MATHEMATICS (EXTENSION 1)

Preliminary Course Assessment Task 3

June 15, 2012

General instructions

- Working time – 50 minutes.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Diagrams shown are not necessarily to scale.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper for submission to examination supervisors.

SECTION I

- Mark your answers to this section on page 3.

SECTION II

- Answer this section in the writing booklets provided.
- Commence each new question on a new page, writing on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NAME: .... **# BOOKLETS USED:** .....

Class (please ✔)

- ◯ 11M3A – Mr Lin/Mr Rezcallah
- ◯ 11M3B – Mr Fletcher
- ◯ 11M3C – Ms Ziaziaris
- ◯ 11M3D – Mr Lucas
- ◯ 11M3E – Mr Lam
- ◯ 11M3F – Mr Berry

Marker's use only.

QUESTION(S)	1-4	5	6	7	8	Total	%
MARKS	$\overline{4}$	$\overline{10}$	$\overline{10}$	$\overline{12}$	$\overline{12}$	$\overline{48}$	

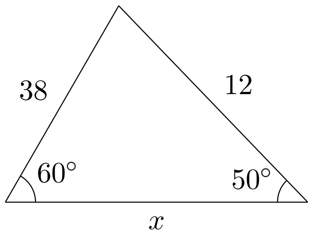
Section I: Objective response

Mark your answer on page 3. **Marks**

1. What is the size of each exterior angle in a regular pentagon? **1**

(A) $72°$ (B) $108°$ (C) $252°$ (D) $540°$

2. Which of the following expressions could be used to calculate the value of x? **1**



(A) $x^2 = 38^2 + 12^2 - 2 \times 12 \times 38 \times \cos 50°$

(B) $x^2 = 38^2 + 12^2 - 2 \times 12 \times 38 \times \cos 60°$

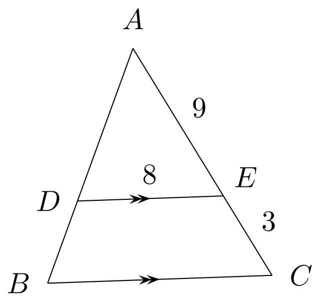
(C) $x^2 = 38^2 + 12^2 - 2 \times 12 \times 38 \times \cos 70°$

(D) $\dfrac{x}{\sin 60°} = \dfrac{38}{\sin 50°}$

3. What is the gradient of any line perpendicular to the interval joining $(-1, 2)$ and $(4, 4)$? **1**

(A) $-\frac{2}{5}$ (B) $-\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{5}{2}$

4. The diagram shows $\triangle ABC$ in which $DE \parallel BC$. $AE = 9$, $EC = 3$ and $DE = 8$. **1**



What is the length of BC?

(A) 2 (B) $2\frac{2}{3}$ (C) 10 (D) $10\frac{2}{3}$

Answer page for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

1 – Ⓐ Ⓑ Ⓒ Ⓓ

2 – Ⓐ Ⓑ Ⓒ Ⓓ

3 – Ⓐ Ⓑ Ⓒ Ⓓ

4 – Ⓐ Ⓑ Ⓒ Ⓓ

Examination continues overleaf. . .

Section II: Short answer

Question 5 (10 Marks) Commence a NEW page. **Marks**

The vertices of a triangle are $A(-1, 0)$, $B(2, 6)$ and $C(x, y)$. Given that C lies on the x axis to the right of A such that $\angle BAC = \angle BCA = \theta$,

(a) Draw a diagram representing the above information. **2**

(b) Find the gradient of AB and hence, find θ correct to the nearest degree. **2**

(c) Show that the equation of the line AB is $y = 2x + 2$. **1**

(d) Explain why BC has a gradient of -2. **1**

(e) If D is the point $(-2, 5)$ on this diagram, find the perpendicular distance of D from the line AB. **2**

(f) Hence, calculate the area of the quadrilateral $ADBC$. **2**

Question 6 (10 Marks) Commence a NEW page. **Marks**

(a) Find the coordinates of the point P that divides the interval AB externally in the ratio $1 : 4$, where A is $(3, 1)$ and B is $(-1, -5)$. **3**

(b) Sketch the region where the following inequalities hold simultaneously: **3**

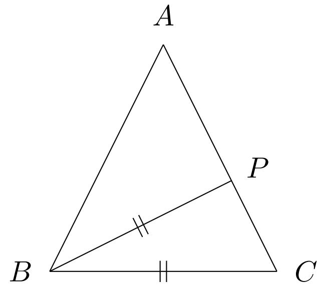
$$\begin{cases} y > x^2 \\ y \leq \sqrt{4 - x^2} \\ x \geq 0 \end{cases}$$

Show all intercepts with the axes but do *not* find the intersection points of the curves.

