrac{3}{2+\sqrt{5}}$. **2**

Question 3 (31 Marks) Commence a NEW page. **Marks**

(a) Solve for x:

i. $x^2 = 2x$. **2**

ii. $3x^2 - 2x - 5 = 0$. **2**

iii. $2^x = \dfrac{1}{8}$. **2**

(b) Expand and simplify:

i. $(2m + n)(3m + 2n)$. **1**

ii. $3b + 2a - 7c - (2c + a + b)$. **1**

(c) Simplify fully:

i. $3a^2b \times 6a^3b^4$. **1**

ii. $\dfrac{30x^3y^2}{(6x^2)^3}$. **1**

(d) Given $v^2 = u^2 - 2as$, find v correct to 3 significant figures if $u = 31.6$, $a = 9.8$ and $s = 40$. **2**

(e) Factorise the following:

i. $k^2 - 7k + 10$. **2**

ii. $27k^3 - 8t^3$. **2**

iii. $6d^2 + 5d - 21$. **2**

(f) Simplify:

i. $\dfrac{1}{a} + \dfrac{c}{b}$. **1**

ii. $\dfrac{2}{x} - \dfrac{3}{x(x+4)}$. **2**

iii. $\dfrac{(x^2 - 4)(3x + 1)}{(9x^2 - 1)(x + 2)}$. **2**

(g) Solve simultaneously: $\begin{cases} y = 11x - 28 \\ y = x^2 - 4 \end{cases}$ **2**

(h) Solve:

i. $|3x + 2| < x - 3$ **3**

ii. $\dfrac{2}{x - 1} \leq 3$ **3**

Question 4 (19 Marks) Commence a NEW page. **Marks**

(a) On separate sets of axes, sketch the functions $y = |x|$ and $y = \dfrac{x^2}{|x|}$. Explain the difference between them. **3**

(b) Is $f(x) = x^3 - x$ an even or odd function? Explain your answer. **2**

(c) A function is defined as follows:

$$f(x) = \begin{cases} x+2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 2 \\ -2 & \text{if } x > 2 \end{cases}$$

i. Find $f(-2) + f(3) + f\left(\sqrt{3}\right)$. **1**

ii. Sketch the graph of this function. **3**

(d) Find the value of a for this continuous, piecewise defined function: **1**

$$f(x) = \begin{cases} 2 - x & \text{if } x < 2 \\ x^2 + a & \text{if } x \ge 2 \end{cases}$$

(e) Sketch the graph of $f(x) = \dfrac{x+4}{x-1}$, showing all important features. **4**

(f) i. Sketch the graph of $y = x^2 - 6$, labelling all intercepts. **1**

ii. On the same set of axes, carefully sketch the graph of $y = |x|$. **1**

iii. Find the x coordinates of the two points where the graphs intersect. **2**

iv. Hence solve the inequality $x^2 - 6 \le |x|$. **1**

End of paper.

Question 1

(a) (3 marks) – (ii) & (iii) are functions.

(b) i. (1 mark)

$$g(4) = \sqrt{4} + 3 \times 4 = \boxed{14}$$

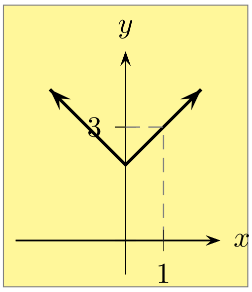
ii. (1 mark)

$$g(x^2) = \sqrt{x^2} + 3x^2 = \boxed{x + 3x^2}$$

(c) i. (2 marks)

✓ [1] for shape

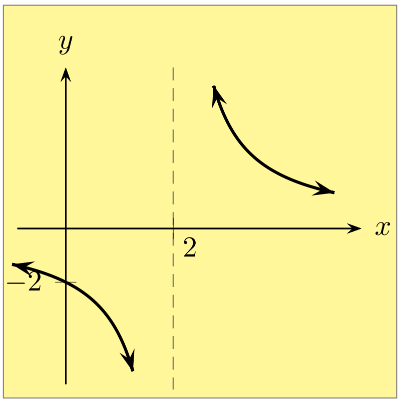
✓ [1] for showing 2 points.



ii. (2 marks)

