100 Years
NORTH SYDNEY BOYS HIGH SCHOOL
1912 - 2012

# MATHEMATICS (EXTENSION 1)

Preliminary Course Assessment Task 2

April 4, 2012

**General instructions**

- Working time – 1 hour and 15 minutes.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.

**SECTION I**

- Mark your answers on the answer sheet provided (numbered as page 9)

**SECTION II**

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

**NAME:** ..... **# BOOKLETS USED**: .....

**Class** (please ✔)

- ◯ 11M3A – Mr Lin/Mr Rezcallah
- ◯ 11M3B – Mr Fletcher
- ◯ 11M3C – Ms Ziaziaris
- ◯ 11M3D – Mr Lucas
- ◯ 11M3E – Mr Lam
- ◯ 11M3F – Mr Berry

Marker's use only.

| **QUESTION(S)** | 1-10 | 11 | 12 | 13 | 14 | 15 | 16 | **Total** | % |
|---|---|---|---|---|---|---|---|---|---|
| **MARKS** | /10 | /15 | /15 | /15 | /12 | /12 | /12 | /91 | |

## Section I: Objective response

Mark your answers on the multiple choice sheet (numbered as page 9) provided. **Marks**

**1.** If $y = |x + 4|$, then $y = 0$ for **1**

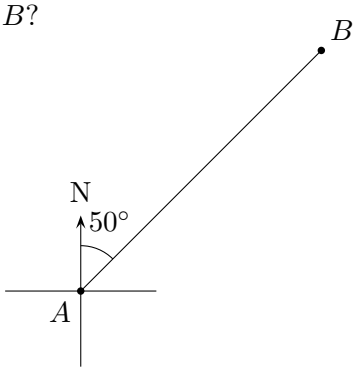
(A) $x = -4$ (C) $x = 4$

(B) $x = 0$ (D) no real values of $x$

**2.** If $\sin\theta = \frac{1}{2}$ and $90^\circ \leq \theta \leq 180^\circ$, then $\cos\theta =$ **1**

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

**3.** The bearing of Town $B$ from Town $A$ is 050°. What is the bearing of Town $A$ from Town $B$? **1**




(A) 050°

(B) 040°

(C) 220°

(D) 230°

**4.** $(x + 1)(x - 3)(x - 5) > 0$ if **1**

(A) $-1 < x < 3$ only (C) $-1 < x < 3$ or $x > 5$

(B) $x < -1$ or $3 < x < 5$ (D) $3 < x < 5$ only

**5.** The linear function $f(x) = 6 - 2x$ has range $R = \{y : -4 \leq y \leq 12\}$. **1**

The domain is thus

(A) $D = \{x : x \in \mathbb{R}\}$ (C) $D = \{x : -3 \leq x \leq 5\}$

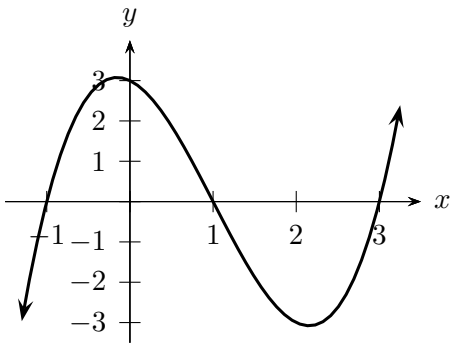
(B) $D = \{x : -5 \leq x \leq 3\}$ (D) none of the above

**6.** $f(x - 2) + 4$ shifts the original curve of $f(x)$ **1**

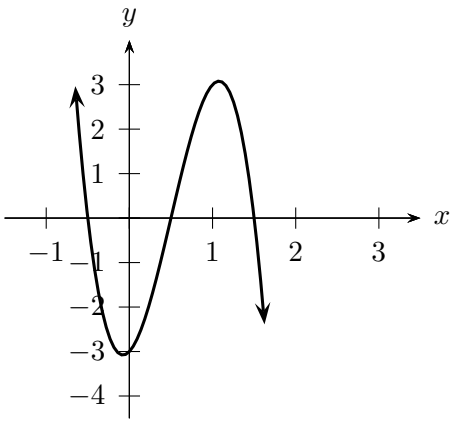
(A) $\leftarrow$ 2 units and $\downarrow$ 4 units (C) $\rightarrow$ 2 units and $\downarrow$ 4 units

