100 Years

NORTH SYDNEY BOYS HIGH SCHOOL

1912 ~ 2012

MATHEMATICS (EXTENSION 1)

Preliminary Course Assessment Task 3

Friday June 14, 2013

General instructions

- Working time – 50 minutes.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Diagrams shown are not necessarily to scale.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper for submission to examination supervisors.

SECTION I

- Mark your answers to this section on page 3.

SECTION II

- Answer this section in the writing booklets provided.
- Commence each new question on a new page, writing on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NAME: .... **# BOOKLETS USED**: .....

Class (please ✔)

- ◯ 11M3A – Mr Ireland
- ◯ 11M3B – Mr Berry
- ◯ 11M3C – Ms Ziaziaris
- ◯ 11M3D – Mr Lam
- ◯ 11M3E – Mr Lowe
- ◯ 11M3F – Mr Fletcher

Marker's use only.

QUESTION(S)	1-5	6	7	8	9	10	11	**Total**	%
MARKS	$\overline{5}$	$\overline{9}$	$\overline{11}$	$\overline{11}$	$\overline{9}$	$\overline{10}$	$\overline{14}$	$\overline{69}$	

Section I

5 marks
Attempt Question 1 to 5
Mark your answers to this section on page 3 **Marks**

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1. What is the domain of $y = \log_{10}(-x)$? **1**

(A) $\mathbb{R}$ (B) $x > 0$ (C) $x < 0$ (D) $x \neq 0$

2. Which of the following represents the inverse of $y = 3^x$? **1**

(A) $x = 3^y$

(B) $y = \log_3 x$

(C) $y = \dfrac{\log_{10} x}{\log_{10} 3}$

(D) All of the above

3. Which of the following terms describes the condition of three or more points lying on the same straight line? **1**

(A) collinear (B) concurrent (C) connected (D) cointerior

4. Which of the following terms describes three or more lines passing through the same point? **1**

(A) collinear (B) concurrent (C) connected (D) cointerior

5. What is the value of k if the line that joins $(0, 5)$ and $(k, k + 8)$ is parallel to the line $2x - y + 7 = 0$? **1**

(A) $k = 1$ (B) $k = 2$ (C) $k = 3$ (D) $k = 4$

Answer grid for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ
5 – Ⓐ Ⓑ Ⓒ Ⓓ

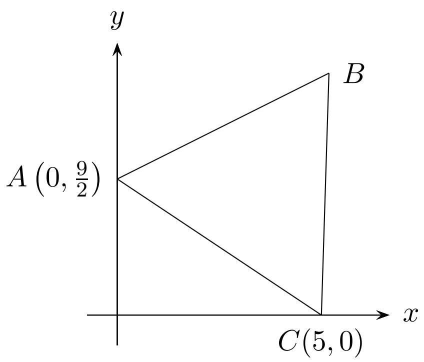
Examination continues overleaf...

Section II

64 marks
Attempt Question 6 to 9
Write your answers on the writing booklets provided.

Question 6 (9 Marks) Commence a NEW page. **Marks**

The lines AB and CB have equations $x - 2y + 9 = 0$ and $4x - y - 20 = 0$ respectively.



NOT TO SCALE

(a) Find the x coordinate of the point B. **2**

(b) Show that the equation of the line AC is $9x + 10y - 45 = 0$. **2**

(c) Calculate the exact distance of AC. **2**

(d) Calculate the shortest distance between the point B and the line AC. Hence, find the area of $\triangle ABC$. **3**

Examination continues overleaf...

Question 7 (11 Marks) Commence a NEW page. **Marks**

(a) Find the coordinates of R if it divides the interval joining $P(-4,5)$ and $Q(-1,9)$ externally in the ratio $3:5$. **2**

(b) By showing appropriate working, find the number of times that the line $6x+8y-37=0$ meets the circle $x^2+y^2=16$. **3**

(c) Find the equation of a line through the intersection of the lines **3**

$$\begin{cases} 2x+3y+5=0 \\ 3x-4y-3=0 \end{cases}$$

that also passes through the point $\left(\frac{1}{2},2\right)$.

(d) Sketch the region represented by the inequalities **3**

$$\begin{cases} y \geq 0 \\ 4x+3y \leq 12 \\ x-y+2>0 \end{cases}$$

Your sketch should be approxmately half a page in size.

Examination continues overleaf...

