Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2013 Year 11 Task 1

Mathematics
Extension 1

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)
- O Mr Berry
- O Mr Fletcher
- O Mr Ireland
- O Mr Lowe
- O Mr Lucas
- O Mrs Ziaziaris

Student Name:________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total %
Mark	/10	/12	/11	/10	/13	/56	/100

Question 1 (10 marks) Start a new page. **Marks**

(a) Evaluate $\dfrac{7\cdot62\times10^{3}}{5\cdot169\times10^{-4}}$ giving your answer in scientific notation correct to two significant figures. 2

(b) Write $0\cdot2\dot{3}\dot{4}$ as a fraction in lowest terms, showing all working. 2

(c) Simplify $3\sqrt{32}+2\sqrt{50}-8\sqrt{18}$ 2

(d) If $(a+\sqrt{3})^2=b+10\sqrt{3}$, where a and b are integers, then find the value of a and b. 2

(e) If $x=\sqrt{2}-1$ then show that $x-\dfrac{1}{x}$ is rational. 2

Question 2 (12 marks) Start a new page.

(a) Factorise the following expressions:

(i) $6x^2-5x-21$ 2

(ii) $500+4x^3$ 2

(iii) $1-a^7$ 2

(iv) $x^2-9y^2+7x+21y$ 2

(b) Simplify the following:

(i) $\dfrac{7-x}{3}-\dfrac{3-4x}{8}$ 2

(ii) $\dfrac{6^n}{2^{n+1}+2^n}$ 2

Question 3 (11 marks) Start a new page.
Solve the following equations and inequalities:

(a) $(3x+2)^2=25$ 2

(b) $3^{-2x}=\dfrac{1}{27}$ 2

(c) $|3x+4|=10$ 2

(d) $|2x-5|>7$ 2

(e) $|x-3|=4x+2$ 3

Question 4 (10 marks) Start a new page.

Sketch the graphs of the following relations. Use separate number planes and show all important features:

(a) $y=-\sqrt{9-x^2}$ **2**

(b) $y=|2x-1|$ **2**

(c) $y=1-2^x$ **2**

(d) $y=\dfrac{x^3-x}{x^2-1}$ **2**

(e) $y^2=x-9$ **2**

Question 5 (13 marks) Start a new page.

(a) Prove that the function $f(x)=x^4-2x^2+3$ is even. **2**

(b) A function $f(x)$ is defined piece-wise by the rule:

$$f(x)=\begin{cases}4, & x\le 0\\ (x-2)^2, & 0<x\le 3\\ 1, & x>3\end{cases}$$

(i) Evaluate $f(2)+f[f(-2)]$ **2**

(ii) What is the range of $f(x)$? **1**

(c) Find $f(x)$ if $f(x-1)=2x+9$. **1**

(d) Find the domain and range of $y=\sqrt{x+1}+\sqrt{x-1}$. **2**

(e) (i) Sketch the graph of $y=\dfrac{x-1}{x+1}$ showing any asymptotes and intercepts. **3**

(ii) Using your sketch, or otherwise, solve the inequality $|x+1|>\dfrac{x-1}{x+1}$. **2**

END OF EXAMINATION

2013 Year 11 Ext. 1 – Task 1 {SUGGESTED SOLUTIONS}

$\boxed{\text{Q1.}}$

(a) $\dfrac{7.62 \times 10^{3}}{5.169 \times 10^{-4}} = 1.474\,172\,95 \times 10^{7}$

$= 1.5 \times 10^{7}$ (2 sig. figs.) ✓✓

(b) Let $x = 0.2\dot{3}\dot{4}$

$\therefore\ x = 0.2343434\ldots$

$100x = 23.4343434\ldots$ ✓ method

Thus $99x = 23.2$

$x = \dfrac{232}{990}$

$= \dfrac{116}{495}$ ✓ correct answer

(c) $3\sqrt{32} + 2\sqrt{50} - 8\sqrt{18} = 12\sqrt{2} + 10\sqrt{2} - 24\sqrt{2}$ ✓