(B) $\leftarrow$ 2 units and $\uparrow$ 4 units (D) $\rightarrow$ 2 units and $\uparrow$ 4 units

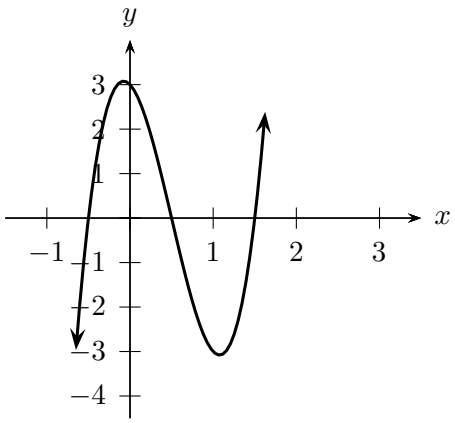
**7.** The graph of $y = f(x)$ is shown. The graph which best represents $1 - f(2x)$ is **1**



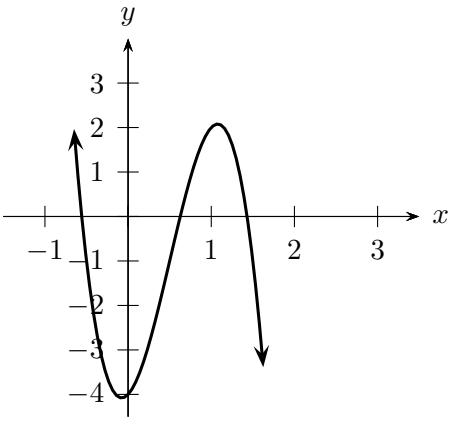

(A)



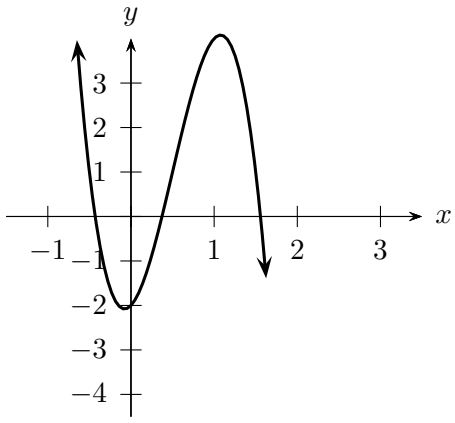

(C)



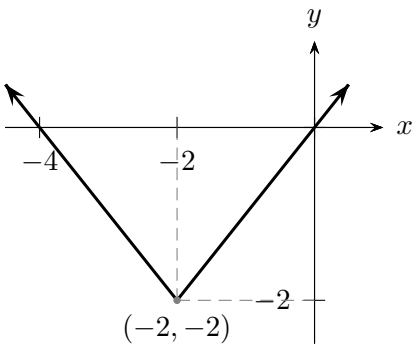

(B)




(D)



**8.** The rule that defines the function shown in the diagram is **1**




(A) $y = |x - 2| - 2$

(B) $y = |x - 2| + 2$

(C) $y = |x + 2| - 2$

(D) $y = |x + 2| + 2$

**9.** The equation of the vertical asymptotes to $f(x) = \dfrac{x^2 - 4}{x^2 + 2x - 3}$ are **1**

(A) $x = -1$, $x = 3$

(B) $x = 1$, $x = -3$

(C) $x = -1$, $x = -3$

(D) $x = 1$, $x = 3$

**10.** A circle with centre $(a, -2)$ and radius 5 has the equation **1**

$$x^2 - 6x + y^2 + 4y = b$$

where $a$ and $b \in \mathbb{R}$. The value of $a$ and $b$ are respectively:

(A) $-3$ and 38

(B) 3 and 12

(C) $-3$ and $-8$

(D) $-3$ and 0

**Examination continues overleaf...**

## Section II: Short answer

**Question 11** (15 Marks) Commence a NEW page. **Marks**

(a) Simplify $|3| - |-4| + |3 - 4|$ **1**

(b) For the formula $a = \sqrt{\dfrac{2V^2 - 1}{y}}$, find $a$ correct to 4 significant figures given $V = 640$ and $y = 9.65$. **2**