(c) Fnd the equation of the line passing through the intersection of $2x - y + 1 = 0$ and $x + 3y - 1 = 0$, and perpendicular to the line $3x - 2y + 4 = 0$. **4**

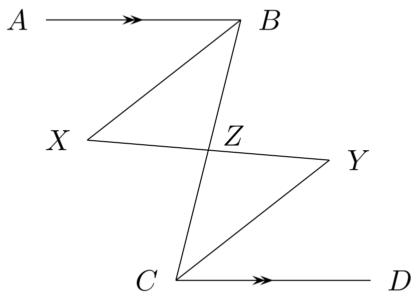
Question 7 (12 Marks) Commence a NEW page. **Marks**

(a) $\triangle ABC$ is isosceles with $AB = AC$, P lies on AC such that $\angle ABP = 3\angle PBC$ and $BP = BC$. **3**



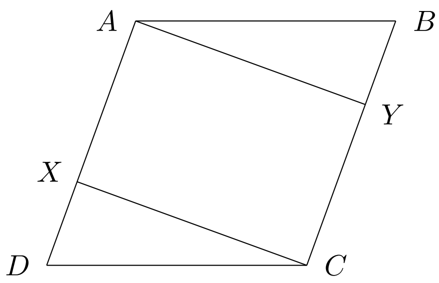
By letting $\angle CBP = x$ or otherwise, find the value of $\angle CBP$.

(b) In this diagram, $BX = CY$, AB is parallel to CD. XB bisects $\angle ABC$ and YC bisects $\angle BCD$. **3**



Show that Z is the midpoint of BC.

(c) $ABCD$ is a rhombus. X and Y lie on AD and BC such that $XC \perp AD$ and $AY \perp BC$.



i. Prove that $\triangle ABY \equiv \triangle DXC$. **2**

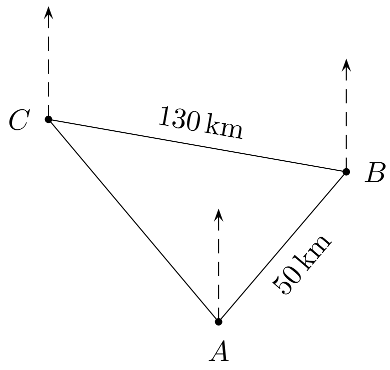
ii. If the side length of the rhombus is 10 cm and $\angle DAB = 120°$, find the length of the diagonal AC. **2**

iii. Find the area of the rectangle $AYCX$. **2**

Examination continues overleaf...

Question 8 (12 Marks) Commence a NEW page. **Marks**

(a) A ship sails 50 km from port A to port B on a bearing of 063°. It then sails 130 km from port B to port C on a bearing of 296°.



i. Show, with full working, that $\angle ABC = 53°$. **1**

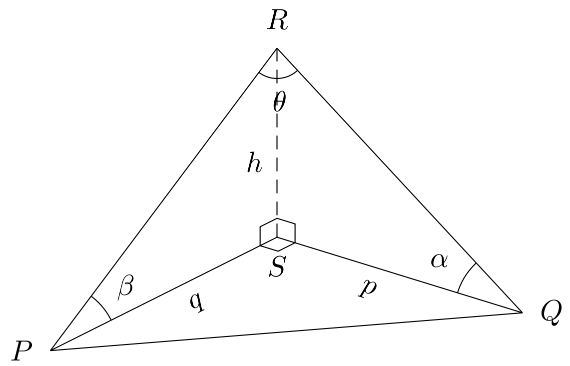
ii. Find AC to the nearest kilometre. **2**