$= -2\sqrt{2}$ ✓

(d) $(a+\sqrt{3})^2 = b + 10\sqrt{3}$

$\therefore\ a^2 + 3 + 2a\sqrt{3} = b + 10\sqrt{3}$

$\therefore\ 2a = 10 \quad \therefore\ a = 5$

$\therefore\ b^2 = 25 + 3 = 28$ Thus $a = 5$, $b = 28$ ✓✓

(e) $x = \sqrt{2} - 1$

$\therefore\ x - \dfrac{1}{x} = \sqrt{2} - 1 - \dfrac{1}{\sqrt{2}-1}$

$= \sqrt{2} - 1 - \dfrac{1}{\sqrt{2}-1} \cdot \dfrac{\sqrt{2}+1}{\sqrt{2}+1}$ ✓

$= \sqrt{2} - 1 - (\sqrt{2}+1)$

$= -2$, which is rational. ✓

/10

$\boxed{Q2}$ (a)

(i) $6x^2 - 5x - 21 = (3x-7)(2x+3)$ ✓ ✓ (one for each factor)

(ii) $500 + 4x^3 = 4(125 + x^3)$

$= 4(5+x)(25 - 5x + x^2)$ ✓✓

(iii) $1 - a^7 = (1-a)(1+a+a^2+a^3+a^4+a^5+a^6)$ ✓✓

(iv) $x^2 - 9y^2 + 7x + 21y = (x-3y)(x+3y) + 7(x+3y)$ ✓

$= (x+3y)(x-3y+7)$ ✓

(b)

(i) $\dfrac{7-x}{3} - \dfrac{3-4x}{8} = \dfrac{8(7-x) - 3(3-4x)}{24}$ ✓

$= \dfrac{56 - 8x - 9 + 12x}{24}$

$= \dfrac{4x + 47}{24}$ ✓

(ii) $\dfrac{6^n}{2^{n+1} + 2^n} = \dfrac{2^n \cdot 3^n}{2^n(2^1+1)}$ ✓ for correct numerator or denominator

$= \dfrac{3^n}{3}$

$= 3^{n-1}$ ✓

/12

Q3

(a) $(3x+2)^2 = 25$

$\therefore\ 3x+2 = 5$ or -5

$\therefore\ 3x = 3$ or -7

$\therefore\ x = 1$ or $-\frac{7}{3}$ ✓✓

(b) $3^{-2x} = \frac{1}{27} = 3^{-3}$

$\therefore\ -2x = -3 \qquad \therefore\ x = \frac{3}{2}$ ✓✓

(c) $|3x+4| = 10$

$\therefore\ 3x+4 = 10$ or $3x+4 = -10$

$3x = 6 \qquad 3x = -14$

$x = 2 \qquad x = -\frac{14}{3}$

$\therefore\ x = 2$ or $-\frac{14}{3}$ ✓✓

(d) $|2x-5| > 7$ ✓

$\therefore\ 2x-5 > 7$ or $2x-5 < -7$

$2x > 12 \qquad 2x < -2$

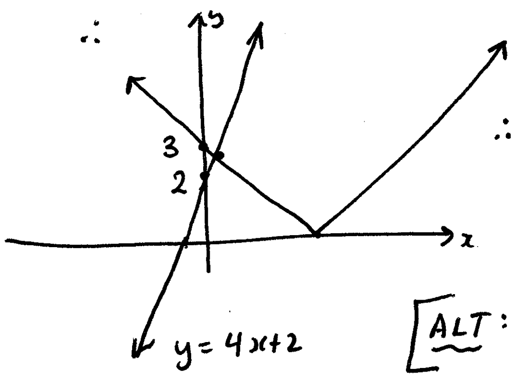
$x > 6 \qquad x < -1$

Thus:

$x > 6$ or $x < -1$ ✓ (penalise "and")

(e) $|x-3| = 4x+2$

$\therefore$



Clearly, only 1 solution.

