Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2013 Year 11 Task 2

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Fletcher
- O Mr Ireland
- O Mr Lowe
- O Mr Lucas
- O Mrs Ziaziaris

Student Name:_____

(To be used by the exam markers only.)

Question No	1-6	7	8	9	10	11	12	Total	Total %
Mark	/6	/9	/9	/9	/9	/9	/9	/60	/100

Student Name : _____

SECTION 1: MULTIPLE CHOICE

Select the alternative A, B, C or D that best answers the question and indicate your choice by shading the appropriate letter.

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D

Multiple Choice – Select the most correct answer

1. Solve for x: $2(3x - 2) = x + 4(x - 1)$

(A) $-\frac{8}{11}$ (B) 0

(C) 8 (D) All real x

2. Select the appropriate values of x for the inequation:

$$(1 - x)(x - 4) < 0$$

(A) $x < 1$ (B) $x > -1$

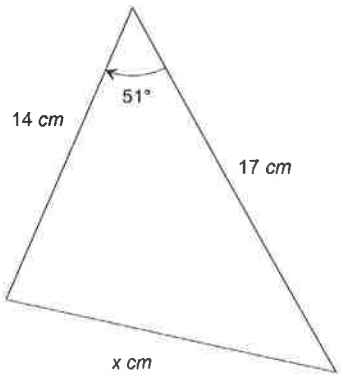
(C) $1 < x < 4$ (D) $x < 1$ or $x > 4$

3. Which of the following trigonometric graphs does not have x – intercepts?

(A) $y = \sin x$ (B) $y = \tan x$

(C) $y = \cot x$ (D) $y = \sec x$

4. What is the length of x in the diagram below?



(A) $\sqrt{14^2 + 17^2 + 476\cos 51°}$ (B) $\sqrt{14^2 + 17^2 - 476\cos 51°}$

(C) $\sqrt{14^2 + 17^2 - 476\sin 51°}$ (D) $\sqrt{14^2 + 17^2 + 476\sin 51°}$

5. How many solutions are there to the equation $\sin x = 0$ for $0 \leq x \leq 360°$

(A) 0 (B) 1

(C) 2 (D) 3

6. Solve for x: $\sin(x - 60°) = -\frac{1}{2}$, for $0° \leq x \leq 180°$

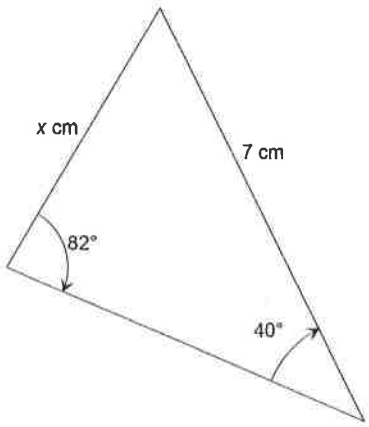
(A) 30° (B) 60°

(C) 90° (D) 210°

Extended Response – Show full working out

Question 7

(a) Find the value of x to 1 decimal place



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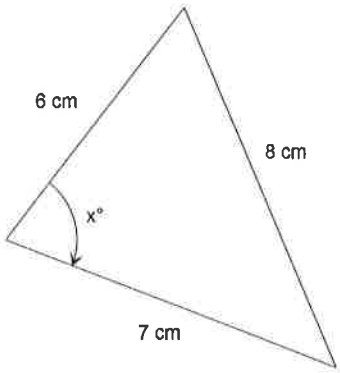
(b) Find the value of x to the nearest degree



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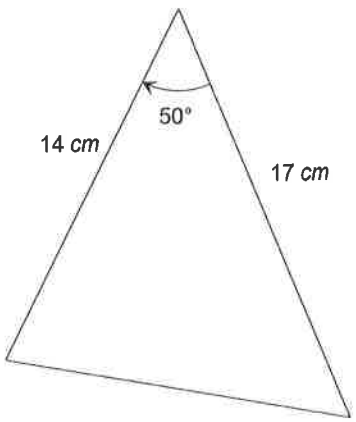
(c) Find the area of the triangle below (answer to 1 decimal place).