(c) Solve $7x^2 - 4x - 1 = 0$, giving your answer in exact form. **2**

(d) Express $0.1\overline{234}$ as a fraction in simplest form. **3**

(e) If $p = 38.6 \times 10^{-5}$ and $q = 471.3 \times 10^{-8}$, find the value of $pq$ in scientific notation correct to 2 decimal places. **2**

(f) Factorise $8t^3 - 1\,000$ fully. **2**

(g) Solve $\left|\dfrac{x+1}{3} - \dfrac{x+2}{4}\right| = 5$ **3**

**Question 12** (15 Marks) Commence a NEW page. **Marks**

(a) Factorise fully:

i. $4a^2 - 9(b+2)^2$ **3**

ii. $t^3 + t^2 - t - 1$ **4**

(b) Solve for $x$:

i. $x^2 < 3x$ **2**

ii. $|2x - 3| > 1$ **3**

iii. $8^{2x-1} = 16^{x+1} \times 4^{-x}$ **3**

**Question 13** (15 Marks) Commence a NEW page. **Marks**

(a) Expand $\left(5-2\sqrt{3}\right)^2$. **2**

(b) Rewrite in simplest form, with a rational denominator: **2**

$$\frac{1}{2\sqrt{3}-1}+\frac{3}{\sqrt{3}+1}$$

(c) Simplify $\dfrac{p^2+2p}{3p^2+5p-2}\div\dfrac{9p+3}{9p^2-1}$ fully. **3**

(d) Solve for $a$ and $b$ if **4**

$$a+b+2\sqrt{3}=4+\sqrt{2a-2b}$$

(e) Solve the inequation $\dfrac{1}{1-x}\leq\dfrac{1}{1+x}$. **4**

**Question 14** (12 Marks) Commence a NEW page. **Marks**

(a) Write the exact value of

i. $\cos 120^\circ$. **2**

ii. $\operatorname{cosec} 675^\circ$ **3**

(b) Solve for $x$: $\sin 4x=\cos(6x+50)$. **2**

(c) Show that $\tan x-\tan x\sin^2 x=\sin x\cos x$. **2**

(d) Solve for $\theta$, where $0^\circ\leq\theta\leq 360^\circ$: **3**

$$2\cos^2\theta+3\cos\theta-2=0$$

**Question 15** (12 Marks) Commence a NEW page. **Marks**

(a) In $\triangle PQR$, $PQ = 20\,\text{cm}$, $QR = 22\,\text{cm}$ and $\angle PRQ = 15°$. Find the value(s) for $\angle RPQ$ and $PR$. **4**

(b) $ABCD$ is a parallelogram with $AB = 8\,\text{cm}$, $BC = 5\,\text{cm}$ and $\angle DAB = 60°$. Calculate the exact area of the parallelogram. **3**

(c) A passenger on a small cruise ship observed, whilst passing under the Harbour Bridge, that the angle of elevation to the pylons are 47° and 28°. It is known that the distance between the pylons is 503 m. (Assume the pylons are equal in height and elevation.)

i. By drawing an appropriate diagram, labelling sides and assigning variables, show that the height $h$ of the pylons is given by **3**

$$h = \frac{503 \tan 47° \tan 28°}{\tan 47° + \tan 28°}$$

ii. Hence or otherwise, find the height of the pylon in centimetres, using scientific notation correct to 4 significant figures. **2**

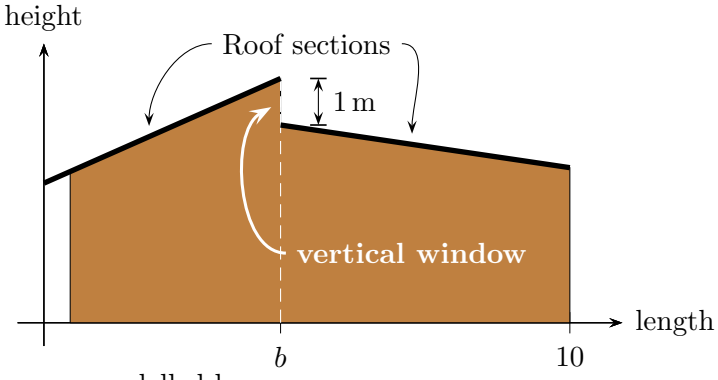
**Examination continues overleaf...**

**Question 16** (12 Marks) Commence a NEW page. **Marks**

(a) Expand and fully simplify **2**

$$(a-5)^2 + (a-4)^2 + \cdots + a^2 + \cdots + (a+4)^2 + (a+5)^2$$

(b) Two roof sections of a building are separated by a vertical window as shown in the diagram. **3**