Question 8 (11 Marks) Commence a NEW page. **Marks**

(a) Simplify fully:

i. $\log_3 \left(27\sqrt{3}\right)$. **2**

ii. $\log_5 a + \frac{1}{2}\log_5 b - 2\log_5 c$. **2**

(b) Solve for x: $\log\left(x^2+7\right) - \log x = 3\log 2$. **3**

(c) i. Sketch the curves $y = \log_2 x$ and $y = \log_2(2x)$ on the same set of axes. **3**

ii. What is the difference in the y values between these two curves? **1**

Question 9 (9 Marks) Commence a NEW page. **Marks**

(a) Solve for x:

i. $2^x = \dfrac{1}{16}$. **1**

ii. $8^{2x-1} = 16^{x+3}$. **3**

(b) Simplify fully:

i. $\dfrac{6^{2-2n} \times 4^n}{9^{1-n}}$. **3**

ii. $\dfrac{4^{n+1} + 4^{n-1}}{4^{n-1} + 4^n}$. **2**

Examination continues overleaf...

Question 10 (10 Marks) Commence a NEW page. **Marks**

(a) Evaluate:

i. $\displaystyle\lim_{x\to 2} \frac{3x}{x^2+x}$. **1**

ii. $\displaystyle\lim_{x\to 0} \frac{3x}{x^2+x}$. **2**

iii. $\displaystyle\lim_{x\to \infty} \frac{6x^3+3x^2-2x+7}{4x^3+x}$. **2**

(b) i. Differentiate $y = x^2 - x$ from first principles. **3**

ii. Hence or otherwise, find the equation of the normal to $y = x^2 - x$ at $(2, 2)$. **2**

Question 11 (14 Marks) Commence a NEW page. **Marks**

(a) Differentiate the following with respect to x:

i. $5x^3 - 14x^2 - 7$. **1**

ii. $x^{\frac{1}{3}}\sqrt{x}$. **2**

iii. $\dfrac{5x}{x^2+1}$. **2**

iv. $\dfrac{x^3+3x^2-2x+1}{3x^2}$. **2**

v. $\left(2x^2+3\right)^5(x-1)$ (Express your answer in fully factored form) **3**

(b) Differentiate the following with respect to t:

i. $S = ut + \dfrac{1}{2}at^2$. **2**

ii. $V = \dfrac{\pi t(t-S)}{3}$. **2**

End of paper.

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Suggested Solutions

Section I

1. (C) **2.** (D) **3.** (A) **4.** (B) **5.** (C)

Section II

Question 6 (Ziaziaris)

(a) (2 marks)

$$\begin{cases} x - 2y + 9 = 0 & (1) \\ 4x - y - 20 = 0 & (2) \end{cases}$$

Substitute (2) into (1):

$$\begin{aligned} x - 2(4x - 20) + 9 &= 0 \\ x - 8x + 49 &= 0 \\ 7x &= 49 \\ x &= 7 \end{aligned}$$

(b) (2 marks)

$$m_{AC} = \frac{\frac{9}{2} - 0}{0 - 5} = -\frac{9}{10}$$

Hence line AC has equation

$$\begin{aligned} y &= -\frac{9}{10}x + \frac{9}{2} \\ 10y &= -9x + 45 \\ 9x + 10y - 45 &= 0 \end{aligned}$$

(c) (2 marks)

$$\begin{aligned} AC &= \sqrt{5^2 + \left(\frac{9}{2}\right)^2} \\ &= \sqrt{\frac{181}{4}} = \frac{\sqrt{181}}{2} \end{aligned}$$

(d) (3 marks)

$$B(7,8) \to AC$$

$$\begin{aligned} d_\perp &= \frac{|(9)(7) + (10)(8) - 45|}{\sqrt{9^2 + 10^2}} \\ &= \frac{|63 + 80 - 45|}{\sqrt{181}} \\ &= \frac{98}{\sqrt{181}} \end{aligned}$$

Hence area of $\triangle ABC$ is

$$A = \frac{1}{2} \times \frac{98}{\sqrt{181}} \times \frac{\sqrt{181}}{2} = \frac{49}{2}$$

Question 7 (Fletcher)

(a) (2 marks)

$$\begin{array}{cc} P(-4,5) & Q(-1,9) \\ 3 & -5 \end{array}$$

$$R\left(\frac{(-3)(-1) + (-4)(5)}{3-5}, \frac{(3)(9) + (5)(-5)}{3-5}\right)$$
$$= \left(-\frac{17}{2}, -1\right)$$

(b) (3 marks)
Line is $6x + 8y - 37 = 0$, find distance to $(0,0)$:

$$d_\perp = \frac{|0 + 0 - 37|}{\sqrt{6^2 + 8^2}} = 3.7$$

As the radius of the circle is 4, and the distance of the line to the centre of the circle $(0,0)$ is only 3.7, therefore the line will cut the circle twice.