✓ [1] for shape

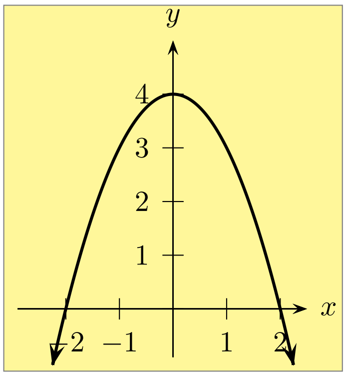
✓ [1] for asymptotes.



iii. (2 marks)

✓ [1] for shape

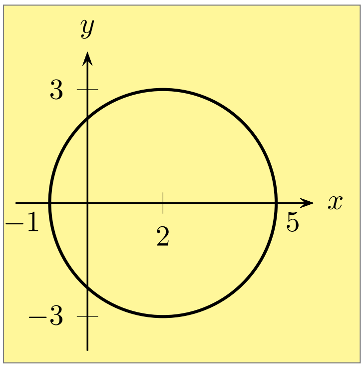
✓ [1] for intercepts.



iv. (2 marks)

✓ [1] for shape.

✓ [1] for centre.

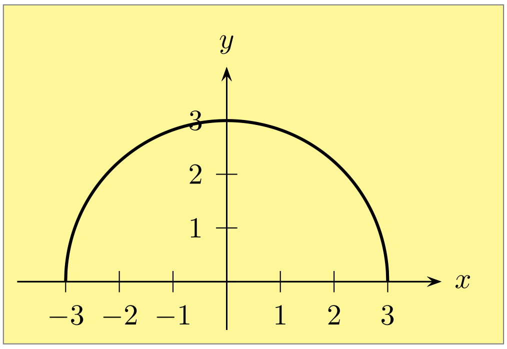


(d) i. (2 marks)

$$\boxed{D = \{x : -3 \le x \le 3\}}$$

$$\boxed{R = \{y : 0 \le y \le 3\}}$$

ii. (1 mark)



(e) (1 mark)

$$f(x) = \sqrt{x-4} + \sqrt{7-x}$$

Domain for $y = \sqrt{x-4}$:

$$x - 4 \ge 0$$
$$\therefore x \ge 4$$

Domain for $y = \sqrt{7-x}$:

$$y = \sqrt{7-x} = \sqrt{-(x-7)}$$
$$-(x-7) \ge 0$$
$$x - 7 \le 0$$
$$x \le 7$$

Hence the domain of $y = \sqrt{x-4} + \sqrt{7-x}$ is

$$\boxed{D = \{x : 4 \le x \le 7\}}$$

Question 2

(a) (2 marks)

✓ [1] for $1000x$.

✓ [1] for final answer.

$$\begin{aligned} x &= 0.37\overline{878} \\ 10x &= 3.787\overline{878} \\ 1000x &= 378.7\overline{878} \\ \therefore 990x &= 375 \\ \Aboxed{x &= \frac{375}{990} = \frac{25}{66}} \end{aligned}$$

(b) (1 mark)

$$\begin{aligned} 2\sqrt{6} &= \sqrt{24} \\ \therefore \ & \boxed{x = 24} \end{aligned}$$

(c) (1 mark)

$$\begin{aligned} \sqrt{18} + \sqrt{8} - \sqrt{2} &= 3\sqrt{2} + 2\sqrt{2} - \sqrt{2} \\ &= \boxed{4\sqrt{2}} \end{aligned}$$

(d) i. (2 marks)

✓ [1] for expanding correctly.

✓ [1] for simplifying expanded answer.

$$\begin{aligned} \left(2\sqrt{3} + \sqrt{5}\right)^2 &= 12 + 5 + 4\sqrt{15} \\ &= \boxed{17 + 4\sqrt{15}} \end{aligned}$$

ii. (1 mark)

$$\left(\sqrt{7} - 3\right)\left(\sqrt{7} + 3\right) = 7 - 9 = \boxed{-2}$$

(e) i. (1 mark)

$$\frac{1}{2\sqrt{3}} {}^{\times\sqrt{3}}_{\times\sqrt{3}} = \boxed{\frac{\sqrt{3}}{6}}$$

ii. (2 marks)

✓ [1] for multiplying by conjugate of denominator in numerator & denominator.

✓ [1] for final answer.

$$\begin{aligned} \frac{3}{2+\sqrt{5}} {}^{\times 2-\sqrt{5}}_{\times 2-\sqrt{5}} &= \frac{6 - 3\sqrt{5}}{4 - 5} \\ &= \boxed{3\sqrt{5} - 6} \end{aligned}$$

Question 3

(a) i. (2 marks)

✓ [1] for changing subject to "0".