The roof sections are modelled by

$$\text{height} = \begin{cases} \frac{1}{2} \times \text{length} + 3 & 0 \leq \text{length} < b \\ -\frac{1}{6} \times \text{length} + 5 & b \leq \text{length} \leq 10 \end{cases}$$

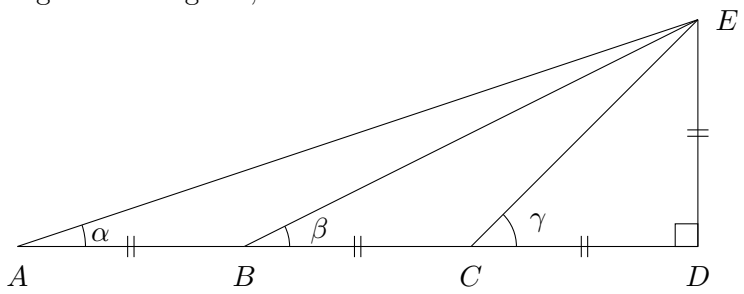
The vertical window has height 1 m. Find the value of $b$.

(c) For the function $f(x) = |x^2 - 9| + 3$

i. State the domain and range. **2**

ii. Sketch the function. **2**

(d) By referring to the diagram,




i. Find the value of $\gamma$. **1**

ii. By using $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$, show that $\alpha + \beta = \gamma$. **2**

**End of paper.**

## Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

**NAME:** .....

**Class** (please ✔)

◯ 11M3A – Mr Lin/Mr Rezcallah

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**1** – Ⓐ Ⓑ Ⓒ Ⓓ

**2** – Ⓐ Ⓑ Ⓒ Ⓓ

**3** – Ⓐ Ⓑ Ⓒ Ⓓ

**4** – Ⓐ Ⓑ Ⓒ Ⓓ

**5** – Ⓐ Ⓑ Ⓒ Ⓓ

**6** – Ⓐ Ⓑ Ⓒ Ⓓ

**7** – Ⓐ Ⓑ Ⓒ Ⓓ

**8** – Ⓐ Ⓑ Ⓒ Ⓓ

**9** – Ⓐ Ⓑ Ⓒ Ⓓ

**10** – Ⓐ Ⓑ Ⓒ Ⓓ

# Suggested Solutions

## Section I

**1.** (A) **2.** (B) **3.** (D) **4.** (C) **5.** (C)
**6.** (D) **7.** (D) **8.** (C) **9.** (B) **10.** (B)

## Section II

**Question 11** (Ziaziaris)

(a) (1 mark)

$$|3| - |-4| + |3-4| = 3 - 4 + 1 = 0$$

(b) (2 marks)

✓ [1] for magnitude.
✓ [1] for 4 significant figures.

$$a = 291.4$$

(c) (2 marks)

✓ [1] for substituting into quadratic formula.
✓ [1] for final answer.

$$\begin{aligned} 7x^2 - 4x - 1 &= 0 \\ x &= \frac{-(-4) \pm \sqrt{4^2 - 4(7)(-1)}}{2 \times 7} \\ &= \frac{4 \pm \sqrt{16+28}}{14} \\ &= \frac{4 \pm \sqrt{44}}{14} \\ &= \frac{2 \pm \sqrt{11}}{7} \end{aligned}$$

(d) (3 marks)

✓ [1] for multiplying by correct power of 10.
✓ [1] for subtraction
✓ [1] for final answer.

$$\begin{aligned} \text{Let } \underset{\times 1\,000}{x} &= \underset{\times 1\,000}{0.1\overline{234}} \quad (1) \\ 1000x &= 123.4\overline{234} \quad (2) \end{aligned}$$

(2) − (1)

$$\begin{aligned} \underset{\div 999}{999x} &= \underset{\div 999}{123.3} \\ \therefore x &= \frac{123.3}{999} = \frac{1\,233}{9\,990} = \frac{137}{1\,110} \end{aligned}$$