$\therefore$ We have

$4x+2 = -x+3$

$5x = 1$

$\therefore\ x = \frac{1}{5}$ ✓✓✓ (needs correct working for full marks)

[ALT: Candidates may go via $x-3 = 4x+2$ or $x-3 = -4x-2$, but must test & discard $x = -5/3$]

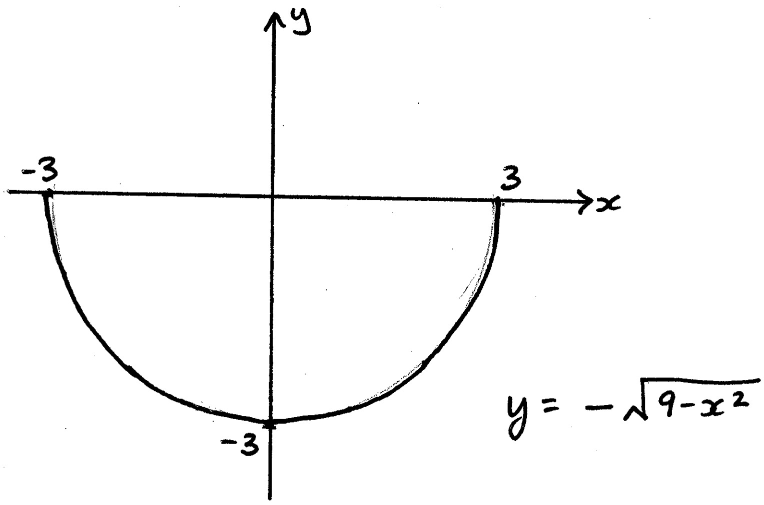
11

2013 Y11 Ext.1 – T.1

$\boxed{\text{Q4}}$

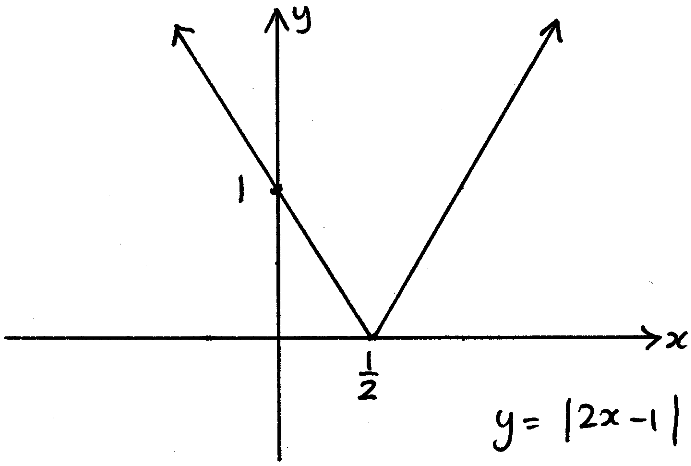
(*) delete 1 mark (total) if axes unlabelled.

(a)



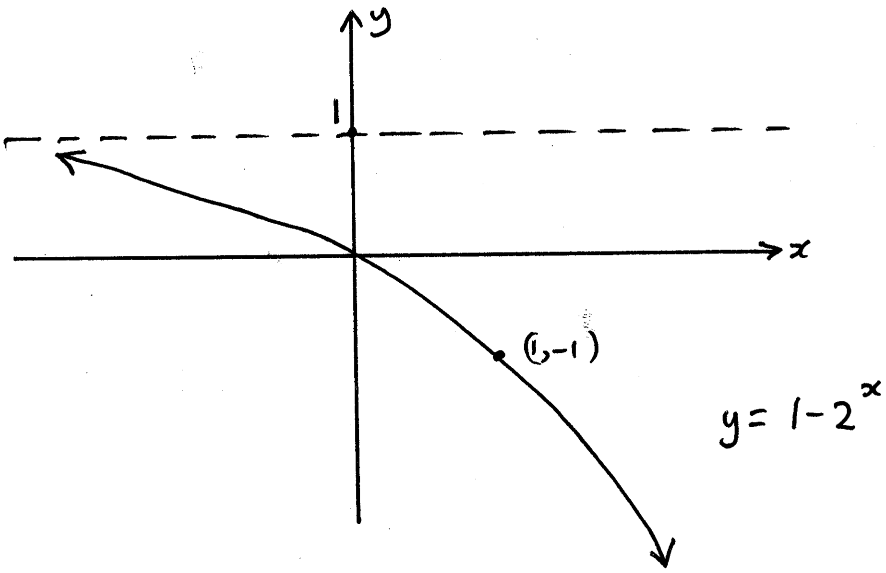
✓ shape/curve
✓ intercepts

(b)



