Ext 1

NORTH SYDNEY BOYS
HIGH SCHOOL

2014 Year 11 Task 3

Mathematics
Extension 1

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:

(Please tick or highlight)

- O Mr Berry
- O Mr Ireland
- O Mr Lam
- O Mr Lin
- O Mr Weiss
- O Mrs Ziaziaris

Student Name:______________________________

(To be used by the exam markers only.)

Question No	1-5	6	7	8	9	10	11	Total	Total %
Mark	$\overline{5}$	$\overline{5}$	$\overline{8}$	$\overline{8}$	$\overline{8}$	$\overline{8}$	$\overline{8}$	$\overline{50}$	$\overline{100}$

Section I: Objective Response (5 marks)

Mark your answers for Questions 1 to 5 by shading the multiple choice box below :

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Marks

1 Which value of m makes the points $(4,-3)$, $(0,m)$ and $(-2,5)$ collinear? 1

(A) $m=1$

(B) $m=\frac{7}{3}$

(C) $m=4$

(D) $m=-\frac{1}{2}$

2 The value of $\lim\limits_{x\to\infty} \frac{3x-2x^2}{5x^2+2x-1}$ is: 1

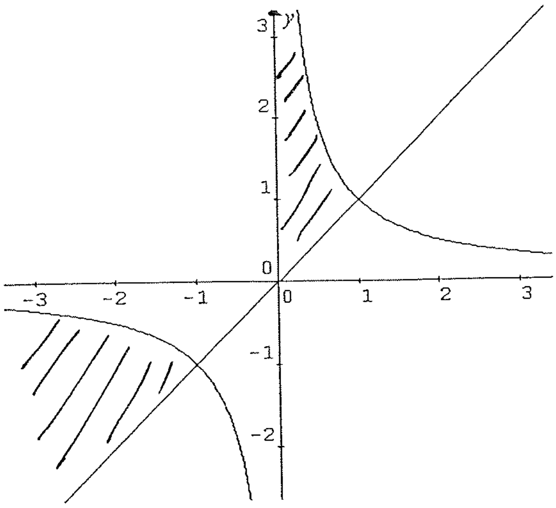
(A) $\frac{3}{2}$

(B) $\frac{3}{5}$

(C) $\frac{-2}{5}$

(D) $\frac{1}{2}$

3



The shaded area below represents which inequalities? 1

(A) $y \leq \frac{1}{x}$ and $x - y \geq 0$

(B) $y \leq \frac{1}{x}$ and $x - y \leq 0$

(C) $y \geq \frac{1}{x}$ and $x - y \geq 0$

(D) $y \geq \frac{1}{x}$ and $x - y \leq 0$

4 For what values of x is the curve $f(x) = x^3 + x^2$ concave down? 1

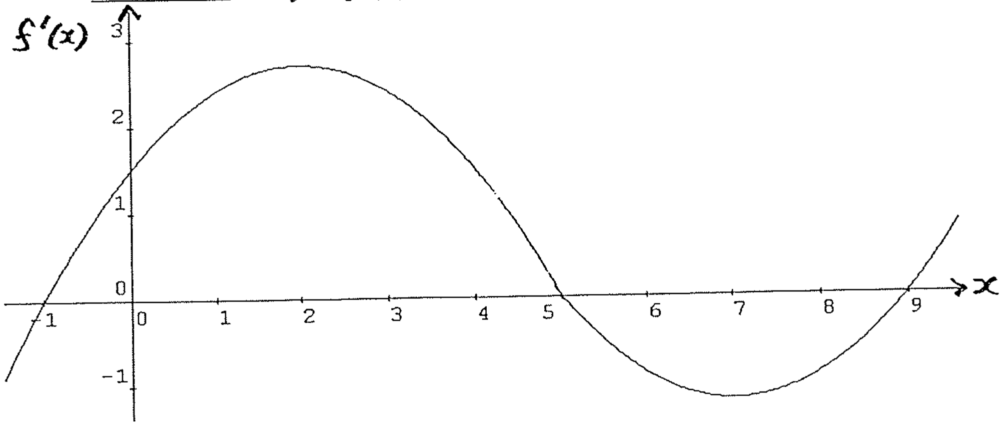
(A) $x < -\frac{1}{3}$

(B) $x > -\frac{1}{3}$

(C) $x < -3$

(D) $x > 3$

5 The graph of the <u>derivative</u> of $y = f(x)$ is shown:



A maximum turning point of $y = f(x)$ occurs at: 1

(A) $x = -1$

(B) $x = 2$

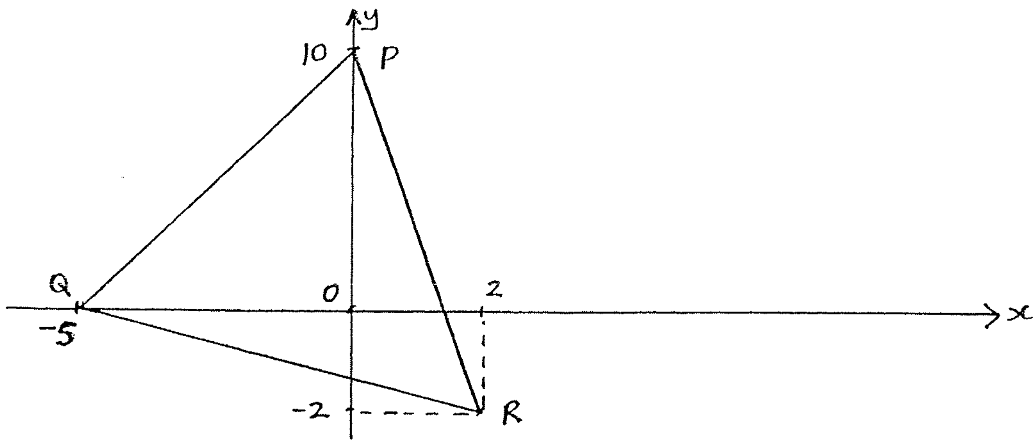
(C) $x = 5$

(D) $x = 7$

Section II: Short Answer

Question 6 (5 marks) Commence a NEW page **Mark**

In the following diagram, ΔPQR has vertices $P(0,10)$, $Q(-5, 0)$, and $R(2,-2)$.