(e) (2 marks)

✓ [1] for magnitude.
✓ [1] for correct scientific notation.

$$\left(38.6 \times 10^{-5}\right) \times \left(471.3 \times 10^{-8}\right) = 1.82 \times 10^{-9}$$

(f) (2 marks)

✓ [−1] for each error (including not being fully factorised)

$$\begin{aligned} 8t^3 - 1\,000 &= 8(t^3 - 125) \\ &= 8(t-5)(t^2 + 5t + 25) \end{aligned}$$

(g) (3 marks)

✓ [1] for simplifying fraction inside abs val.
✓ [1] for each correct solution.

$$\begin{aligned} \left|\frac{x+1}{3} - \frac{x+2}{4}\right| &= 5 \\ \left|\frac{4x+4-3x-6}{12}\right| &= 5 \\ \left|\underset{\times 12}{\frac{x-2}{12}}\right| &= \underset{\times 12}{5} \\ \therefore |x-2| &= 60 \\ x - 2 &= \pm 60 \\ \therefore x &= 2 \pm 60 = -58 \text{ or } 62 \end{aligned}$$

**Question 12** (Lucas)

(a) i. (3 marks)

✓ [−1] for each error, including not fully simplified to the last step.

$$\begin{aligned} 4a^2 - 9(b+2)^2 &= (2a - 3(b+2))\,(2a + 3(b+2)) \\ &= (2a - 3b - 6)\,(2a + 3b + 6) \end{aligned}$$

ii. (4 marks)

✓ [−1] for each error, including not fully factorised + simplified to the last step.

$$\begin{aligned} t^3+t^2-t-1 &= t^2(t+1)-(t+1) \\ &= (t+1)(t^2-1) \\ &= (t+1)(t-1)(t+1) \\ &\left(= (t+1)^2(t-1)\right) \end{aligned}$$

(b) i. (2 marks)

✓ [1] for $x(x-3)<0$

✓ [1] for final answer.

✓ [0] if student cancels one of the $x$ terms.

$$\begin{aligned} x^2 &< 3x \\ x^2-3x &< 0 \\ x(x-3) &< 0 \\ \therefore\ 0 &< x < 3 \end{aligned}$$

ii. (3 marks)

✓ [1] for correct cases.

✓ [1] for each correct solution.

$$|2x-3|>1$$

| $2x-3>1$ | $2x-3<-1$ |
|---|---|
| $2x>4$ | $2x<2$ |
| $x>2$ | $x<1$ |

$$\therefore\ x>2 \text{ or } x<1$$

iii. (3 marks)

✓ [1] for converting to base 2

✓ [1] for correct expansion of indices

✓ [1] for final answer.

$$\begin{aligned} 8^{2x-1} &= 16^{x+1}\times 4^{-x} \\ \left(2^3\right)^{2x-1} &= \left(2^4\right)^{x+1}\times\left(2^2\right)^{-x} \\ 2^{6x-3} &= 2^{4x+4}\times 2^{-2x} = 2^{2x+4} \\ 6x-3 &= 2x+4 \\ 4x &= 7 \\ \therefore\ x &= \frac{7}{4} \end{aligned}$$

**Question 13** (Fletcher)

(a) (2 marks)

✓ [−1] for each error.

$$\begin{aligned} \left(5-2\sqrt{3}\right)^2 &= 25-20\sqrt{3}+12 \\ &= 37-20\sqrt{3} \end{aligned}$$

(b) (2 marks)

✓ [−1] for each error.

$$\begin{aligned} &\frac{1}{2\sqrt{3}-1}\overset{\times 2\sqrt{3}+1}{\times 2\sqrt{3}+1}+\frac{3}{\sqrt{3}+1}\overset{\times\sqrt{3}-1}{\times\sqrt{3}-1} \\ &= \frac{2\sqrt{3}+1}{12-1}+\frac{3\sqrt{3}-3}{3-1} \\ &= \frac{2\sqrt{3}+1}{11}\overset{\times 2}{\times 2}+\frac{3\sqrt{3}-3}{2}\overset{\times 11}{\times 11} \\ &= \frac{4\sqrt{3}+2+33\sqrt{3}-33}{22} \\ &= \frac{37\sqrt{3}-31}{22} \end{aligned}$$

(c) (3 marks)

✓ [1] for factorising.