✓ shape
✓ intercepts

(c)

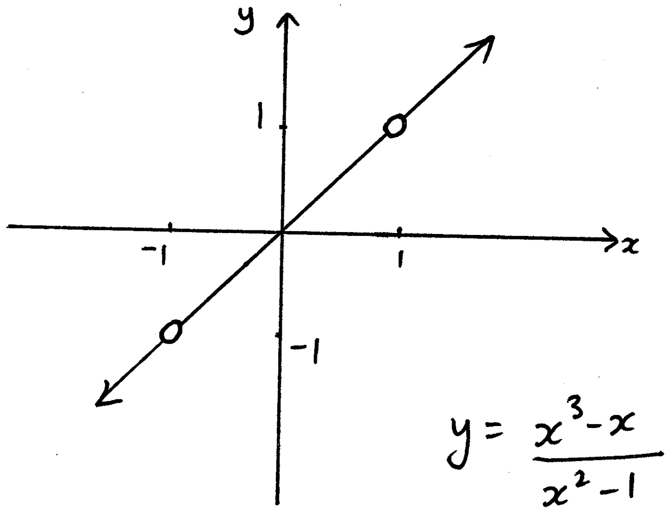


✓ curve
✓ asymptote, point

$\boxed{Q4}$ - continued

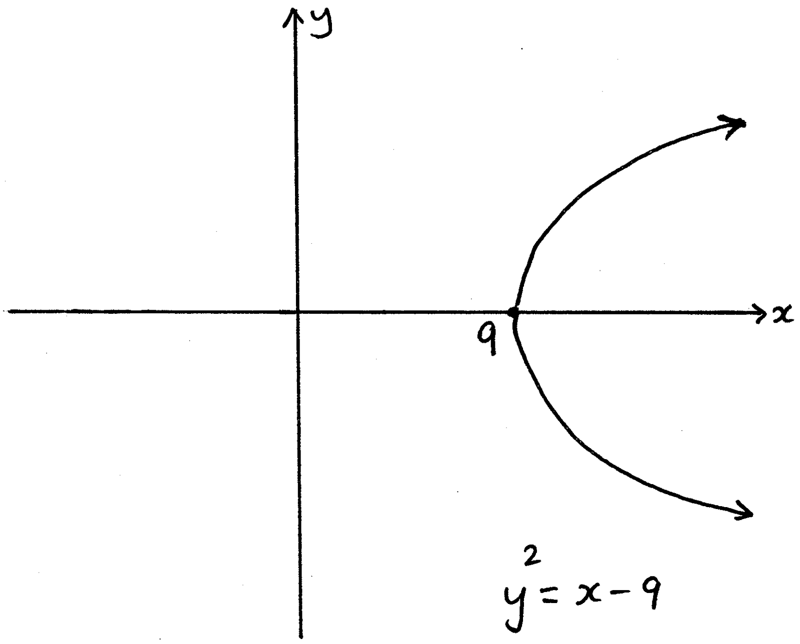
(d) $y = \dfrac{x^3 - x}{x^2 - 1} = \dfrac{x(x^2-1)}{x^2-1}$

$\therefore\ y = x \quad (x \neq -1, 1)$



✓ 'curve'
✓ holes

(e)



✓ curve
✓ intercept

/10

$\boxed{Q5}$

(a) $f(x) = x^4 - 2x^2 + 3$

$\therefore f(-x) = (-x)^4 - 2(-x)^2 + 3$

$= x^4 - 2x^2 + 3$

$= f(x)$

$\therefore f(x)$ is even.