(i) Show that PQ has length $5\sqrt{5}$ units. 1

(ii) Show that PQ has equation $2x-y+10=0$. 1

(iii) Show that the perpendicular distance from R to PQ is $\dfrac{16}{\sqrt{5}}$ units. 1

(iv) Find the coordinates of point S such that $PQRS$ is a parallelogram. 1

(v) Find the area of parallelogram $PQRS$. 1

Question 7 (8 marks) Commence a NEW page **Mark**

(a) Point P divides the interval AB externally in the ratio $1:3$. 2
If A has coordinates $(-1,1)$ and B is $(3,-1)$, what are Ps coordinates?

(b) The acute angle between the lines $3x-y+7=0$ and $kx-y+1=0$ is $45°$.
Find the possible values of k. 2

(c) Sketch the region where the inequalities $x+y<2$ and $y\le\sqrt{4-x^2}$ both hold true. 4

Question 8 (8 marks) Commence a NEW page **Mark**

Differentiate the following with respect to x.

(i) $y = \dfrac{x^2}{2\sqrt{x}}$ 2

(ii) $y = \dfrac{1+x}{1-x}$ 2

(iii) $y = (3x+4)^3$ 2

(iv) $y = \sqrt{5-2x}$ 2

Question 9 (8 marks) Commence a NEW page **Mark**

Consider the curve $y = 2x^3 + 3x^2 - 36x + 4$.

(i) Find the stationary points and determine their nature. 4

(ii) Find any points of inflexion. 2

(iii) Sketch the curve (x-intercepts not needed). 2

Question 10 (8 marks) Commence a NEW page **Mark**

(a) The line $2x + my + 1 = 0$ is tangent to the circle $x^2 + y^2 = \dfrac{1}{9}$.

Find the possible values of m. 3

(b) For what values of x is the function $y = \dfrac{1}{4}x^4 - 2x^2 + 3$ an increasing function? 2

(c) Find the equation of the tangent to $y = x^3 - 2x$ at the point $(2, 4)$. 2

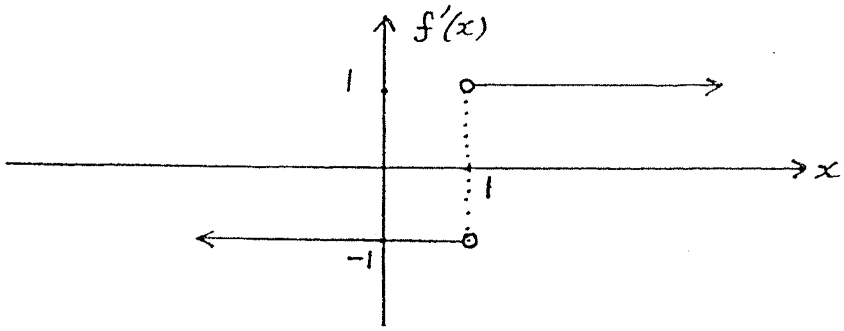
(d) Evaluate the following limit: $\lim\limits_{x \to 10} \dfrac{x^2 - 100}{x - 10}$ 1

Question 11 (8 marks) Commence a NEW page **Mark**

(a) The curve $y = ax^2 + bx + 2$ has a minimum turning point at $(-1, 1)$.