(c) (3 marks)

$$2x + 3y + 5 + k(3x - 4y - 3) = 0$$

As the line passes through $\left(\frac{1}{2}, 2\right)$,

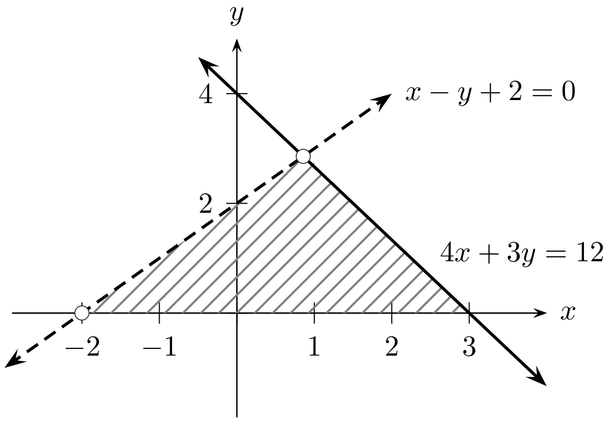
$$\begin{aligned} 1 + 6 + 5 + k\left(\frac{3}{2} - 8 - 3\right) &= 0 \\ 12 - \frac{19}{2}k &= 0 \\ k &= \frac{24}{19} \end{aligned}$$

Hence the line is

$$\underbrace{2x + 3y + 5 + \frac{24}{19}(3x - 4y - 5)}_{\times 19} = \underset{\times 19}{0}$$
$$38x + 57y + 95 + 72x - 96y - 72 = 0$$
$$110x - 39y + 23 = 0$$

(d) (3 marks)

- ✓ [1] for boundary lines
- ✓ [1] for open circles
- ✓ [1] for region



Question 8 (Berry)

(a) i. (2 marks)

$$\log_3\left(27\sqrt{3}\right) = \log_3\left(3^3 3^{\frac{1}{2}}\right)$$
$$= \log_3\left(3^{\frac{7}{2}}\right) = \frac{7}{2}$$

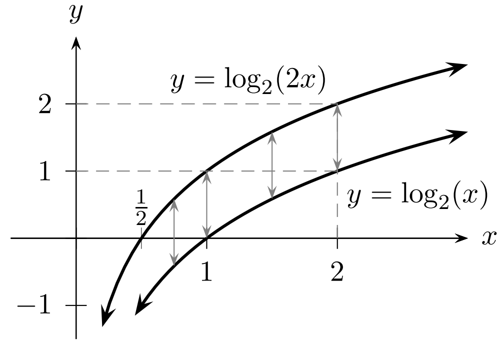
ii. (2 marks)

$$\log_5 a + \frac{1}{2}\log_5 b - 2\log_5 c$$
$$= \log_5 a + \log_5\left(b^{\frac{1}{2}}\right) + \log_5\left(c^2\right)$$
$$= \log_5\left(\frac{ab^{\frac{1}{2}}}{c^2}\right)$$

(b) (3 marks)

$$\log\left(x^2+7\right) - \log x = 3\log 2$$
$$\log\left(\frac{x^2+7}{x}\right) = \log\left(2^3\right)$$
$$\frac{x^2+7}{x} = 8$$
$$x^2 + 7 = 8x$$
$$x^2 - 8x + 7 = 0$$
$$(x-7)(x-1) = 0$$
$$\therefore x = 1, 7$$

(c) i. (3 marks)



ii. (1 mark)

$$\Delta y = \log_2(2x) - \log_2 x$$
$$= \log_2\left(\frac{2x}{x}\right)$$
$$= \log_2 2 = 1$$

Question 9 (Lowe)

(a) i. (1 mark)

$$2^x = \frac{1}{16} = 2^{-4}$$
$$x = -4$$

ii. (3 marks)

$$8^{2x-1} = 16^{x+3}$$
$$\left(2^3\right)^{2x-1} = \left(2^4\right)^{x+3}$$
$$2^{6x-3} = 2^{4x+12}$$
$$6x - 3 = 4x + 12$$
$$2x = 15$$
$$x = \frac{15}{2}$$

(b) i. (3 marks)

$$\frac{6^{2-2n} \times 4^n}{9^{1-n}} = \frac{\cancel{3^{2-2n}} \times 2^{2-2n} \times 2^{2n}}{\cancel{3^{2-2n}}}$$
$$= 2^{2-2n+2n} = 4$$

ii. (2 marks)

$$\frac{4^{n+1} + 4^{n-1}}{4^{n-1} + 4^n} = \frac{4^{n-1}\left(4^2+1\right)}{4^{n-1}\left(1+4\right)}$$
$$= \frac{17}{5}$$

Question 10 (Ireland)

(a) i. (1 mark)

$$\lim_{x\to 2}\frac{3x}{x^2+x}=\frac{6}{6}=1$$

ii. (2 marks)

$$\begin{aligned}\lim_{x\to 0}\frac{3x}{x^2+x}&=\lim_{x\to 0}\frac{3\cancel{x}}{\cancel{x}(x+1)}\\&=\frac{3}{1}=3\end{aligned}$$

iii. (2 marks)

$$\begin{aligned}&\lim_{x\to\infty}\frac{6x^3+3x^2-2x+7}{4x^3+x}\,\frac{\div x^3}{\div x^3}\\&=\lim_{x\to\infty}\frac{6+\frac{3}{x}-\frac{2}{x^2}+\frac{7}{x^3}}{4+\frac{1}{x^2}}\\&=\frac{3}{2}\end{aligned}$$

