Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2011 Year 11 Task 3

Mathematics

Extension 1

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Ms Collins
- O Mr Fletcher/Ms Everingham
- O Mr Ireland
- O Mr Lam
- O Mr Weiss

Student Name:___________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total %
Mark	/13	/10	/11	/11	/12	/57	/100

Question 1 (13 marks) Start a new page. **Marks**

(a) Differentiate the following with respect to x:

(i) $y = x\sqrt{x}$ **2**

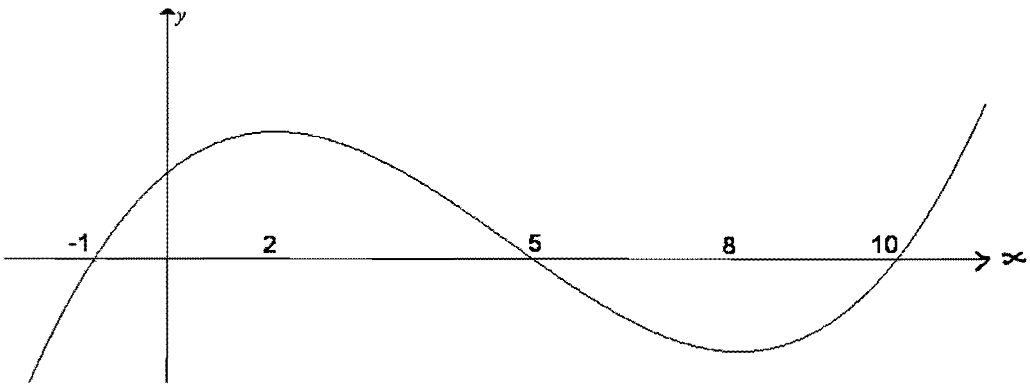
(ii) $y = (2-3x^2)^4$ **2**

(iii) $y = 2x^5(x+3)^9$ **2**

(iv) $y = \dfrac{x^2}{x-2}$ **2**

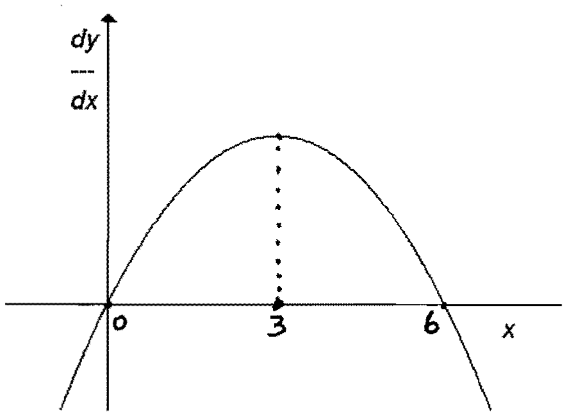
(b) Differentiate from first principles the function $f(x) = x^2 + 5x$ **3**

(c) The graph of $y = f(x)$ is shown below. Draw a sketch of $y = f'(x)$. **2**



Question 2 (10 marks) Start a new page.

(a) The curve $y = f(x)$ has gradient function $\dfrac{dy}{dx}$. The graph of $\dfrac{dy}{dx}$ as a function of x is shown:

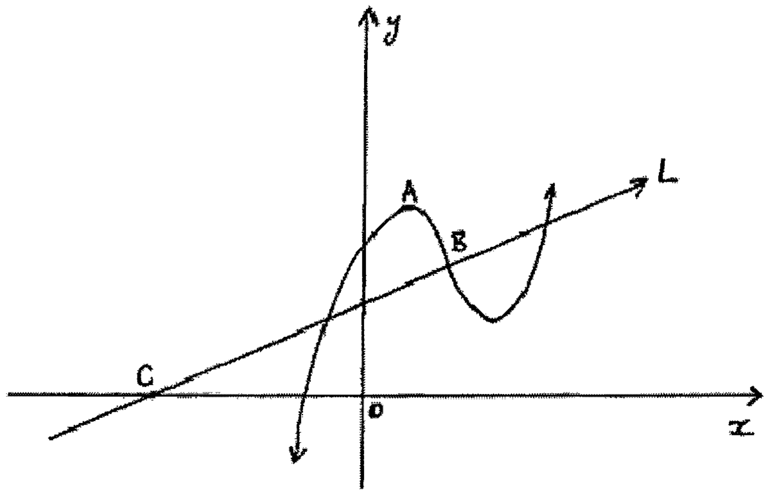


Find where the curve $y = f(x)$

(i) has a maximum turning point; **1**

(ii) is concave down. **1**

(b)



The diagram shows a sketch of $y = x^3 - 6x^2 + 9x + 4$. The curve has a local maximum at *A* and a point of inflexion at *B*. The line *L* is a normal to the curve at point *B*.