Evaluate a and b. 3

(b) Sketch a possible function which has the gradient function shown below: 2



(c) Differentiate from first principles the function $f(x) = (x+1)^2$. 3

END OF EXAM

2014 Y11 Ext. 1 — Test 3 Suggested Solutions

Q1 $\dfrac{5-m}{-2-0} = \dfrac{m+3}{0-4}$

$-2m-6 = -20+4m$

$m = \dfrac{14}{6} = \dfrac{7}{3}$ $\therefore$ (B) ✓

Q2 $\lim\limits_{x \to \infty} \dfrac{3x-2x^2}{5x^2+2x-1}$

$= \lim\limits_{x \to \infty} \dfrac{\frac{3}{x}-2}{5+\frac{1}{x}-\frac{1}{x^2}} = -\dfrac{2}{5}$ $\therefore$ (C) ✓

Q3 Test $P(\frac{1}{2},1) \rightarrow \left.\begin{array}{l} y \le \frac{1}{x} ✓ \\ x<y ✓ \end{array}\right\}$ $\therefore \rightarrow$ (B) ✓

Test $Q(-2,-1) \rightarrow \left.\begin{array}{l} y \le \frac{1}{x} ✓ \\ x<y ✓ \end{array}\right\}$ $\therefore \nearrow$

Q4 $f'(x) = 3x^2+2x$

$f''(x) = 6x+2$

$f''<0 \therefore 6x+2<0 \therefore x<-\frac{1}{3}$ $\therefore$ (A) ✓

Q5 $x=5$ $\therefore$ (C) ✓

/5

Q6 (i) $PQ^2 = 5^2 + 10^2$

$= 125$

$\therefore PQ = \sqrt{125} = 5\sqrt{5}$ ✓

(ii) $m = \frac{10}{5} = 2$

y-int. $= 10$

$\therefore y = 2x + 10$

ie $2x - y + 10 = 0.$ ✓

(or alternative methods).

(iii) $d = \frac{|2(2) + (-1)(-2) + 10|}{\sqrt{2^2 + (-1)^2}}$

$= \frac{|4 + 2 + 10|}{\sqrt{4+1}} = \frac{16}{\sqrt{5}}$ ✓

(iv) S is to R as P is to Q,

$\therefore S = (2+5, -2+10) = (7, 8)$ ✓

(v) $A_{PQRS} = PQ \times d$

$= 5\sqrt{5} \times \frac{16}{\sqrt{5}}$

$= 80$ units^2. ✓

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Q7 (a)

$A(-1, 1)$ $\quad$ $B(3, -1)$

$1 : -3$

$$P = \left[\frac{(-3)(-1) + 1(3)}{1 + (-3)}, \frac{(-3)(1) + 1(-1)}{1 + (-3)} \right]$$