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(d) In ΔABC, AC = 30 cm, BC = 20 cm and $\angle$BAC =40°. Find the value *(s)* of $\angle$ ABC to the nearest minute.

3

Question 8

(a) Find the exact values of (express your answer in simplest form):

(i) $\cos 150°$ **1**

(ii) $\dfrac{\sec 45° + \cos 30°}{\operatorname{cosec} 210°}$ **2**

(b) Find θ to the nearest minute for $0^o \leq \theta \leq 360^o$:

(i) $\sin\theta = -0.234$ **1**

(ii) $\sin\theta = 4\cos\theta$ **2**

(iii) $4\cos 2\theta = 2$ **3**

Question 9

(a) State the period and amplitude and sketch

$y = 2\cos 3x, \quad$ for $0° \leq x \leq 360^o$ **3**

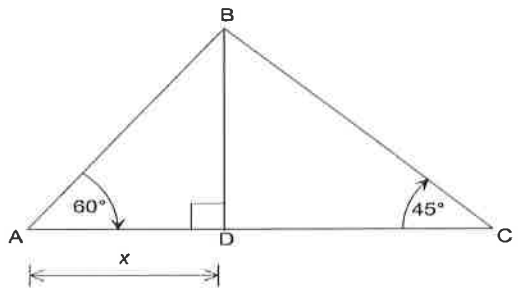
showing all relevant features.

(b) Solve for x:

$$\tan(2x + 20°) = \cot(3x - 60°)$$ **2**

(c) Given that $\tan\theta = 3.5$ and $\cos\theta < 0$, find the exact value of $\sin\theta$ **2**

(d) In the diagram below AD = x. Find the length of BC in terms of x.



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Question 10

(a) Find the points of intersection of: $y = (x-2)^2$ and $y = 3x - 6$. **3**

(b) Solve for x and sketch your solution on a number line:

$\dfrac{5}{2-x} \leq 3$ **3**

(c) Solve for x:

$|x+1| + |x-2| = 5$ **3**

Question 11

(a) A is 5 km N37°W from a lighthouse and B is 12 km S53°W from the same lighthouse.

(i) Find the distance from A to B. **2**

(ii) Find the bearing of B from A. **2**

(b) Simplify:

$1 - \sin\theta\cos(90° - \theta)$ **2**

(c) Prove the identity:

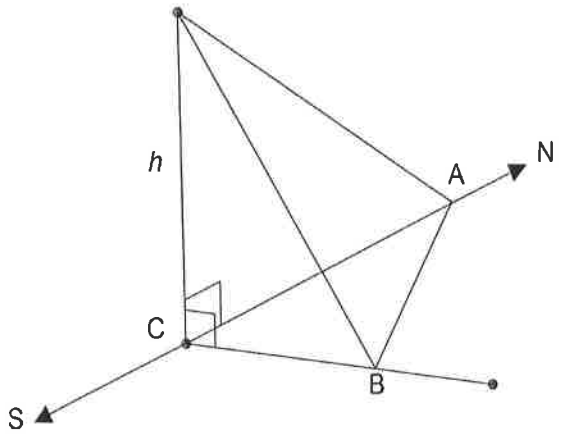
$$(\cot\theta + \operatorname{cosec}\theta)^2 = \frac{1+\cos\theta}{1-\cos\theta}$$ **3**

Question 12

(a) Solve for θ where $0° \leq \theta \leq 360°$

$2\tan^2\theta = 5\sec\theta - 4$ **3**

(b) Amy walks away from a telegraph pole in a northerly direction. When she arrives at Point A the angle of elevation to the top of the pole is 23°. Brenda walks in a bearing of $032°T$ from the same tower until she reaches Point B and notices that the angle of elevation to the top of the pole is 17°. The distance between A and B is 55m. Let h be the height of the pole and Point C be the base of the pole.