✓ [1] for cancelling.

✓ [1] for final answer.

$$\begin{aligned} &\frac{p^2+2p}{3p^2+5p-2}\div\frac{9p+3}{9p^2-1} \\ &= \frac{\cancel{p(p+2)}}{\cancel{(3p-1)(p+2)}}\times\frac{\cancel{(3p-1)}\cancel{(3p+1)}}{\cancel{3(3p+1)}} \\ &= \frac{p}{3} \end{aligned}$$

(d) (4 marks)

✓ [1] for equating rational & irrational parts.

✓ [1] for setting up simultaneous equations.

✓ [1] for each of $a$ and $b$.

$$a+b+2\sqrt{3}=4+\sqrt{2a-2b}$$

Rational parts:

$$\begin{aligned} a+b &= 4 \\ 2a+2b &= 8 \qquad (13.1) \end{aligned}$$

Irrational parts:

$$\sqrt{2a-2b} = 2\sqrt{3} = \sqrt{12}$$
$$\therefore 2a - 2b = 12 \qquad (13.2)$$

(13.2) − (13.1)

$$-4b = 4$$
$$\therefore b = -1$$
$$\therefore a = 5$$

(e) (4 marks)

✓ [1] for multiplying by square of denominator on each side.

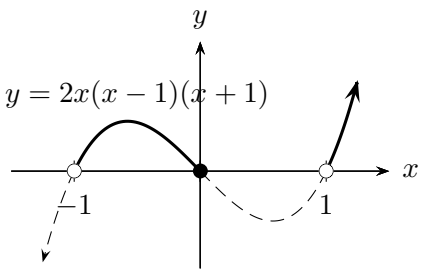
✓ [2] for all critical points. [−1] for each error.

✓ [1] for correct $<$, $\leq$ and $>$ sign.

$$\underset{\times(1-x)^2(1+x)^2}{\frac{1}{1-x}} \leq \underset{\times(1-x)^2(1+x)^2}{\frac{1}{1+x}}$$

Note $x \neq \pm 1$

$$(1-x)(1+x)^2 \leq (1+x)(1-x)^2$$
$$(1+x)(1-x)^2 - (1-x)(1+x)^2 \geq 0$$
$$(1+x)(1-x)\Big((1-x) - (1+x)\Big) \geq 0$$
$$-2x(1+x)(1-x) \geq 0$$
$$2x(x-1)(x+1) \geq 0$$




$$\therefore -1 < x \leq 0 \text{ or } x > 1$$

**Question 14** (Lam)

(a) i. (2 marks)

✓ [1] for correct sign.

✓ [1] for correct magnitude.

$$\cos 120^\circ = -\frac{1}{2}$$

ii. (3 marks)

✓ [1] for subtracting 360°

✓ [1] for correct sign.

✓ [1] for correct magnitude.

$$\operatorname{cosec} 675^\circ = \frac{1}{\sin 315^\circ} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

(b) (2 marks)

✓ [1] for realising the angles are complementary.

✓ [1] for final answer.

$$\sin 4x = \cos(6x + 50)$$
$$4x + (6x + 50) = 90$$
$$10x + 50 = 90$$
$$10x = 40$$
$$x = 4$$

(c) (2 marks)

✓ [−1] for each error.

$$\begin{aligned}&\tan x - \tan x \sin^2 x\\ &= \tan x\left(1 - \sin^2 x\right)\\ &= \frac{\sin x}{\cancel{\cos x}} \times \cos\cancel{^2} x\\ &= \sin x \cos x\end{aligned}$$

(d) (3 marks)

✓ [1] for factorising correctly.