(b) Sketch the graph of $y = 1 - \sin 2x$ for $-180° \leq x \leq 180°$. **2**

(c) A triangular pyramid has the following properties:

- $QS = p$
- $RS = h$
- $\angle RQS = \alpha$
- $PS = q$
- $\angle PRQ = \theta$
- $\angle RPS = \beta$

Also, RS is perpendicular to the plane containing $\triangle PQS$ which is right angled at S.



i. Show that $p = h \cot \alpha$ and obtain a similar expression for q. **2**

ii. Show that $\cos \theta = \dfrac{h^2}{\sqrt{(p^2 + h^2)(q^2 + h^2)}}$. **3**

iii. Hence, show that $\cos \theta = \sin \alpha \sin \beta$. **2**

End of paper.

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Suggested Solutions

Section I

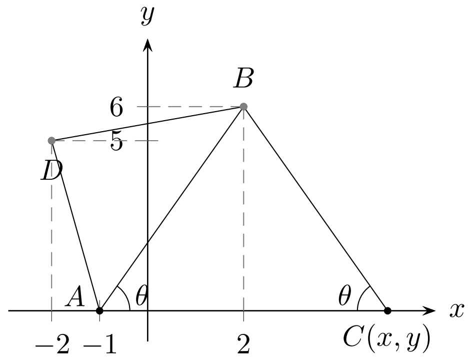
1. (A) **2.** (C) **3.** (B) **4.** (D)

Section II

Question 5 (Lam)

(a) (2 marks)

✓ [−1] for each missing point.



D should be filled in upon reaching part (e), but do not penalise if missing.

(b) (2 marks)

✓ [1] for gradient.

✓ [1] for θ.

$$m_{AB} = \frac{6-0}{2+1} = \frac{6}{3} = 2$$
$$\therefore \tan\theta = 2$$
$$\theta \approx 63^\circ$$

(c) (1 mark)

Equation of AB:

$$\frac{y-0}{x+1} = 2$$
$$y = 2(x+1) = 2x+2$$

Alternatively, use $B(2,6)$ as point in point-gradient formula.

(d) (1 mark)

As $\angle BCA = 63^\circ$, its supplement is 117°.

$$m_{BC} = \tan 117^\circ = -\tan 63^\circ = -2$$

Alternatively, $\triangle ABC$ is isosceles, hence $C(5,0)$ and use gradient formula to demonstrate $m_{BC} = -2$.

(e) (2 marks)

✓ [−1] for each error.

$$2x - y + 2 = 0 \quad \rightarrow \quad D(-2,5)$$
$$d_\perp = \frac{|2(-2)+(-1)(5)+2|}{\sqrt{2^2+(-1)^2}}$$
$$= \frac{|-7|}{\sqrt{5}} = \frac{7}{\sqrt{5}}$$

(f) (2 marks)

✓ [−1] for each error.

$$A_{ABCD} = A_{\triangle ABD} + A_{\triangle ABC}$$
$$A_{\triangle ABC} = \frac{1}{2}\times 6\times 6 = 18$$

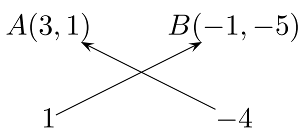
Find length AB:

$$AB = \sqrt{(2-(-1))^2+(6-0)^2}$$
$$= \sqrt{3^2+6^2} = \sqrt{45} = 3\sqrt{5}$$
$$A_{\triangle ABD} = \frac{1}{2}\times d_\perp \times AB$$
$$= \frac{1}{2}\times\frac{7}{\cancel{\sqrt{5}}}\times 3\cancel{\sqrt{5}}$$
$$= \frac{21}{2}$$
$$\therefore A_{ABCD} = \frac{21}{2} + 18 = \frac{57}{2}$$

Question 6 (Ziaziaris)

(a) (3 marks)

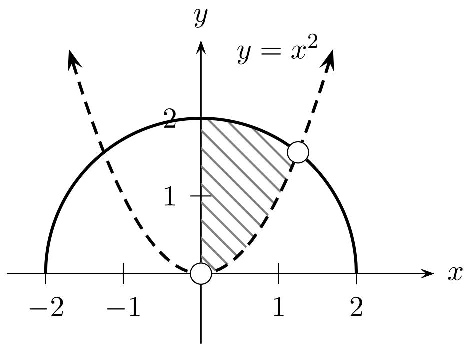
✓ [−1] for each error.



$$x_B = \frac{-4(3)+(-1)}{1-4} = \frac{-13}{-3} = \frac{13}{3}$$
$$y_B = \frac{-4(1)+(-5)}{1-4} = \frac{-9}{-3} = 3$$
$$\therefore B\left(\frac{13}{3}, 3\right)$$

(b) (3 marks)

✓ [−1] for each error.