(i) Copy the diagram above into your answer booklet and clearly mark all of the information given. **1**

(ii) Find expressions for AC and BC in terms of h **2**

(iii) Hence, or otherwise, find the height h of the tower to the nearest metre **3**

Multiple Choice

1. B
2. D
3. D
4. B
5. D
6. A

Question 7

(a) $\frac{x}{\sin 40^\circ} = \frac{7}{\sin 82^\circ}$ ✓ correct sine rule

$\therefore x = \frac{7 \sin 40^\circ}{\sin 82^\circ}$

$\therefore x = 4.5 \text{ cm}$ (1 d.p.) ✓ answer

(b) $\cos x^\circ = \frac{6^2 + 7^2 - 8^2}{2 \times 6 \times 7}$ ✓ correct cosine rule

$x^\circ = \cos^{-1}\left(\frac{1}{4}\right)$

$\therefore x^\circ = 76^\circ$ (nearest degree) ✓ correct answer

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(c) $A = \frac{1}{2} \times 14 \times 17 \times \sin 50^\circ$ ✓ correct formula

$= 91.2 \text{ cm}^2$ (1 d.p.) ✓ correct answer

(d) $\frac{\sin B}{30} = \frac{\sin 40^\circ}{20}$ ✓ correct substitution into formula

$\sin B = \frac{30 \sin 40^\circ}{20}$

$\therefore B = 74^\circ 37'$ (nearest minute) ✓ acute answer

or $B = 105^\circ 23'$ (nearest minute) ✓ obtuse answer

Question 8

(a)(i) $\cos 150° = -\sqrt{3}/2$

✓ correct answer

(a)(ii) $\dfrac{\sqrt{2} + \sqrt{3}/2}{(-2)}$

$= \dfrac{-2\sqrt{2} - \sqrt{3}}{4}$

✓ at least 2 correct exact values

✓ correct answer

(b)(i) $\theta = 193°32'$

or

$\theta = 346°28'$

✓ both answers

(b)(ii) $\sin\theta = 4\cos\theta$

$\therefore \tan\theta = 4$

✓ $\tan\theta = 4$

$\therefore \theta = 75°58'$

or

$\theta = 255°58'$

✓ both answers

(b)(iii) $4\cos 2\theta = 2$

$\therefore \cos 2\theta = 1/2$

$\therefore 2\theta = 60°, 300°, 420°, 660°$

$\therefore \theta = 30°, 150°, 210°, 330°$

✓ 2 answers for 2θ

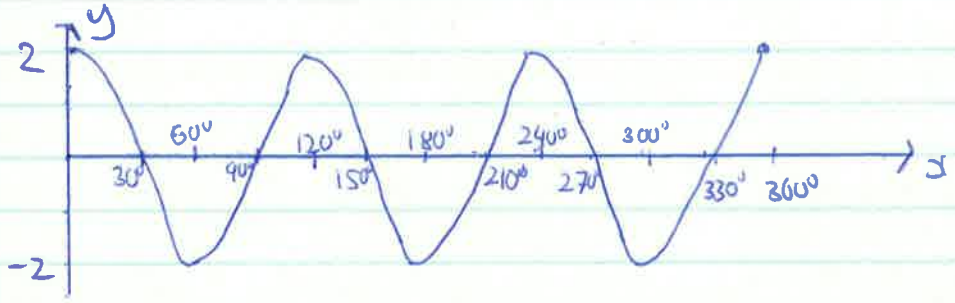
✓ 4 answers for 2θ

✓ all correct answers

Question 9

(a) period = 120°

amplitude = 2



✓ period

✓ amplitude

✓ graph

(b) $\tan(2x + 20°) = \cot(3x - 60°)$

$\therefore 2x + 20° = 90 - (3x - 60°)$

$2x + 20° = 90 - 3x + 60°$

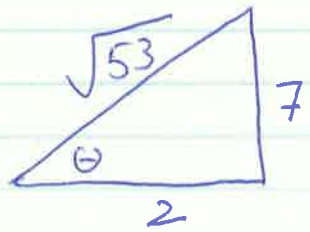
$5x = 130°$

$x = 26°$

✓ complementary angles

✓ correct answer

(c) $\tan\theta = 7/2$



$\tan\theta > 0$, $\cos < 0 \rightarrow$ 3rd quadrant

$\therefore \sin\theta = -\frac{7}{\sqrt{53}}$

✓ correct value

✓ correct sign

7mm A4

9.(d) $\tan 60° = BD/x$

$\therefore BD = x \tan 60°$

$BD = \sqrt{3}\,x$ ✓ length BD

$\sin 45° = BD/BC$

$\therefore BC = \dfrac{BD}{(1/\sqrt{2})}$

$\therefore BC = (\sqrt{3}\,x)(\sqrt{2})$

$\therefore BC = \sqrt{6}\,x$ ✓ correct answer

Question 10

(a) $y = (x-2)^2$ & $y = 3x-6$

$\therefore (x-2)^2 = 3x-6$ ✓ simultaneous equations

$x^2 - 4x + 4 = 3x - 6$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$\therefore x = 5$ or $x = 2$ ✓ correct x-values