✓ [1] for final solutions.

$$\begin{aligned} x^2 &= 2x \\ x^2 - 2x &= 0 \\ x(x-2) &= 0 \\ \therefore \ & \boxed{x = 0, 2} \end{aligned}$$

ii. (2 marks)

✓ [1] for correct factorisation.

✓ [1] for final solutions.

$$\begin{aligned} 3x^2 - 2x - 5 &= 0 \\ (3x-5)(x+1) &= 0 \\ \therefore \ & \boxed{x = \frac{5}{3}, -1} \end{aligned}$$

iii. (2 marks)

✓ [1] for identifying $\frac{1}{8} = 2^{-3}$.

✓ [1] for final solution.

$$\begin{aligned} 2^x &= \frac{1}{8} = 2^{-3} \\ \therefore \ & \boxed{x = -3} \end{aligned}$$

(b) i. (1 mark)

$$(2m+n)(3m+2n) = \boxed{6m^2 + 7mn + 2n^2}$$

ii. (1 mark)

$$\begin{aligned} & 3b + 2a - 7c - (2c + a + b) \\ &= 3b + 2a - 7c - 2c - a - b \\ &= \boxed{a + 2b - 9c} \end{aligned}$$

(c) i. (1 mark)

$$3a^2b \times 6a^3b^4 = \boxed{18a^5b^5}$$

ii. (1 mark)

$$\frac{30x^3y^2}{(6x^2)^3} = \frac{\cancel{6} \times 5 \times \cancel{x^3}y^2}{6^2 \times \cancel{6} \times \cancel{x^6}_{x^3}} = \boxed{\frac{5y^2}{36x^3}}$$

(d) (2 marks)

✓ [1] for $v^2 = 214.56$.

✓ [1] for v to 3 s.f.

$$\begin{aligned} v^2 &= u^2 - 2as \\ v^2 &= (31.6)^2 - 2 \times 9.8 \times 40 \\ &= 214.56 \\ \therefore\ & \boxed{v = 14.6 \text{ (3 s.f.)}} \end{aligned}$$

(e) i. (2 marks)

$$k^2 - 7k + 10 = \boxed{(k-5)(k-2)}$$

ii. (2 marks)

$$27k^3 - 8t^3 = \boxed{(3k-2t)(9k^2 + 6kt + 4t^2)}$$

iii. (2 marks)

$$6d^2 + 5d - 21 = \boxed{(3d+7)(2d-3)}$$

(f) i. (1 mark)

$$\frac{1}{a} + \frac{c}{b} = \boxed{\frac{b+ac}{ab}}$$

ii. (2 marks)

✓ [1] for creating a common denominator.

✓ [1] for final answer.

$$\begin{aligned} \frac{2}{x} - \frac{3}{x(x+4)} &= \frac{2(x+4)}{x(x+4)} - \frac{3}{x(x+4)} \\ &= \frac{2x+8-3}{x(x+4)} \\ &= \boxed{\frac{2x+5}{x(x+4)}} \end{aligned}$$

iii. (2 marks)

✓ [1] for factorising fully.

✓ [1] for final answer.

$$\begin{aligned} &\frac{(x^2-4)(3x+1)}{(9x^2-1)(x+2)} \\ &= \frac{\cancel{(x+2)}(x-2)\cancel{(3x+1)}}{(3x-1)\cancel{(3x+1)}\cancel{(x+2)}} \\ &= \boxed{\frac{x-2}{3x-1}} \end{aligned}$$

(g) (2 marks)

$$\begin{cases} y = 11x - 28 \\ y = x^2 - 4 \end{cases}$$

Equating,

$$\begin{aligned} x^2 - 4 &= 11x - 28 \\ x^2 - 11x + 24 &= 0 \\ (x-3)(x-8) &= 0 \\ \therefore x &= 3, 8 \\ \therefore y &= 5, 60 \end{aligned}$$

Hence the point of intersection is $\boxed{(3,5) \text{ and } (8,60)}$.

(h) i. (3 marks)

✓ [2] for finding x intercepts $-\frac{5}{2}$ and $\frac{1}{4}$.