(i) Find the coordinates of point *A* and verify it is a maximum. 3

(ii) Find the coordinates of point *B* and verify it is an inflexion. 2

(iii) Find the equation of the normal (line *L*). 3

Question 3 (11 marks) Start a new page.

(a) Evaluate:

(i) $log_4(\frac{1}{\sqrt{2}})$ 2

(ii) $log_6 12$ (to 2 decimal places) 2

(b) Using $log_a 2 = 0{\cdot}387$ and $log_a 3 = 0{\cdot}613$,

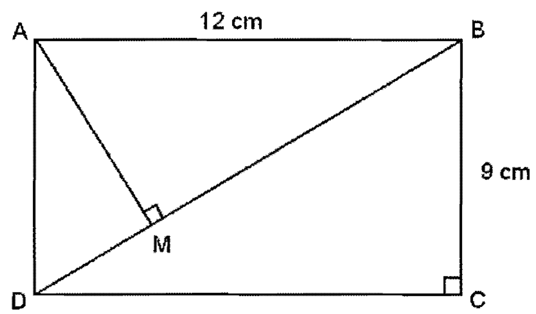
find the value of $log_a 12$. 2

(c) Sketch $y = log_2(2x - 2)$ 2

(d) Solve the equation $log_2 x + log_2(x + 7) = 3$ 3

Question 4 (11 marks) Start a new page.

(a)

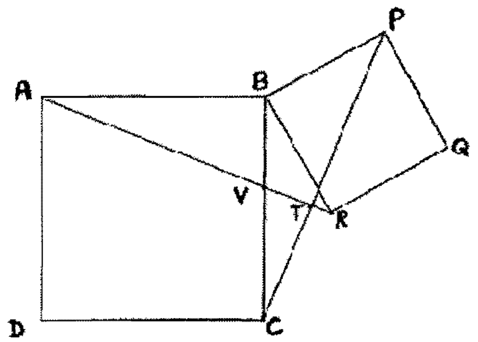


ABCD is a rectangle. *AB* = 12 cm, *BC* = 9 cm, and *AM* is perpendicular to *BD*. Copy the diagram to your answer sheet.

(i) Prove $\Delta ABM \; ||| \; \Delta DBA$ 2

(ii) Hence find the length of *AM*. 3

(b) In the diagram, $ABCD$ and $BPQR$ are squares. AR intersects PC at T, and BC at V.



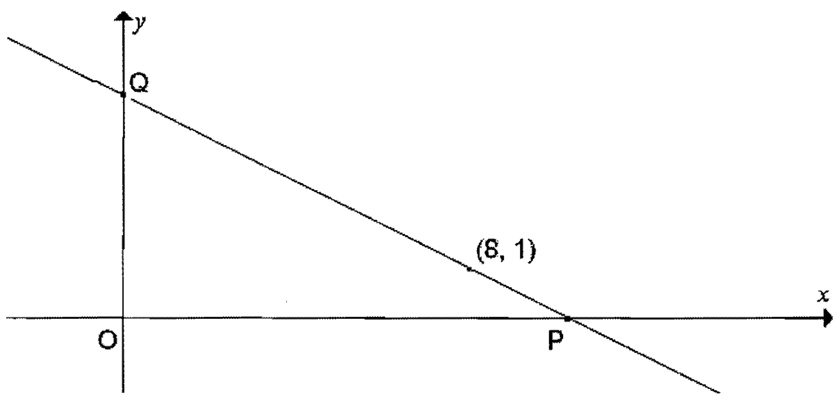
Copy the diagram to your answer sheet.

(i) Prove that $\Delta ABR \equiv \Delta CBP$ **3**

(ii) Use this to prove $AR \perp PC$ **3**

Question 5 (12 marks) Start a new page.

(a) A line is drawn through the point (8,1) to cut the positive x-axis at P and the positive y-axis at Q. The gradient of PQ is m.



(i) Find the equation of PQ in terms of m. **1**

(ii) Show that the area of ΔOPQ is $A = \frac{1}{2}(16 - 64m - \frac{1}{m})$ **3**

(iii) Find the minimum area of ΔOPQ **4**

(b) A particle moves along the x-axis so that its displacement x, in metres, is given by $x = 4 + 6t - t^2$ where t is the time in seconds.