$y = 3\times5 - 6 = 9$ or $y = 3\times2 - 6 = 0$

$\therefore$ intersection at $(5,9)$ & $(2,0)$ ✓ correct points

(b) $\dfrac{5}{2-x} \leq 3$

$\dfrac{5(2-x)^2}{(2-x)} \leq 3(2-x)^2, \quad x \neq 2$ ✓ any valid method

$5(2-x) \leq 3(2-x)^2$

$5(2-x) - 3(2-x)^2 \leq 0$

$(2-x)(5-3(2-x)) \leq 0$

$(2-x)(5-6+3x) \leq 0$

$(2-x)(3x-1) \leq 0$ ✓ points $\frac{1}{3}$ & 2

$\therefore x \leq \frac{1}{3}$ or $x \geq 2$ ✓ correct inequations

(c) <u>case 1 $x \leq -1$</u>

$-(x+1) - (x+2) = 5$

$-x - 1 - x + 2 = 5$

$-2x + 1 = 5$

$-2x = 4$

$\therefore x = -2$

✓ any correct method

✓ $x = -2$

<u>case 2 $-1 \leq x \leq 2$</u>

$(x+1) - (x-2) = 5$

$x + 1 - x + 2 = 5$

$\therefore 3 = 5$

$\therefore$ no solutions

<u>case 3 $x \geq 2$</u>

$(x+1) + (x-2) = 5$

$\therefore 2x - 1 = 5$

$2x = 6$

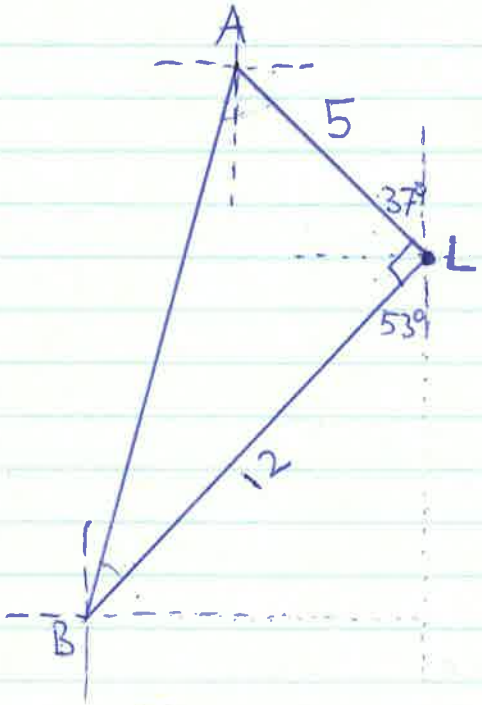
$\therefore x = 3$

✓ $x = 3$

$\therefore$ solutions are $x = -2$ or $x = 3$

Question 11

(a)(i)



$AB = \sqrt{5^2 + 12^2}$ ✓ pythagoras

$= 13\,km$ ✓ answer.

(ii) $\angle BAL = \theta$

$\therefore \tan\theta = 12/5$

$\therefore \theta = 67°23'$ (nearest minute) ✓ finding $\angle A$

$\therefore$ Bearing $= 180° + 67°23' - 37°$

$= 210°23'\,T$ (nearest minute) ✓ correct answer

(b) $1 - \sin\theta \cos(90-\theta)$

$= 1 - \sin^2\theta$ ✓ $\cos(90-\theta) = \sin\theta$

$= \cos^2\theta$ ✓ correct answer

(c) $(\cot\theta + \operatorname{cosec}\theta)^2$

$= \cot^2\theta + 2\cot\theta \operatorname{cosec}\theta + \operatorname{cosec}^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta} + 2\,\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta} + \frac{1}{\sin^2\theta}$ ✓ correctly converting ratios