✓ [1] for final solution.

$$|3x+2| < x - 3$$

Squaring both sides,

$$\begin{aligned} (3x+2)^2 &< (x-3)^2 \\ (3x+2)^2 - (x-3)^2 &< 0 \\ \big[(3x+2) - (x-3)\big]\big[(3x+2) + (x-3)\big] &< 0 \\ (2x+5)(4x-1) &< 0 \\ \therefore\ \boxed{-\tfrac{5}{2} < x < \tfrac{1}{4}} \end{aligned}$$

Testing for $x = 0$ (which falls within the interval $-\frac{5}{2} < x < \frac{1}{4}$):

$$\begin{aligned} |3(0) + 2| &= 2 \\ 0 - 3 &= -3 \end{aligned}$$

but $2 \not< -3$. Hence there are $\boxed{\text{no real solutions}}$.

Alternatively, examine cases:

- $3x + 2 \geq 0$, i.e. $x \geq -\frac{2}{3}$

$$\begin{aligned} 3x + 2 &< x - 3 \\ 2x &< -5 \\ x &< -\frac{5}{2} \end{aligned}$$

But there is no solution such that $x \geq -\frac{2}{3}$ and $x < -\frac{5}{2}$. Hence no solution to this branch.

- $3x + 2 < 0$, i.e. $x < -\frac{2}{3}$.

$$\begin{aligned} -(3x + 2) &< x - 3 \\ -3x - 2 &< x - 3 \\ 4x &> 1 \\ x &> \frac{1}{4} \end{aligned}$$

Again there is no x that exists such that $x > \frac{1}{4}$ and $x < -\frac{2}{3}$. So no solution to this branch.

Hence there are no real solutions.

ii. (3 marks)

✓ [1] for multiplying by square of denominator on both sides.

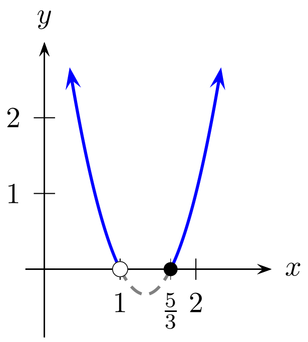
✓ [1] for obtaining $(x-1)(3x-5) \geq 0$.

✓ [1] for final solution.

$$\underset{\times(x-1)^2}{\frac{2}{x-1}} \leq \underset{\times(x-1)^2}{3}$$

Note: $x \neq 1$

$$\begin{aligned} 2(x-1) &\leq 3(x-1)^2 \\ 3(x-1)^2 - 2(x-1) &\geq 0 \\ (x-1)\big(3(x-1) - 2\big) &\geq 0 \\ (x-1)(3x-5) &\geq 0 \end{aligned}$$



As $x \neq 1$,

$$\boxed{x < 1 \text{ or } x \geq \frac{5}{3}}$$

Alternatively,

$$\begin{aligned} \frac{2}{x-1} &\leq 3 \\ \frac{2}{x-1} - \frac{3(x-1)}{x-1} &\leq 0 \\ \frac{2 - 3x + 3}{x-1} &\leq 0 \\ \frac{-3x+5}{x-1} &\leq 0 \end{aligned}$$

Drawing a table of values for $-3x+5$ and $x-1$ and testing values for each of these expressions (values tested are shown in parentheses):

x	(0)	1	(1.1)	$\frac{5}{3}$	(2)
sign of: $-3x+5$	$+$		$+$	0	$-$
sign of: $x-1$	$-$		$+$		$+$
result: $\frac{-3x+5}{x-1}$	$-$		$+$	0	$-$

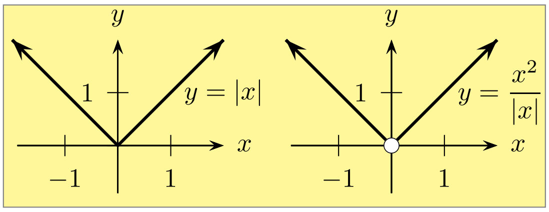
From the table,

$$\frac{-3x+5}{x-1} \leq 0$$

when $x < 1$ or $x \geq \frac{5}{3}$.

Question 4

(a) (3 marks)



The graph of $y = \dfrac{x^2}{|x|}$ is essentially the same as $y = |x|$ but $\boxed{x \neq 0 \text{ for the former}}$.