(i) Express the particle's velocity, v, as a function of time, t. **1**

(ii) When is the particle at rest? **1**

(iii) Find the total distance it moves in the first 7 seconds. **2**

END OF EXAMINATION

Q1 (a)

(i) $y = x\sqrt{x}$

$= x^{\frac{3}{2}}$ ✓ (e.g. uses indices)

$\therefore \frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$ ✓ correct answer

(ii) $y = (2-3x^2)^4$

$\therefore \frac{dy}{dx} = 4 \cdot (2-3x^2)^3 \cdot -6x$

$= -24x\,(2-3x^2)^3$ ✓✓ uses chain rule correctly

(iii) $y = 2x^5(x+3)^9$

$\therefore \frac{dy}{dx} = (x+3)^9 \cdot 10x^4 + 2x^5 \cdot 9(x+3)^8$ ✓✓ correct use of product rule

$= 10x^4(x+3)^9 + 18x^5(x+3)^8$

(iv) $y = \frac{x^2}{x-2}$

$\therefore \frac{dy}{dx} = \frac{(x-2)\cdot 2x - x^2 \cdot 1}{(x-2)^2}$ ✓✓ correct use of the quotient rule

$= \frac{x^2 - 4x}{(x-2)^2}$

(b) $f(x) = x^2 + 5x$

$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ✓ correct start

$= \lim_{h\to 0} \frac{(x+h)^2 + 5(x+h) - [x^2+5x]}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{5x} + 5h - \cancel{x^2} - \cancel{5x}}{h}$ ✓ correct working

$= \lim_{h\to 0} (2x + h + 5) = 2x+5$ ✓ correct answer

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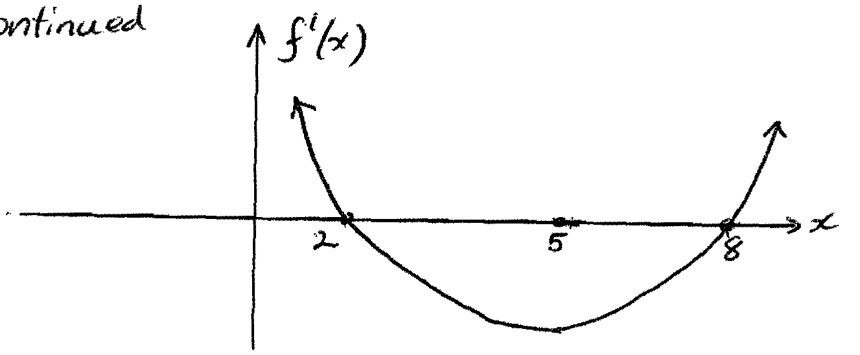
Q1 - continued

(c)



✓ correct shape

✓ both x-intercepts

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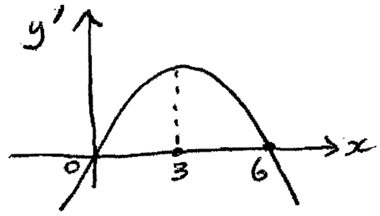
Q2 (a)



(i) Turning points when $y' = 0$ $\therefore$ at $x = 0, 6$

<u>Test</u>:

x	$-\varepsilon$	0	$+\varepsilon$
y'	$-$	0	$+$
	\	—	/

x	$6-\varepsilon$	6	$6+\varepsilon$
y'	$+$	0	$-$
	/	—	\

$\therefore$ max. turning point at $x = 6$ ✓ for $x=6$

(ii) Concave down when $\dfrac{d^2y}{dx^2} < 0$

i.e. when $\dfrac{dy}{dx}$ is decreasing.

$\therefore$ concave down when $x > 3$ ✓ for $x>3$

(b) $y = x^3 - 6x^2 + 9x + 4$

(i) $y' = 3x^2 - 12x + 9$ $\qquad y'' = 6x - 12$

$= 3(x^2 - 4x + 3)$ $\qquad = 6(x-2)$

$= 3(x-3)(x-1)$ ✓ correct derivative

$\therefore$ Turning points when $y' = 0$ $\therefore$ at $x = 1, 3$.

$\begin{cases} y''(1) = 6(1-2) < 0 \\ y''(3) > 0 \end{cases}$ $\therefore$ max. turning point at $x = 1$. ✓ testing performed

Thus $A = (1, 8)$ ✓ coordinates of A

Q2 - continued

(b) (ii) Possible inflexions when $y''=0$

$\therefore 6(x-2)=0 \quad \therefore x=2$

Test:

x	1.9	2	2.1
y''	$-$	0	$+$

Changes concavity $\therefore$ inflexion at $x=2$.