$= \frac{\cos^2\theta + 2\cos\theta + 1}{\sin^2\theta}$

$= \frac{(\cos\theta + 1)^2}{1 - \cos^2\theta}$ ✓ everything in terms of $\cos\theta$

$= \frac{(\cos\theta + 1)(\cos\theta + 1)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1 + \cos\theta}{1 - \cos\theta}$, as required ✓ final answer

Question 12

(a) $2\tan^2\theta = 5\sec\theta - 4$

$\therefore 2(\sec^2\theta - 1) = 5\sec\theta - 4$

$\therefore 2\sec^2\theta - 2 = 5\sec\theta - 4$

$2\sec^2\theta - 5\sec\theta + 2 = 0$

$(\sec\theta - 2)(2\sec\theta - 1) = 0$

$\therefore \sec\theta = 2$ or $\sec\theta = \frac{1}{2}$

as $|\sec\theta| \geq 1$

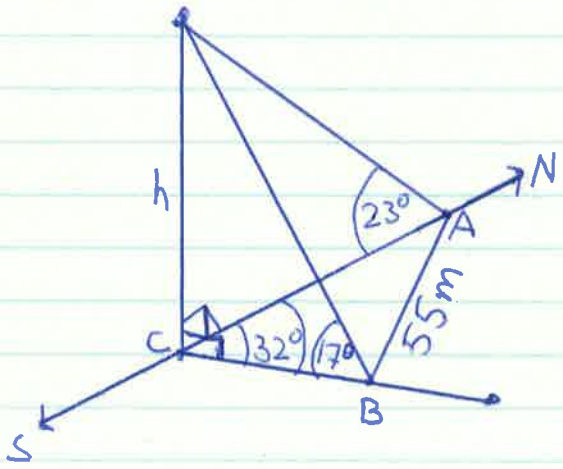
$\therefore \theta = 60°$ or $\theta = 300°$

✓ quadratic equation in $\sec\theta$

✓ values for $\sec\theta$

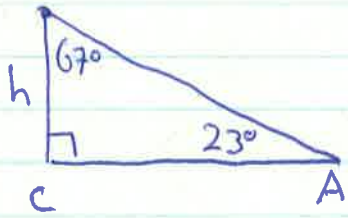
✓ values for θ

(b)(i)



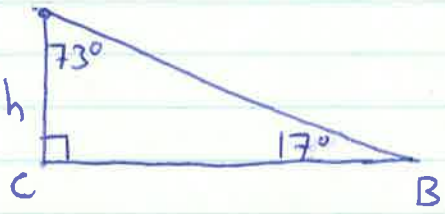
✓ all information shown

(b)(ii)



$\tan 67° = AC/h$

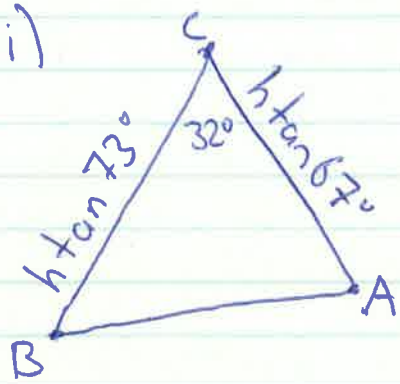
$\therefore AC = h\tan 67°$ ✓ AC



$\tan 73° = BC/h$

$\therefore BC = h\tan 73°$ ✓ BC

(b)(iii)



$55^2 = h^2\tan^2 73° + h^2\tan^2 67° - 2(h\tan 73°)(h\tan 67°)(\cos 32°)$ ✓ correct cosine rule

$55^2 = h^2(\tan^2 73° + \tan^2 67° - 2\tan 73°\tan 67°\cos 32°)$

$\therefore h^2 = \dfrac{55^2}{\tan^2 73° + \tan^2 67° - 2\tan 73°\tan 67°\cos 32°}$ ✓ making h^2 the subject

$\therefore h = 31$ m (nearest m) ✓ final answer