(b) (2 marks)

$$f(x) = x^3 - x$$

$$f(-x) = (-x)^3 - (-x) = -x^3 + x \qquad \Big| \qquad -f(x) = -(x^3 - x) = -x^3 + x$$

As $\boxed{f(-x) = -f(x), \text{ the function is odd}}$.

(c) i. (1 mark)

$$f(x) = \begin{cases} x + 2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 2 \\ -2 & \text{if } x > 2 \end{cases}$$

$$f(-2) = -2 + 2 = 0$$

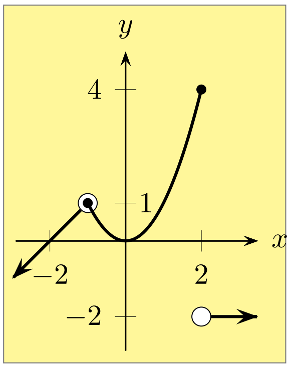
$$f(3) = -2$$

$$f\left(\sqrt{3}\right) = \left(\sqrt{3}\right)^2 = 3$$

$$\begin{aligned} \therefore f(-2) + f(3) + f\left(\sqrt{3}\right) &= 0 - 2 + 3 \\ &= \boxed{1} \end{aligned}$$

ii. (3 marks)

✓ [1] for each piece of the graph.



(d) (1 mark)

$$f(x) = \begin{cases} 2 - x & \text{if } x < 2 \\ x^2 + a & \text{if } x \ge 2 \end{cases}$$

$$f(2^-) = 2 - 2 = 0$$

$$\therefore f(2) = 2^2 + a = 0$$

$$\therefore \boxed{a = -4}$$

(e) (4 marks)

✓ [2] for hyperbola.

✓ [2] for asymptotes.

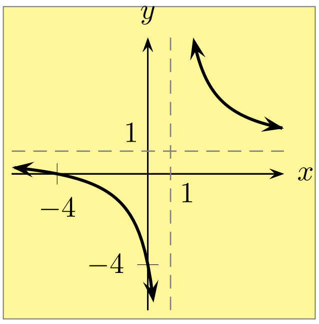
$$\begin{aligned} y &= \frac{x+4}{x-1} = \frac{x-1+5}{x-1} \\ &= \frac{x-1}{x-1} + \frac{5}{x-1} = 1 + \frac{5}{x-1} \end{aligned}$$

Confirming the asymptotes:

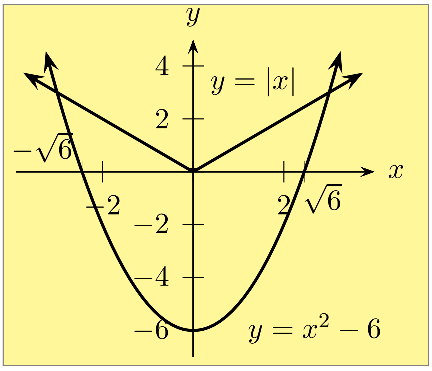
$$\underset{-1}{y} = \underset{-1}{1} + \frac{5}{x-1}$$

$$(y-1)(x-1) = 5$$

$$\therefore y \neq 1 \quad x \neq 1$$



(f) i. (1 mark)



ii. (1 mark) – see above.

iii. (2 marks)

$$x^2 - 6 = |x|$$

Case 1: $x \geq 0$

$$\begin{aligned} x^2 - 6 &= x \\ x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ \therefore \boxed{x = 3 \text{ or } -2} \end{aligned}$$

Testing values:

- $x = -2$:

$$\begin{aligned} |-2| &= 2 \\ (-2)^2 - 6 &\neq 2 \quad ✗ \end{aligned}$$

- $x = 3$:

$$\begin{aligned} |3| &= 3 \\ 3^2 - 6 = 9 - 6 &= 3 \quad ✓ \end{aligned}$$

Case 2: $x < 0$:

$$\begin{aligned} x^2 - 6 &= -x \\ x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ x &= -3, 2 \end{aligned}$$

Testing values:

- $x = -3$:

$$\begin{aligned} |-3| &= 3 \\ (-3)^2 - 6 &= 3 \quad ✓ \end{aligned}$$

- $x = 2$:

$$\begin{aligned} |2| &= 2 \\ 2^2 - 6 = 4 - 6 &= -2 \quad ✗ \end{aligned}$$

$$\therefore \boxed{x = -3, 3}$$

Note it is possible to find the points of intersection graphically.

iv. (1 mark)

$$\boxed{-3 \leq x \leq 3}$$