Thus $B=(2,6)$

✓ correct inflexion point (x-value)

✓ Tests for change in concavity.

(iii) $y' = 3(x-3)(x-1)$

$\therefore y'(2) = 3(-1)(1) = -3$

$\therefore m_T$ at B is -3

$\therefore m_N$ at B is $\frac{1}{3}$.

Thus normal is $y-6 = \frac{1}{3}(x-2)$

i.e. $3y-18 = x-2$

$\therefore \quad x-3y+16=0$

(or $y=\frac{1}{3}x+\frac{16}{3}$)

✓ Correct m_N

✓✓ Correct equation (using correct B coordinates)

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Q3 (a)

(i) $\log_4\left(\frac{1}{\sqrt{2}}\right) = \log_4\left(2^{-\frac{1}{2}}\right)$

$= \log_4\left(\left[4^{\frac{1}{2}}\right]^{-\frac{1}{2}}\right)$

$= \log_4\left(4^{-\frac{1}{4}}\right)$

$= -\frac{1}{4}$

✓ correct start

✓ correct answer

(ii) $\log_6 12 = \dfrac{\log_{10} 12}{\log_{10} 6}$

$\doteqdot 1.39 \quad (2\text{ d.p.})$

✓ Change of base formula

✓ correct answer (don't penalise d.p.'s)

$\boxed{Q3}$ - continued.

(b) $\log_a 12 = \log_a(2^2 \cdot 3)$

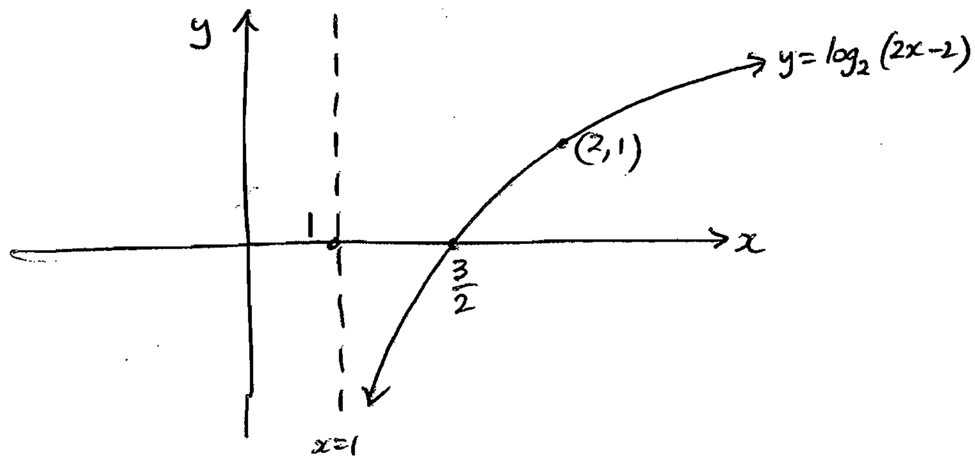
$= 2\log_a 2 + \log_a 3$ ✓

$= 1.387$ ✓

(c) $y = \log_2(2x-2)$

$= \log_2 2[x-1]$

[Note: curve is $\log_2 x$ shifted to right & raised up.
Thus vertical asymptote at $x=1$.
Also, curve $=0$ when $2x-2=1$ $\therefore x=\frac{3}{2}$ is x-int.]



✓ correct shape & asymptote

✓ correct x-int. OR other point on curve.

(d) $\log_2 x + \log_2(x+7) = 3$

$\therefore \log_2 x(x+7) = 3$

$\therefore x(x+7) = 2^3 = 8$ ✓ uses log laws correctly

$\therefore x^2+7x-8=0$

$(x+8)(x-1)=0$

$x=-8$ or $x=1$ ✓ gets $-8, 1$

But $x>0$, $\therefore$ solution is $x=1$ ✓ correct solution

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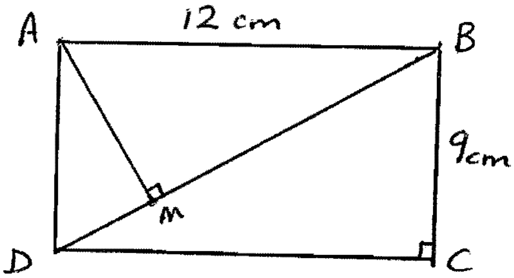
Q4 (a)



(i) In ΔABM, ΔDBA,

$\angle ABM = \angle DBA$ (common angle)

$\angle AMB = 90° = \angle DAB$ (angle of rectangle)

✓ correct reasoning

$\therefore \Delta ABM \, ||| \, \Delta DBA$ (equiangular)

✓ correct similarity condition [* no abbrev.]