(c) (4 marks)
Required line will be in the form

$$\begin{aligned} 2x - y + 1 + k(x + 3y - 1) &= 0 \\ (2 + k)x + (-1 + 3k)y + (1 - k) &= 0 \end{aligned}$$

Gradient:

$$\begin{aligned} m = \frac{2+k}{1-3k} &= -\frac{2}{3} \\ \therefore 3(2+k) &= -2(1-3k) \\ 6 + 3k &= -2 + 6k \\ 3k &= 8 \\ \therefore k &= \frac{8}{3} \end{aligned}$$

Hence the line is

$$\begin{aligned} \underbrace{2x - y + 1}_{\times 3} + \underbrace{\frac{8}{3}(x + 3y - 1)}_{\times 3} &= \underset{\times 3}{0} \\ 6x - 3y + 3 + 8x + 24y - 8 &= 0 \\ 14x + 21y - 5 &= 0 \end{aligned}$$

Alternatively, find the point of intersection $\left(-\frac{2}{7}, \frac{3}{7}\right)$ and use point gradient formula.

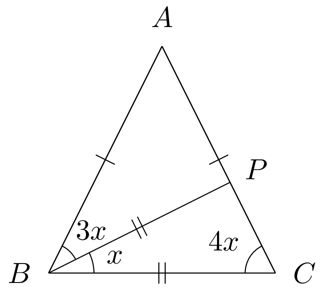
Question 7 (Lin)

(a) (3 marks)

✓ [1] for $\angle ACB = 4x$, with reason.

✓ [1] for $\angle CPB = 4x$, with reason.

✓ [1] for final answer.

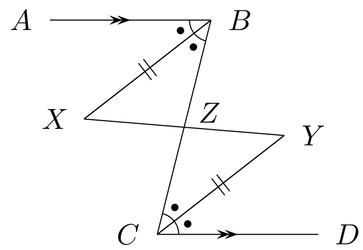


- As $\angle CBP = x$, then $\angle PBA = 3x$ (given).
- $\therefore \angle BCP = 4x$ (base $\angle$ of isos $\triangle ABC$)
- $\therefore \angle CPB = 4x$ (base $\angle$ of isos $\triangle CPB$)
- Hence $4x + 4x + x = 180°$ ($\angle$ sum of $\triangle PBC$)

$$\begin{aligned} 9x &= 180° \\ x &= 20° \end{aligned}$$

(b) (3 marks)

✓ [−1] for each missing line or reason.



In $\triangle XBZ$ and $\triangle YCZ$,

- $XB = YC$ (given)
- $\angle XZB = \angle YZC$ (vertically opp)
- $\angle ABZ = \angle DCB$
  (alternate $\angle$, $AB \parallel CD$)

$\therefore \triangle XBZ \equiv \triangle YCZ$ (AAS)
Hence $BZ = CZ$ (corresponding sides of congruent $\triangle$) and Z is the midpoint of BC.

(c) i. (2 marks)
In $\triangle ABY$ and $\triangle DXC$:

- $AB = CD$
  (opp. sides of rhombus)
- $\angle ABY = \angle CDX$
  (opp. $\angle$ of rhombus)
- $\angle AYB = 90° = \angle CXD$ (given)

$\therefore \triangle ABY \equiv \triangle DXC$ (AAS)

ii. (2 marks)

- $\angle ABC = 60°$ (cointerior $\angle$, $AD \parallel BC$)
- $\angle BAC = 60°$ (diagonal bisects $\angle DAB$)