(b) i. (3 marks)

$$\begin{aligned}f(x)&=x^2-x\\f(x+h)&=(x+h)^2-(x+h)\\&=x^2+2xh+h^2-x-h\\\frac{dy}{dx}&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{x^2+2xh+h^2-x-h-\left(x^2-x\right)}{h}\\&=\lim_{h\to 0}\frac{\cancel{h}(2x+h-1)}{\cancel{h}}=2x-1\end{aligned}$$

ii. (2 marks)

$$\begin{aligned}\frac{dy}{dx}=2x-1\Big|_{x+2}&=3\\\therefore m_\perp&=-\frac{1}{3}\\\therefore y-2&=-\frac{1}{3}(x-2)\\3y-6&=-x+2\\x+3y&=8\end{aligned}$$

Question 11 (Lam)

(a) i. (1 mark)

$$\frac{d}{dx}\left(5x^3-14x^2-7\right)=15x^2-28x$$

ii. (2 marks)

$$\begin{aligned}x^{\frac{1}{3}}\sqrt{x}&=x^{\frac{1}{3}}\times x^{\frac{1}{2}}\\&=x^{\frac{1}{3}+\frac{1}{2}}=x^{\frac{5}{6}}\\\frac{d}{dx}\left(x^{\frac{5}{6}}\right)&=\frac{5}{6}x^{-\frac{1}{6}}\quad\left(=\frac{5}{6\sqrt[6]{x}}\right)\end{aligned}$$

Alternatively If product rule is used (slow & cumbersome):

$$\frac{d}{dx}\left(x^{\frac{1}{3}}\times\sqrt{x}\right)=\frac{\sqrt[3]{x}}{2\sqrt{x}}+\frac{\sqrt{x}}{3\sqrt[3]{x^2}}$$

[0] awarded if answer is not correct for inefficient method that leads to nowhere.

iii. (2 marks)

$$\begin{aligned}y&=\frac{5x}{x^2+1}\\u=5x&\quad v=\left(x^2+1\right)\\u'=5&\quad v'=2x\\\frac{dy}{dx}&=\frac{5\left(x^2+1\right)-5x(2x)}{\left(x^2+1\right)^2}\\&=\frac{5x^2+5-10x^2}{\left(x^2+1\right)^2}\\&=\frac{-5x^2+5}{\left(x^2+1\right)^2}\end{aligned}$$

iv. (2 marks)

$$\begin{aligned} y &= \frac{x^3+3x^2-2x+1}{3x^2} \\ &= \frac{1}{3}x + 1 - \frac{2}{3}x^{-1} + \frac{1}{3}x^{-2} \\ \frac{dy}{dx} &= \frac{1}{3} + \frac{2}{3}x^{-2} - \frac{2}{3}x^{-3} \end{aligned}$$

Alternatively (and far more cumbersome), use quotient rule to obtain. Marks not awarded if numerator still as like terms that can be grouped together, or easy terms to cancel in the fraction.

$$\begin{aligned} u &= x^3+3x^2-2x+1 & v &= 3x^2 \\ u' &= 3x^2+6x-2 & v' &= 6x \end{aligned}$$

$$\frac{dy}{dx} = \frac{x^3+2x-2}{3x^3}$$

v. (3 marks)

$$y = \left(2x^2+3\right)^5(x-1)$$

$$\begin{aligned} u &= \left(2x^2+3\right)^5 & v &= x-1 \\ u' &= 5\left(2x^2+3\right)^4 \times 4x & v' &= 1 \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= uv' + vu' \\ &= \left(2x^2+3\right)^5 + 5 \times 4x \times (x-1)\left(2x^2+3\right)^4 \\ &= \left(2x^2+3\right)^4\left[\left(2x^2+3\right) + 4x(5x-5)\right] \\ &= \left(2x^2+3\right)^4\left(2x^2+3+20x^2-20x\right) \\ &= \left(2x^2+3\right)^4\left(22x^2-20x+3\right) \end{aligned}$$

(b) i. (2 marks)

$$\begin{aligned} S &= ut + \frac{1}{2}at^2 \\ \frac{dS}{dt} &= u + at \end{aligned}$$

(NB. resembles $s = ut + \frac{1}{2}at^2$ and $v = u + at$ in physics formulae)

ii. (2 marks)

$$\begin{aligned} V &= \frac{\pi t(t-S)}{3} = \frac{\pi\left(t^2-St\right)}{3} \\ \frac{dV}{dt} &= \frac{\pi(2t-S)}{3} \end{aligned}$$