(iii) $\dfrac{AM}{AD} = \dfrac{AB}{DB}$ (corresp. sides in proportion in similar Δ's)

$\therefore AM = \dfrac{AD \cdot AB}{DB} = \dfrac{(9)(12)}{DB}$

✓ correct proportion statement / equation

But $DB = \sqrt{9^2+12^2} = 15$

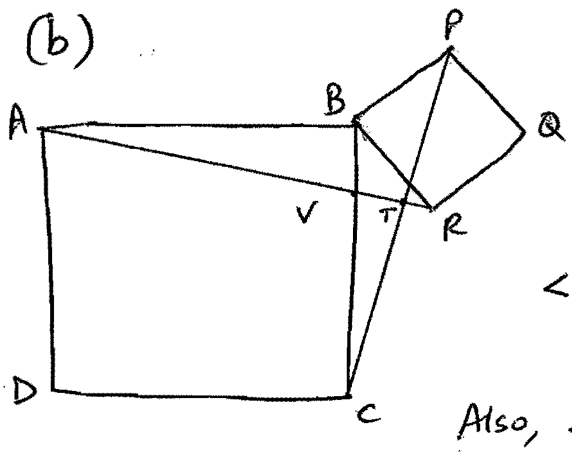
✓ correct DB

$\therefore AM = \dfrac{108}{15}$

$= 7.2$

✓ correct answer

(b)



(i) In ΔABR, ΔCBP,

$AB = CB$ (sides of square ABCD)

$BR = BP$ (sides of square BPQR)

✓ correct sides info.

$\angle ABR = \angle ABC + \angle CBR$

$= 90° + \angle CBR$ ($\angle$ of square)

Also, $\angle CBP = \angle RBP + \angle CBR$

$= 90° + \angle CBR$

$= \angle ABR$

✓ correct angle info.

$\therefore \Delta ABR \equiv \Delta CBP$ (SAS)

✓ (correct (* reasons are needed for full marks)

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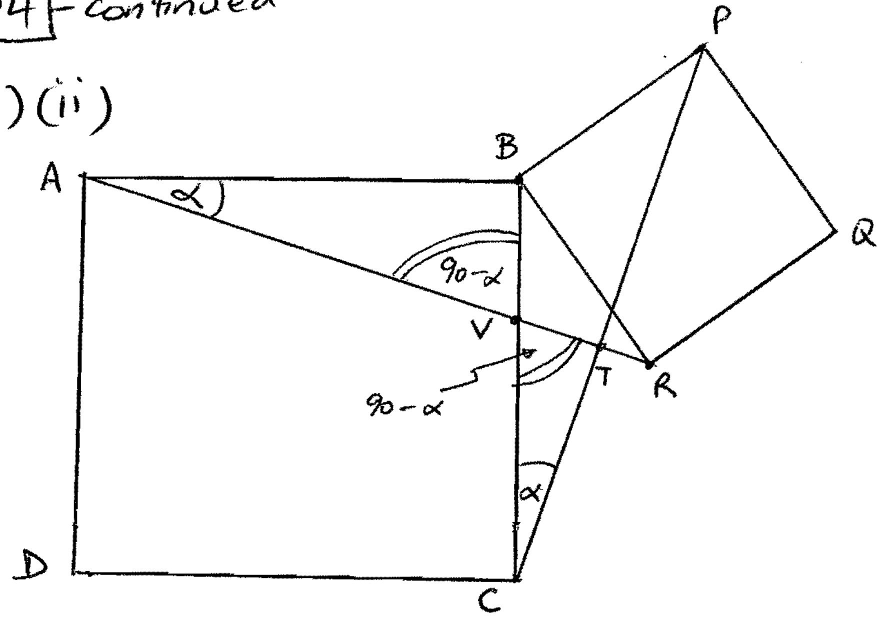
Q4 – continued

(b)(ii)



Let $\angle BAV = \alpha$.