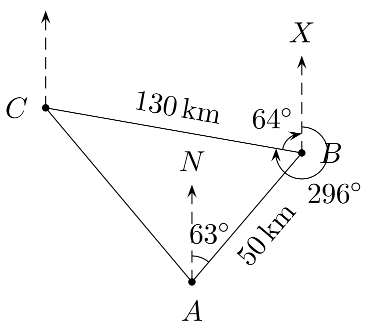
Hence $\triangle ABC$ is equilateral and $AC = 10\,\text{cm}$.

iii. (2 marks)

- $AX = 5\,\text{cm}$ ($\triangle ADC$ equilateral, $CX \perp AD$)
- $XC = \sqrt{10^2 + 5^2} = 5\sqrt{3}$
- $A_{AYCX} = 5 \times 5\sqrt{3} = 25\sqrt{3}$

Question 8 (Berry)

(a) i. (1 mark)



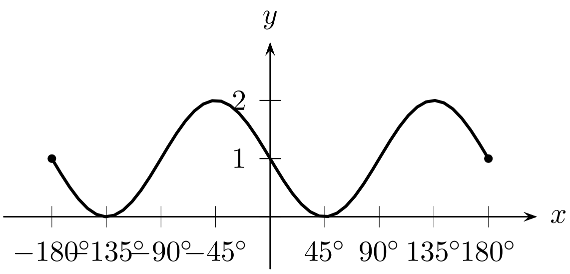
- $\angle XBC = 360° - 296° = 64°$
- $\angle NAB = 63°$.
- $\therefore \angle ABC + 63° + 64° = 180°$ (cointerior $\angle$, $NA \parallel XB$).

$$\angle ABC = 53°$$

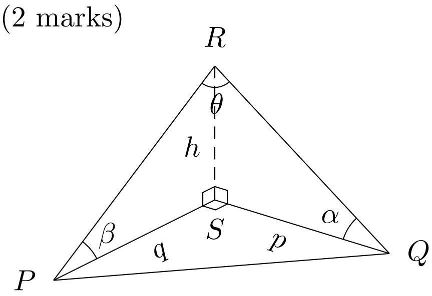
ii. (2 marks)
Using the cosine rule,

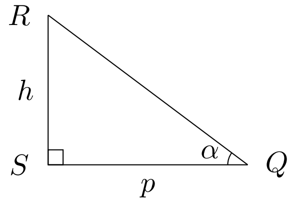
$$AC^2 = 130^2 + 50^2 - 2(130)(50)(\cos 53°)$$
$$\therefore AC = 108\,\text{km}$$

(b) (2 marks)



(c) i. (2 marks)





In $\triangle RSQ$,

$$\frac{h}{p} = \tan\alpha$$
$$\therefore p = h\cot\alpha$$

Similarly in $\triangle RSP$,

$$q = h\cot\beta$$

ii. (3 marks)

✓ [1] for PQ^2.

✓ [1] for substituting into cosine rule.

✓ [1] for final answer.

In $\triangle PSR$,

$$PR^2 = h^2 + q^2$$

In $\triangle QSR$,

$$QR^2 = h^2 + p^2$$

In $\triangle PRQ$,

$$PQ^2 = p^2 + q^2$$

Using the cosine rule in $\triangle PQR$,

$$PQ^2 = PR^2 + QR^2 - 2(PR)(QR)\cos\theta$$
$$\cancel{p^2 + q^2} = \left(h^2 + \cancel{q^2}\right) + \left(h^2 + \cancel{p^2}\right) - 2\sqrt{\left(h^2+q^2\right)\left(h^2+p^2\right)}\cos\theta$$
$$\therefore \cancel{2}h^2 = \cancel{2}\sqrt{\left(h^2+q^2\right)\left(h^2+p^2\right)}\cos\theta$$
$$\therefore \cos\theta = \frac{h^2}{\sqrt{\left(h^2+q^2\right)\left(h^2+p^2\right)}}$$

iii. (2 marks)

$$\sin\alpha = \frac{h}{\sqrt{h^2+p^2}} \qquad \sin\beta = \frac{h^2}{\sqrt{h^2+q^2}}$$
$$\therefore \sin\alpha\sin\beta = \frac{h^2}{\sqrt{\left(p^2+q^2\right)\left(q^2+h^2\right)}} = \cos\theta$$