✓ [1] for each correct solution.

$$2\cos^2\theta + 3\cos\theta - 2 = 0$$

Let $m = \cos\theta$,

$$2m^2 + 3m - 2 = 0$$
$$(2m-1)(m+2) = 0$$
$$2m = 1 \qquad m = -2$$
$$\therefore 2\cos\theta = 1 \qquad \cos\theta = -2$$

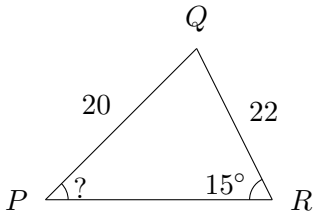
As $-1 \leq \cos\theta \leq 1$, second solution is invalid.

$$\therefore \cos\theta = \frac{1}{2}$$
$$\theta = 60^\circ, 300^\circ$$

**Question 15** (Berry)

(a) (4 marks)

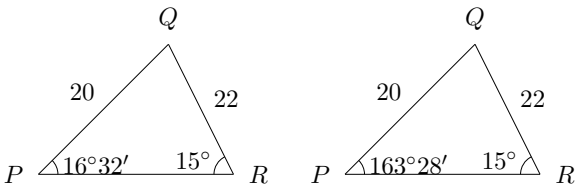
✓ [1] for each correct value of $\angle RPQ$ and $PR$.




$$\frac{\sin\angle RPQ}{22} = \frac{\sin 15^\circ}{20}$$
$$\sin\angle RPQ = \frac{22\sin 15^\circ}{20}$$
$$\angle RPQ = 16^\circ 32' \text{ or } 163^\circ 28'$$

(there are two solutions as the angle to be found is opposite the longer side)

(c)




(Not to scale)

Using the angle sum of $\triangle$, $\angle PQR = 148^\circ 28'$ or $1^\circ 32'$.

- When $\angle PQR = 148^\circ 28'$,

$$\frac{PR}{\sin 148^\circ 28'} = \frac{20}{\sin 15^\circ}$$
$$PR = \frac{20\sin 148^\circ 28'}{\sin 15^\circ}$$
$$= 40.42\cdots$$

- When $\angle PQR = 1^\circ 32'$,

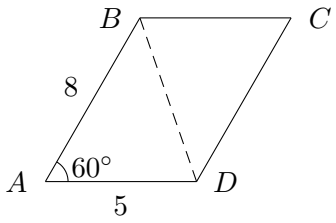
$$\frac{PR}{\sin 1^\circ 32'} = \frac{20}{\sin 15^\circ}$$
$$PR = \frac{20\sin 1^\circ 32'}{\sin 15^\circ}$$
$$= 2.08\cdots$$

(b) (3 marks)

✓ [1] for correct area of triangle via sine formula.

✓ [1] for area of triangle as exact value.

✓ [1] for doubling area of triangle.




Area of $\triangle ABD$:

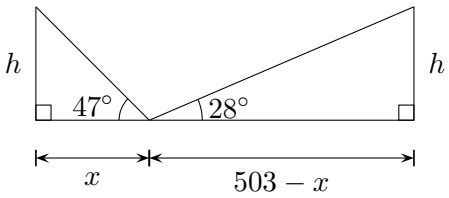
$$A_{\triangle ABD} = \frac{1}{2}\times 5\times 8\times \sin 60^\circ$$
$$= 10\sqrt{3}$$
$$\therefore A_{ABCD} = 20\sqrt{3}$$

i. (3 marks)

✓ [1] for correct diagram

✓ [1] for two equations in $x$, and $503 - x$

✓ [1] for final answer.




Let $x$ be the distance from the souther pylon to the ship (could be from the northern pylon, but this is not important as the solution is identical)

$$\frac{h}{x} = \tan 47^\circ$$
$$\therefore x = \frac{h}{\tan 47^\circ} \qquad (15.1)$$

In the ther triangle,

$$\frac{h}{503 - x} = \tan 28^\circ$$
$$\therefore 503 - x = \frac{h}{\tan 28^\circ}$$
$$x = 503 - \frac{h}{\tan 28^\circ} \qquad (15.2)$$

Equating $x$ in (15.1) and (15.2)

$$\begin{aligned} 503 - \frac{h}{\tan 28^\circ} &= \frac{h}{\tan 47^\circ} \\ 503 &= \frac{h}{\tan 47^\circ} + \frac{h}{\tan 28^\circ} \\ &= h\left(\frac{1}{\tan 47^\circ} + \frac{1}{\tan 28^\circ}\right) \\ &= h\left(\frac{\tan 28^\circ + \tan 47^\circ}{\tan 28^\circ \tan 47^\circ}\right) \end{aligned}$$

Dividing both sides by $\frac{\tan 28^\circ + \tan 47^\circ}{\tan 28^\circ \tan 47^\circ}$,

$$\therefore h = \frac{503 \tan 28^\circ \tan 47^\circ}{\tan 28^\circ + \tan 47^\circ}$$

ii. (2 marks)

✓ [1] for correct number of significant figures.