$$= \left(\frac{6}{-2}, \frac{-4}{-2} \right)$$

$\therefore P = (-3, 2)$. ✓✓

(b) $m_1 = 3$, $m_2 = k$

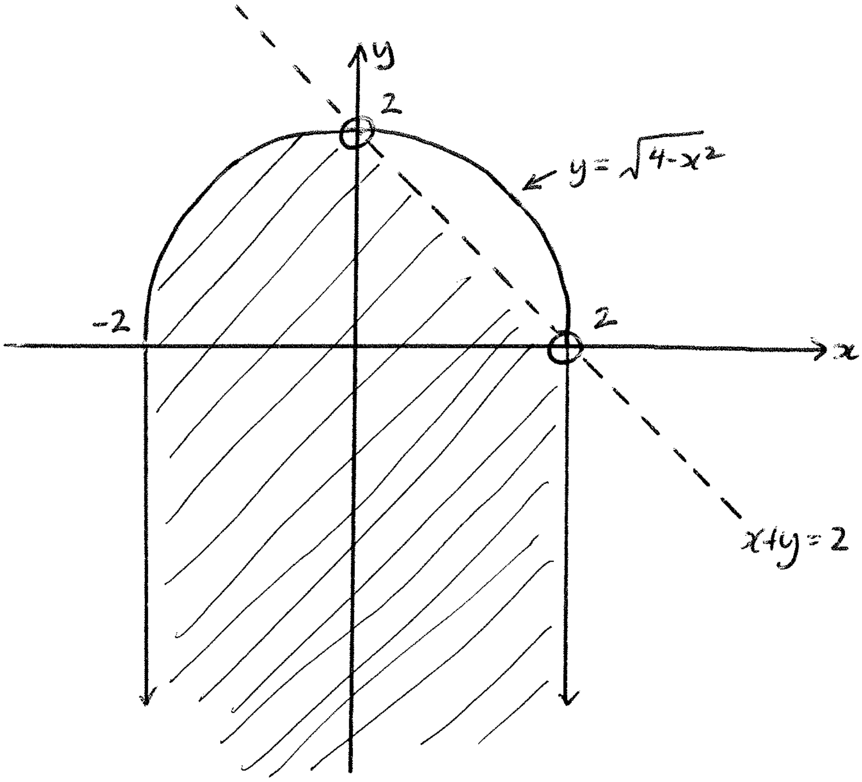
$$\tan 45^\circ = \left| \frac{3 - k}{1 + 3k} \right|$$ ✓

$$\therefore \left| \frac{3-k}{1+3k} \right| = 1$$

$$\therefore \frac{3-k}{1+3k} = 1 \quad \text{or} \quad \frac{3-k}{1+3k} = -1$$

$\therefore k = \frac{1}{2}$ or -2. ✓

(c)



✓ correct semicircle

✓ correct dotted line

✓ correct region

✓ open circles

/8

Q8

(i) $y = \dfrac{x^2}{2\sqrt{x}}$

$= \dfrac{1}{2} \cdot x^{\frac{3}{2}}$

$\therefore y' = \dfrac{1}{2} \cdot \dfrac{3}{2} \cdot x^{\frac{1}{2}}$

$\therefore y' = \dfrac{3}{4} x^{\frac{1}{2}}$ $\left(\text{or } \dfrac{3\sqrt{x}}{4}\right)$. ✓✓

(ii) $y = \dfrac{1+x}{1-x}$

$\therefore y' = \dfrac{(1-x)\cdot 1 - (1+x)\cdot -1}{(1-x)^2}$

$= \dfrac{1-x+1+x}{(1-x)^2}$ $\therefore y' = \dfrac{2}{(1-x)^2}$ ✓✓

(iii) $y = (3x+4)^3$

$\therefore y' = 3(3x+4)^2 \cdot 3$

$\therefore y' = 9(3x+4)^2$. ✓✓

(iv) $y = \sqrt{5-2x}$

$= (5-2x)^{\frac{1}{2}}$

$\therefore y' = \frac{1}{2} \cdot (5-2x)^{-\frac{1}{2}} \cdot -2$

$\therefore y' = -(5-2x)^{-\frac{1}{2}}$

$= \dfrac{-1}{\sqrt{5-2x}}$. ✓✓

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Q9 $y = 2x^3 + 3x^2 - 36x + 4$

(i) $y' = 6x^2 + 6x - 36$ $\qquad y'' = 12x + 6$

$= 6(x^2 + x - 6)$ $\qquad = 6(2x+1)$

$= 6(x+3)(x-2)$

✓ correct y'

Stat. points when $y' = 0$

$\therefore x = -3$ or $x = 2$

$y = 85$ $\qquad y = -40$

✓ correct x coords.

✓ correct y coords.

At $(-3, 85)$, $y'' = 6(2\times -3 + 1) < 0$

$\therefore (-3, 85)$ is a maximum turning point.

At $(2, -40)$, $y'' = 6(4+1) > 0$

$\therefore (2, -40)$ is a minimum turning point.

✓ Tests adequately

(ii)

For inflexions, $y'' = 0$

$\therefore 6(2x+1) = 0$

$x = -\frac{1}{2}$

$y = 22\frac{1}{2}$.