Then $\angle BCP = \alpha$ [corresp. angles in congruent Δ's]

✓ uses the congruence correctly [mandatory]

Then $\angle BVA = 90^\circ - \alpha$ [angle sum in ΔABV]

$\therefore \angle CVT = 90^\circ - \alpha$ [vert. opposite $\angle$'s are equal]

Thus $\angle VTC = 180^\circ - \alpha - (90^\circ - \alpha)$ [angle sum in ΔVTC]

$= 90^\circ$

i.e. $AR \perp PC$

✓✓ correct argument with reasons

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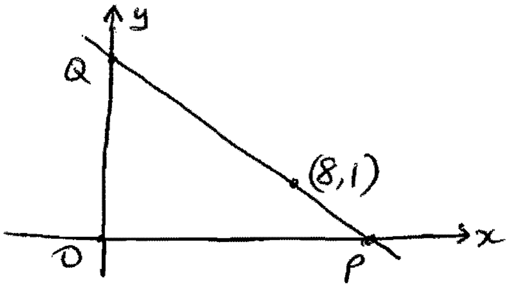
Q5 (a)



(i) $y-1 = m(x-8)$

✓ correct answer

(ii) At Q, $x=0$ $\therefore$ $y-1 = m(-8)$

$y = -8m+1$

✓ correct Distance OQ

At P, $y=0$ $\therefore$ $-1 = mx - 8m$

$x = 8 - \frac{1}{m}$

✓ correct Distance OP

$\therefore$ Area $\Delta OPQ = \frac{1}{2} \cdot (1-8m)(8-\frac{1}{m})$

$= \frac{1}{2}\left(8 - \frac{1}{m} - 64m + 8\right)$

$\therefore A = \frac{1}{2}\left(16 - 64m - \frac{1}{m}\right)$

✓ Derives A correctly

(iii) $\frac{dA}{dm} = \frac{1}{2}\left[-64 + \frac{1}{m^2}\right]$

✓ correct $\frac{dA}{dm}$

For minimum, $\frac{dA}{dm} = 0$ $\therefore$ $\frac{1}{m^2} = 64$

$\therefore$ $m = \pm\frac{1}{8}$

But $m<0$ $\therefore$ $m = -\frac{1}{8}$

✓ for $m = -\frac{1}{8}$

Test: $\frac{d^2A}{dm^2} = \frac{1}{2}\left[\frac{-2}{m^3}\right] = \frac{-1}{m^3}$

$\therefore \quad = -\frac{1}{(\frac{-1}{8})^3} > 0$

$\therefore$ $m = -\frac{1}{8}$ represent a minimum area.

✓ Tests it's a minimum

The minimum area is thus

$A = \frac{1}{2}\left[16 - 64(-\frac{1}{8}) - \frac{1}{-\frac{1}{8}}\right]$

$= \frac{1}{2}\left[16+8+8\right] = 16$ units^2.

✓ correct minimum area

Q5 – continued

(b) $x = 4 + 6t - t^2$

(i) $v = \frac{dx}{dt} = 6 - 2t$ ✓

(ii) "at rest" $\Rightarrow v = 0$

$\therefore 6 - 2t = 0$

$t = 3$ seconds. ✓

(iii) At $t = 0$, $x = 4$

At $t = 3$, $x = 13$

At $t = 7$, $x = -3$

[The particle starts at $x = 4$, moves right until it comes to rest momentarily at $x = 13$, then moves left to reach $x = -3$ after 7 seconds.]

$\therefore$ Total distance $= (13 - 4) + 13 + 3$ ✓ correct working

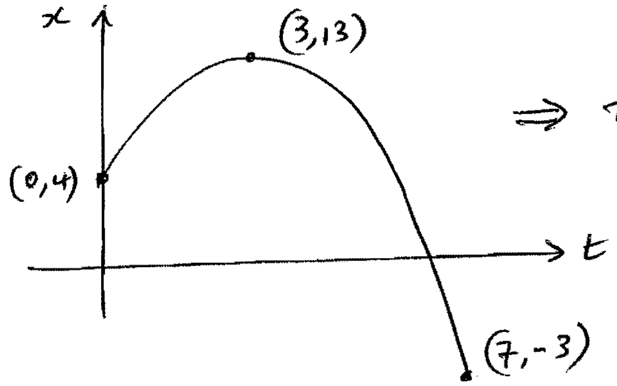
$= 25$ metres ✓ correct answer.

[Alternatively,

$x = -(t^2 - 6t - 4)$

$= -(t^2 - 6t + 9 - 13)$

$\therefore x = -(t-3)^2 + 13$



$\Rightarrow$ Total distance $= 9 + 16 = 25$ m.]

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