✓ [1] for correct scientific notation.

✓ [−1] for metres (required to give cm)

$$h = 178.79\,\text{m} = 1.788 \times 10^4\,\text{cm}$$

**Question 16** (Rezcallah)

(a) (2 marks)

✓ [1] expanding some terms correctly *or* obtaining one term of the answer correct.

✓ [1] correct final answer
([1] only for entire question if no working is provided)

$$\begin{aligned} (a-5)^2 &= a^2 - 10a + 25 \\ (a+5)^2 &= a^2 + 10a + 25 \\ (a-4)^2 &= a^2 - 8a + 16 \\ (a+4)^2 &= a^2 + 8a + 16 \\ &\vdots \end{aligned}$$

**Observations**

- there will be eleven $a^2$ terms (including the single $(a+0)^2$ expansion)
- all $a$ terms are offset;
- Constant terms produces double of sum of squares – $2\left(1^2 + 2^2 + \cdots + 5^2\right)$

$$\begin{aligned} \therefore (a-5)^2 + \cdots + a^2 + \cdots + (a+5)^2 \\ = 11a^2 + 110 \end{aligned}$$

(b) (3 marks)

✓ [2] for correct equation $\frac{1}{2}b + 3 = \left(-\frac{1}{6}b + 5\right) + 1$.

✓ [1] for correct final answer.

- Upper part of the roof, at $b$:

$$h = \frac{1}{2}b + 3$$

- Lower part of the roof, at $b$:

$$h = -\frac{1}{6}b + 5$$

- Relating the upper and lower part (these parts differ by 1 m):

$$\begin{aligned} \frac{1}{2}b + 3 &= \left(-\frac{1}{6}b + 5\right) + 1 \\ \underbrace{\frac{1}{2}b + 3}_{\times 6} &= \underbrace{-\frac{1}{6}b + 6}_{\times 6} \\ 3b + 18 &= -b + 36 \\ 4b &= 18 \\ \therefore b &= \frac{9}{2} \end{aligned}$$

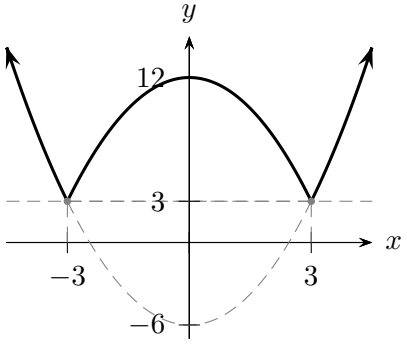
(c) i. (2 marks)

$$\begin{aligned} D &= \{x : x \in \mathbb{R}\} \\ R &= \{y : y \geq 3\} \end{aligned}$$

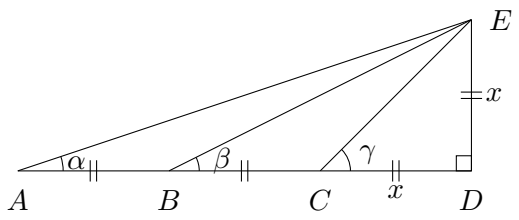
ii. (2 marks)

✓ [1] for correct shape of the curve.

✓ [1] for correct intercepts and critical points with the shape.



(d) i. (1 mark)




$$\tan\gamma = \frac{x}{x} = 1$$
$$\therefore \gamma = 45^\circ$$

ii. (2 marks)

✓ [1] for both $\tan\alpha = \frac{1}{3}$ and $\tan\beta = \frac{1}{2}$ *and* substituting into given formula.

✓ [1] for showing $\tan(\alpha+\beta) = 1 = \tan\gamma$ using the given formula.

$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \\ &= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2} \times \frac{1}{3}\right)} \\ &= \frac{\frac{5}{6}}{1 - \frac{1}{6}} \\ &= 1\end{aligned}$$
$$\therefore \alpha + \beta = 45^\circ$$
$$\therefore \alpha + \beta = \gamma$$