✓ correct inflexion

Test:

x	-1	$-\frac{1}{2}$	0
y''	$6(-1)$	0	$6(1)$

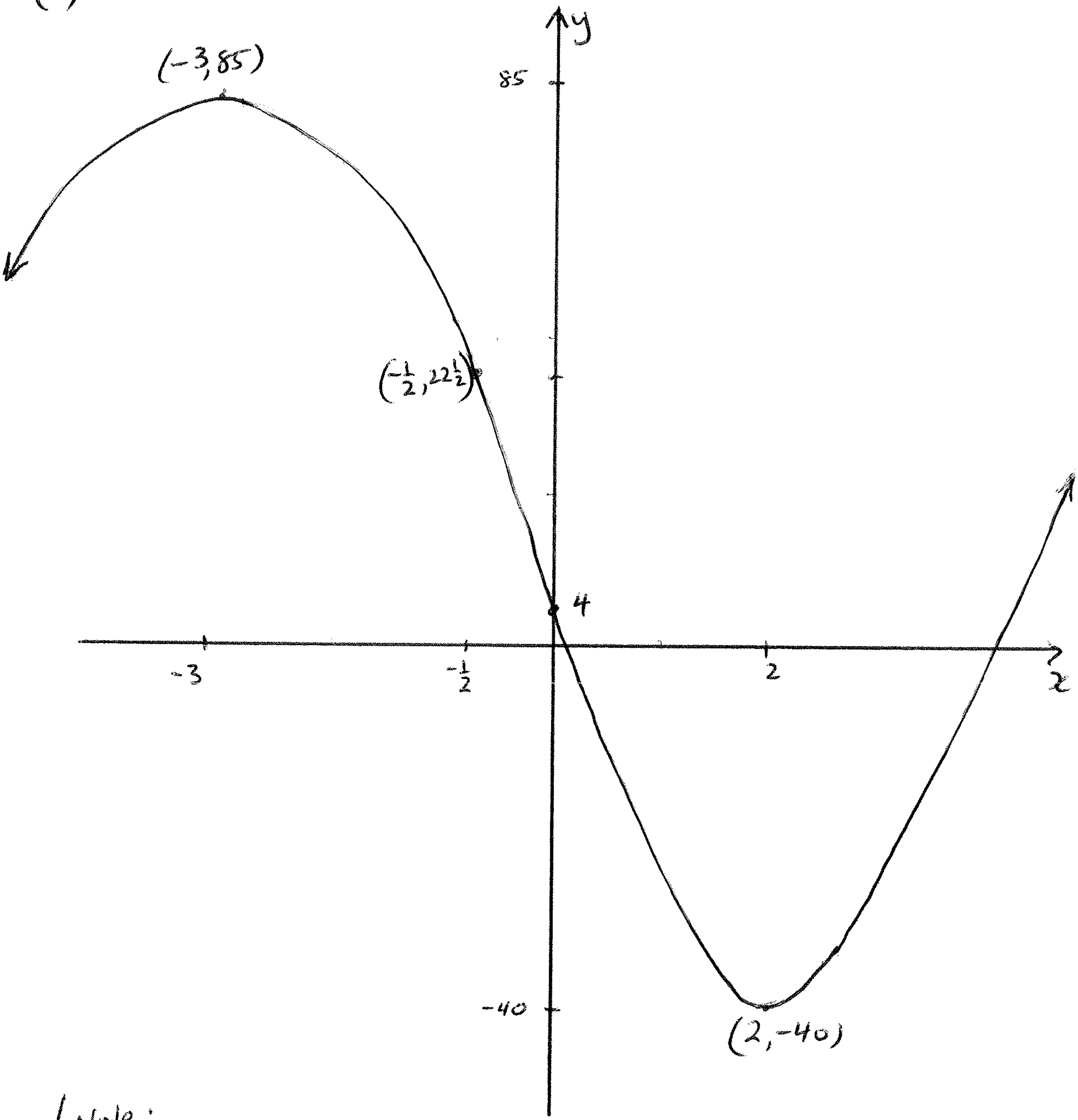
✓ Tests adequately

Change of concavity $\therefore \left(-\frac{1}{2}, 22\frac{1}{2}\right)$ is an inflexion.

P.T.O. →

...[Q9 - ctd.]

(iii)



(Note:
y-int. = 4)

✓✓

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Q10

(a) $\begin{cases} \text{Centre of circle} = (0,0) \\ \text{radius} = \frac{1}{3} \end{cases}$

distance of line from $(0,0)$ is:

$$d = \frac{|2(0) + m(0) + 1|}{\sqrt{2^2 + m^2}} = \frac{1}{\sqrt{4+m^2}}$$

For tangency, $d = \frac{1}{3}$

$\therefore \quad \sqrt{4+m^2} = 3$

$\therefore \quad 4 + m^2 = 9$

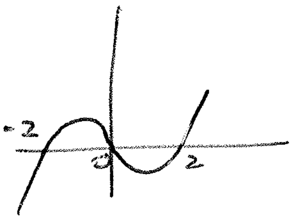
$\therefore \quad m = \pm\sqrt{5}$.