✓✓ correct demonstration

(b)

(i) $f(2) + f[f(-2)] = (2-2)^2 + f(4)$ ✓

$= 0 + 1$ ✓

$= 1$

(ii) $R: \ 0 \le y \le 4$ ✓

(c) $f(x-1) = 2x + 9$

$= 2(x-1) + 11$

$\therefore f(x) = 2x + 11$ ✓

(d) $y = \sqrt{x+1} + \sqrt{x-1}$

$D: \ x \geq 1$ ✓

$R: \ y \geq \sqrt{2}$ ✓

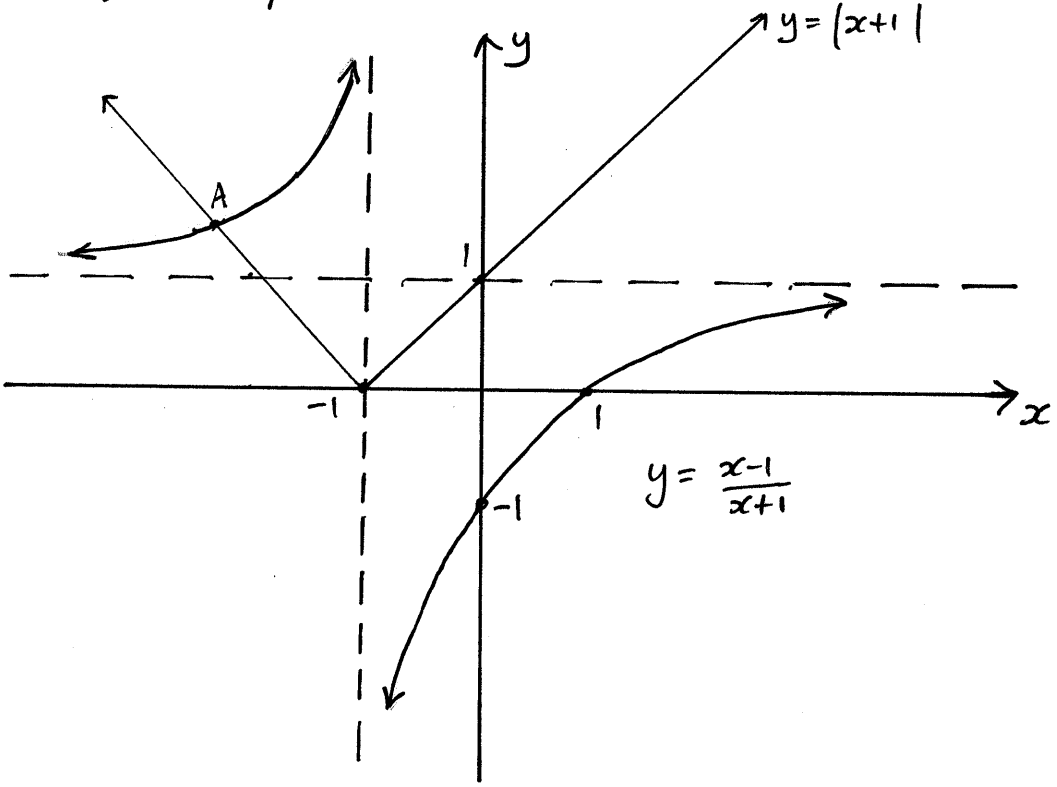
[P.T.O. →]

$\boxed{Q5}$ – continued

(e) (i) $y = \dfrac{x-1}{x+1} = \dfrac{x+1-2}{x+1}$

$\therefore y = 1 - \dfrac{2}{x+1}$

Thus, vertical asymptote is $x = -1$
horizontal asymptote is $y = 1$
y-intercept is -1
x-intercept is 1



✓ curve
✓ asymptotes
✓ intercepts

(ii) By graphing $y = |x+1|$ on the same number plane, we see we need to find point A: $-x-1 = \dfrac{x-1}{x+1}$

$\therefore -x^2 - 2x - 1 = x - 1$

$x^2 + 3x = 0 \qquad \therefore x = -3$ [from the diagram]

$\therefore$ Solution to $|x+1| > \dfrac{x-1}{x+1}$

is therefore $x < -3$ or $x > -1$

✓✓

13