(b) $y = \frac{1}{4}x^4 - 2x^2 + 3$ is increasing if $y' > 0$

$\therefore \quad x^3 - 4x > 0$

$x(x^2 - 4) > 0$

$x(x-2)(x+2) > 0$



$\therefore \quad -2 < x < 0$ or $x > 2$.

(c) $y = x^3 - 2x \Rightarrow y' = 3x^2 - 2$

$\therefore \quad m_T = 3(2)^2 - 2$ at $(2,4)$ $\quad \therefore m_T = 10$

$\therefore$ tangent is $y - 4 = 10(x-2)$.

[ie. $10x - y - 16 = 0$]

(d) $\lim\limits_{x\to 10} \frac{x^2 - 100}{x - 10} = \lim\limits_{x \to 10} \frac{\cancel{(x-10)}(x+10)}{\cancel{x-10}} = 20$.

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Q11 (a) $y = ax^2 + bx + 2$, min. at $(-1,1)$.

$y' = 2ax + b = 0$ for turning point

$\therefore \quad -2a + b = 0 \quad \cdots (1)$

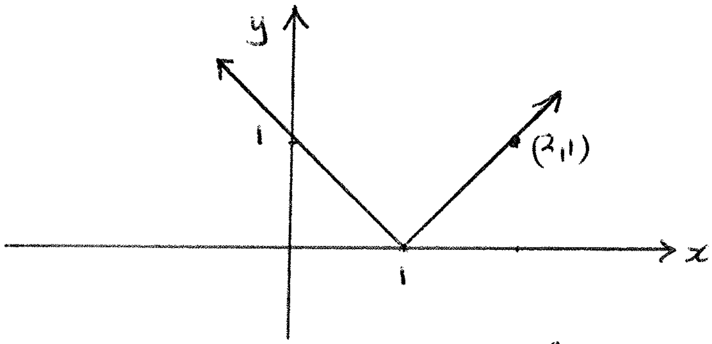
Goes through $(-1,1)$, so $1 = a(-1)^2 + b(-1) + 2$

$\therefore \quad a - b = -1 \quad \cdots (2)$

$(1) + (2) \rightarrow \quad -a = -1, \quad \therefore \quad a = 1, \quad b = 2$. ✓✓✓

[ALT. The curve is clearly a parabola, with vertex $(-1,1)$.
So it can be written $y = A(x+1)^2 + 1$.
Since it goes through $(0,2)$,
$\therefore \quad 2 = A + 1 \quad \therefore A = 1$
Thus $y = (x+1)^2 + 1 = x^2 + 2x + 2 \quad \therefore a = 1, \ b = 2$].

(b)



✓✓

(c) $f'(x) = \lim_{h \to 0} \dfrac{(x+h+1)^2 - (x+1)^2}{h}$ ✓

$= \lim_{h \to 0} \dfrac{(x+h+1+x+1)(\cancel{x}+h+\cancel{1}-\cancel{x}-\cancel{1})}{h}$ ✓

$= \lim_{h \to 0} \dfrac{(2x+h+2)(\cancel{h})}{\cancel{h}}$

$= 2x + 2$. ✓

$\longrightarrow$ P.T.O. for alternative

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Q11 (c) ALTERNATIVELY ...

[1] $f'(x) = \lim_{u \to x} \frac{f(u) - f(x)}{u - x}$

$= \lim_{u \to x} \frac{(u+1)^2 - (x+1)^2}{u - x}$

$= \lim_{u \to x} \frac{(u+1+x+1)(u+1-x-1)}{u-x}$

$= \lim_{u \to x} \frac{(u+x+2)\cancel{(u-x)}}{\cancel{u-x}}$

$= 2x+2.$

ALT. [2] $f'(x) = \lim_{h \to 0} \frac{(x+h+1)^2 - (x+1)^2}{h}$

$= \lim_{h \to 0} \frac{[(x+1)+h]^2 - (x+1)^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{(x+1)^2} + 2(x+1)h + h^2 - \cancel{(x+1)^2}}{h}$

$= \lim_{h \to 0} \frac{2(x+1)h + h^2}{h}$

$= \lim_{h \to 0} 2(x+1) + h$

$= 2(x+1).$